Answer Key

Coulson- Spring 2008

First Exam

Economics 490– Introduction to Econometrics

Answer all of the questions. 5 points per question unless otherwise indicated. No calculators or electronic devices of any kind. For all questions that have a numerical answer, I want a numerical answer. NO SYMBOLS. But you may (in fact, you SHOULD) leave your answers in unreduced form whenever you like. That is, the answer can be something like

$$\frac{2*(35-13)}{6}$$

and you don't have to calculate that this actually becomes 7.333. When I ask for explanations, feel free to be brief.

Part I (80 points; 5 points each): I have 100 observations on two variables, X and Y. In this sample:

$$\sum_{i=1}^{100} X_i = 1200$$

$$\sum_{i=1}^{100} Y_i = 1600$$

$$\sum_{i=1}^{100} (Y_i - \overline{Y})^2 = 10000$$

$$\sum_{i=1}^{100} (X_i - \overline{X})^2 = 8000$$

$$\sum_{i=1}^{100} (X_i - \overline{X})(Y_i - \overline{Y}) = 4000$$

$$\sum_{i=1}^{100} (\hat{Y}_i - \overline{Y})^2 = 5000$$

In the line Y=a+bX+e:

1. Find the least squares estimate $\hat{b}$ $\frac{4000}{8000}$

2. Find the least squares estimate $\hat{a}$ $\frac{1600}{100} - \left(\frac{4000}{8000}\right)\left(\frac{1200}{100}\right)$

3. What is $\sum_{i=1}^{100} \hat{e}_i$? 0

4. What is $\sum_{i=1}^{100} \hat{e}_i^{\,2}$? $10000-5000$

5. What is the prediction of Y given X=15?

$$\left[\frac{1600}{100} - \left(\frac{4000}{8000}\right)\left(\frac{1200}{100}\right)\right] + \frac{4000}{8000} \cdot 15$$

6. If the actual value of Y is 20 when X=15 what is the error term for that observation?

$$20 - \left\{\left[\frac{1600}{100} - \left(\frac{4000}{8000}\right)\left(\frac{1200}{100}\right)\right] + \frac{4000}{8000} \cdot 15\right\}$$

7. What is the predicted increase in Y if X rises from 15 to 16?

$$\frac{4000}{8000}$$

8. What is the estimate of the variance of the error term, σ^2?

$$\frac{10000-5000}{100-2}$$

9. What is the variance of $\hat{b}$?

$$\frac{\frac{10000-5000}{100-2}}{8000}$$

10. What is the variance of $\hat{a}$?

$$\left[\frac{\frac{10000-5000}{100-2}}{8000}\right] \cdot \left(\frac{1200}{100}\right)^2 + \frac{\frac{10000-5000}{100-2}}{100}$$

11 What is the test-statistic for the hypothesis a=5?

$$\frac{\left[\frac{1600}{100} - \left(\frac{4000}{8000}\right)\left(\frac{1200}{100}\right)\right] - 5}{\sqrt{\frac{\frac{10000-5600}{100-2}}{8000} \cdot \left(\frac{1200}{100}\right)^2 + \left(\frac{10000-5000}{100-2}\right)/100}}$$

12 What is the test-statistic for the hypothesis b=0?

$$\frac{4000}{8000}$$
$$\sqrt{\frac{10000-5000}{98}}/8000$$

13. If everything is normally distributed, what is the distribution of those two test-statistics, including degrees of freedom? (They are the same.)

t_{100-2}

14. If I feed the number given in (11) into the t-calculator and it gives me a prob-value of .004, what do I conclude?

Reject H_0 since prob value $< .05$

15 What is the R^2 of this regression?

$$\frac{5000}{10000}$$

16. How and why would you react if I had stated above that $\sum_{i=1}^{100} (\hat{Y}_i - \bar{Y})^2 = 11000$?

Badly. Since then SSR < 0, and that is impossible

Part II (20 points):

1. (10) In the proof of the unbiasedness of $\hat{b}$, there is a step that says:

$$\left(\frac{\sum_i (X_i - \overline{X})a}{\sum_i (X_i - \overline{X})^2} \right) = 0$$

What allows you to say that the expression on the left is indeed equal to zero?

Numerator is $= a \sum (X_i - \overline{X})$ by distributive law and the sum of deviations from mean always $= 0$

2. (10) I claim in class that the least squares estimator for b is "consistent". One half of consistency is covered by the fact that this estimator is unbiased. What is the other half of this definition and how does the formula for the ~~estimator~~ sample variance show that this is confirmed?

Other half of definition = variance $\to 0$

That variance $= \frac{\sigma^2}{\sum (X_i - \overline{X})^2}$

and as n ↑, keep adding positive numbers to the denominator