J. Tybout

Economics 507a: International Trade
Lecture 2

II. The simple mechanics of the 2 x 2 Hecksher-Ohlin model

Last time:
Constant returns, neoclassical technology with no factor intensity reversals

Zero profits: $P_j = c_j = wa_{jL} + ra_{jK}$

Labor market clearing: $L = X_1 a_{1L} + X_2 a_{2L}$

Capital market clearing: $K = X_1 a_{1K} + X_2 a_{2K}$

Profit-max (cost-min): $a_{jL}(w,r)$ and $a_{jK}(w,r)$ solve cost min. problem
a_{jK} / a_{jL} is a monotonic positive function of *w*/*r* for all *j*
$\hat{P}_j = \hat{c}_j = \theta_{Lj}\hat{w} + \theta_{Kj}\hat{r}$,

Details of cost minimization
Drop the *j* subscripts to reduce clutter. Cost minimization per unit output can be characterized using the following LaGrangian:

$$L = (wa_L + ra_K) + \lambda[1 - F(a_L, a_K)] \text{, so}$$

$w = \lambda F_L(a_L, a_K), \quad r = \lambda F_K(a_L, a_K)$, $\frac{w}{r} = \frac{F_L(a_L, a_K)}{F_K(a_L, a_K)}$, and $F_L da_L + F_K da_K = 0$ follows from the constraint. Substituting $w = \lambda F_L(a_L, a_K), \quad r = \lambda F_K(a_L, a_K)$ into $F_L da_L + F_K da_K = 0$ yields $\frac{w}{\lambda} da_L + \frac{r}{\lambda} da_K = 0$. Also, the total differential of unit cost (or price) is: $dP = dC = dw \cdot a_L + dr \cdot a_K + w \cdot da_L + r \cdot da_K$ which simplifies to:

$$dP = dc = dw \cdot a_L + dr \cdot a_K$$

because $\frac{w}{\lambda} da_L + \frac{r}{\lambda} da_K = 0$. (This is an envelope theorem result.) Dividing through by cost and re-arranging yields: $\frac{dP}{P} = \frac{dc}{c} = \theta_L \frac{dw}{w} + \theta_K \frac{dr}{r}$, or in "hat" notation,

$$\hat{P} = \hat{c} = \theta_L \hat{w} + \theta_K \hat{r} ,$$

where θ_j is factor *j*'s share in total cost. Note also that the cost differential implies

$$\frac{\partial c}{\partial w} = a_L \text{ and } \frac{\partial c}{\partial r} = a_K$$

These relationships provided the basis for the Factor price equalization theorem and the Stolper-Samuelson theorem.

The Factor Price Equalization Theorem
When a set of countries faces common global prices, and each country produces both goods, identical factor prices will prevail in all countries.

The Stolper-Samuelson Theorem
An increase in the relative price of one commodity, say commodity 1, generates a more than proportional increase in the price of the factor that that commodity uses intensively. Thus the owners of the factor used intensively enjoy an increase in their real purchasing power. The opposite result holds for the owners of the other factor.

Implication: if opening to trade drives down the price of the good that uses intensively the factor you supply, you stand to lose. (Lobbying groups should line up according to factor rather than industry.)

New material:

D. The Rybcznski Theorem

Rybcznski theorem
(A dual to the Stolper-Samuelson theorem.) ***Assume that the country of interest is producing both goods. Then, expanding one factor stock while holding output prices constant leads to a more than proportionate expansion in the good that uses that factor intensively, and a contraction in the other good.***

Log-differentiating the factor market clearing conditions, we have:

$$\hat{L} = \hat{a}_{L1}\lambda_{L1} + \hat{a}_{L2}\lambda_{L2} + \lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2$$

$$\hat{K} = \hat{a}_{K1}\lambda_{K1} + \hat{a}_{K2}\lambda_{K2} + \lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2$$

Further, if we assume that the conditions for factor price equalization hold, factor intensities are pinned down by global prices:

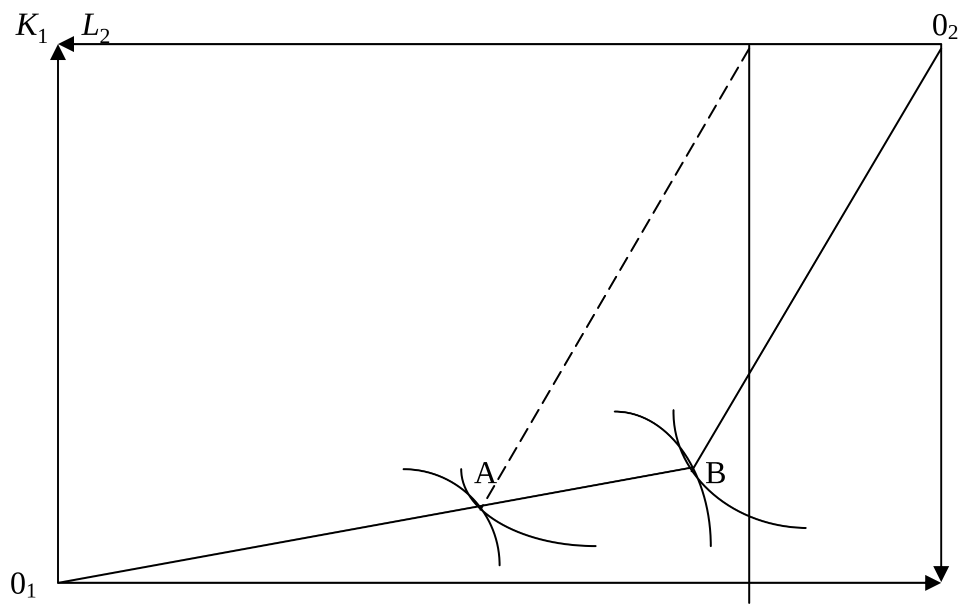
$$\hat{L} = \lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2$$

$$\hat{K} = \lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2,$$

Where λ_{ij} the fraction of factor *i* allocated to good *j*. In vector notation, $\begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix} = \begin{bmatrix} \lambda_{L1} & \lambda_{L2} \\ \lambda_{K1} & \lambda_{K2} \end{bmatrix} \cdot \begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix}$, or more compactly, $\hat{V} = \Lambda \cdot \hat{X}$. Since Λ is invertible when both goods are produced, $\Lambda^{-1} \cdot \hat{V} = \hat{X}$ determines the output adjustments associated with changes in factor stocks. As in the Stolper-Samuelson theorem, the properties of $\Lambda^{-1} = \begin{bmatrix} \lambda_{K2} & -\lambda_{K1} \\ -\lambda_{L2} & \lambda_{L1} \end{bmatrix} \cdot \left(\lambda_{L1}\lambda_{K2} - \lambda_{K1}\lambda_{L2}\right)^{-1}$ establish the Rybcznski claim.

The logic is simple. Suppose the labor force expands, while capital is held fixed. With factor proportions fixed by global prices and factor price equalization, both goods cannot expand. If 2 were to expand and 1 contract, the result would increase demand for capital and reduce demand for labor, exacerbating the disequilibrium. Instead, 1 expands and 2 contracts to free up the necessary capital. (Think of the expansion in *L* as putting incipient downward pressure on the relative price of good 1, attracting a surge of demand for good 1 in global markets.)

To visualize the output adjustments associated with a change in factor stocks, it is convenient to draw an Edgeworth box. Recall that each point in this box represents a division of factors between the two goods, and that rays from the two origins represent the associated factor intensities in the two sectors.



Since we are holding output prices constant, and the country is presumed to be producing both goods, factor prices remain constant too. Thus factor proportions don't change as factor stocks move, and the equilibrium factor allocation shifts from A to B. Because we have assumed constant returns to scale, the percentage increase in the length of the ray from the 0_1 origin represents the percentage increase in production of good 1, and the percentage reduction in the length of the ray from the 0_2 origin represents the percentage shrinkage in production of good 2.

E. Trading equilibria and the Heckscher-Ohlin theorem

Thus far we have considered a single country in isolation, taking world prices as given. Now let's characterize the determination of world prices and the associated trade flows.

<u>The production possibility frontier</u>
First, note that perfect competition implies each country will fully utilize its resources, implying that it will get the maximum amount of good 2 at each level of good 1. Define the associated production possibility frontier (PPF) to be

$$X_2 = h(X_1, K, L)$$

Note:

- So long as production technologies are weakly concave in factor inputs, the PPF will be weakly concave as well.
- The optimization problem that yields this PPF function also implies that the slope of the PPF is $\frac{dX_2}{dX_1} = \frac{\partial X_2}{\partial L_2} / \frac{\partial X_1}{\partial L_1} = \frac{\partial X_2}{\partial K_2} / \frac{\partial X_1}{\partial K_1}$.

Profit-maximizing entrepreneurs will allocate resources in such a way as to maximize the value of its output. Or we can think of production choices as solving:

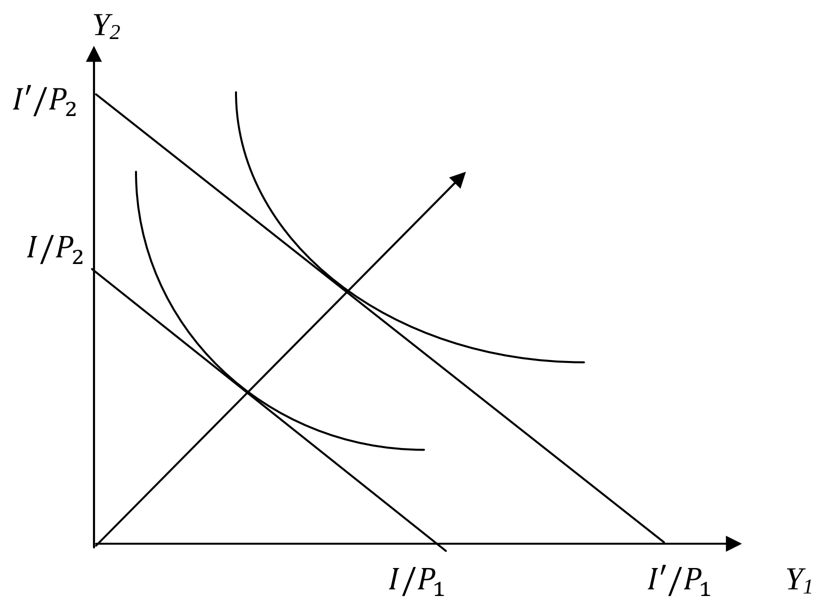
$$max_{X_1}(P_1 X_1 + P_2 X_2) \text{ subject to } X_2 = h(X_1, K, L)$$

This implies that $p = \frac{P_1}{P_2} = \frac{-\partial h}{\partial X_1} = \frac{-\partial X_2}{\partial X_1}$, that is, the relative price of good 1 should match the (negative of the) slope of the PPF.

<u>Consumer behavior</u>
A consumer with income I will, for her part, solve the following utility maximization problem:

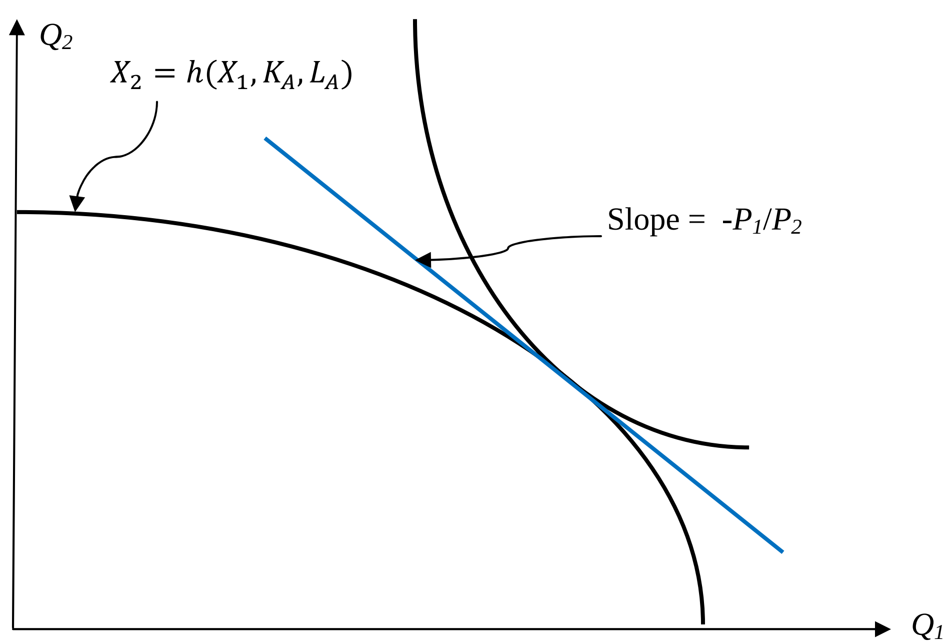
$$max_{X_1, X_2} U(Y_1, Y_2) \text{ subject to } I = P_1 Y_1 + P_2 Y_2$$

From which it follows that they equate relative prices to the marginal rate of substitution (i.e., the slope of their indifference curve), $\frac{P_1}{P_2} = \partial U / \partial Y_1 \, / \partial U / \partial Y_2$. They also exhaust their budget.

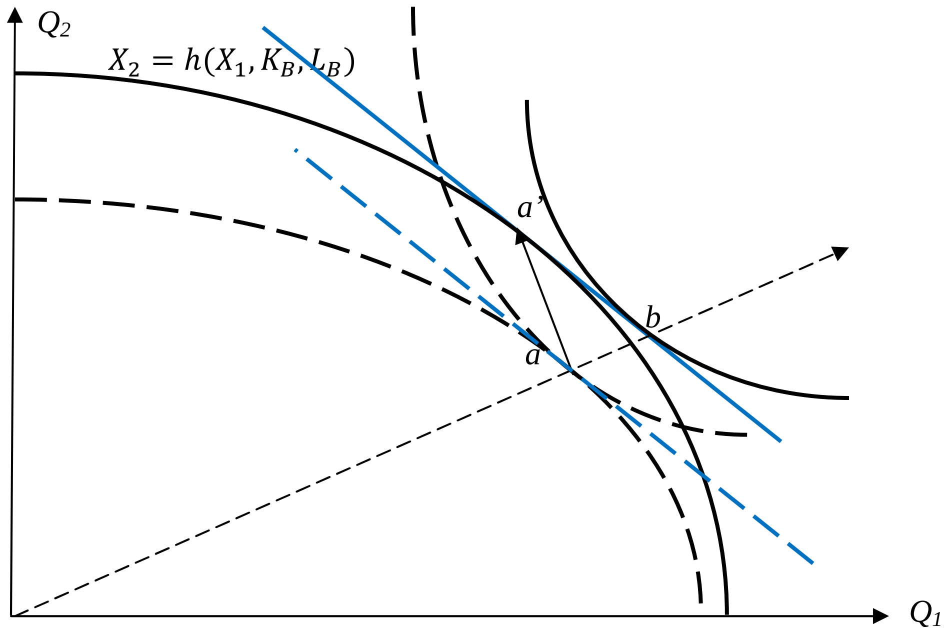


Given our assumption of homothetic tastes, the MRS depends only on the proportions in which goods are consumed and not on budgets. Further given that all consumers have identical tastes, it doesn't matter how income is distributed among them, they will all choose to consume goods 1 and 2 in the same proportions, given that the face the same prices. Accordingly, we can hereafter represent consumer choices with a utility function (and associated set of indifference curves) for a representative consumer, and we can act as if this single consumer has a budget that matches national income.

Autarky prices (that is, prices in the absence of trade) must therefore match the slope of the PPF at a point of tangency with an indifference curve for this representative consumer. For country A, this equilibrium might look like:



Now consider another country, B, with more capital. How will the autarky equilibrium differ? We know from Rybcznski that if the same output prices prevailed in that country, it would produce more good 2 and less good 1, so the PPF would something like:

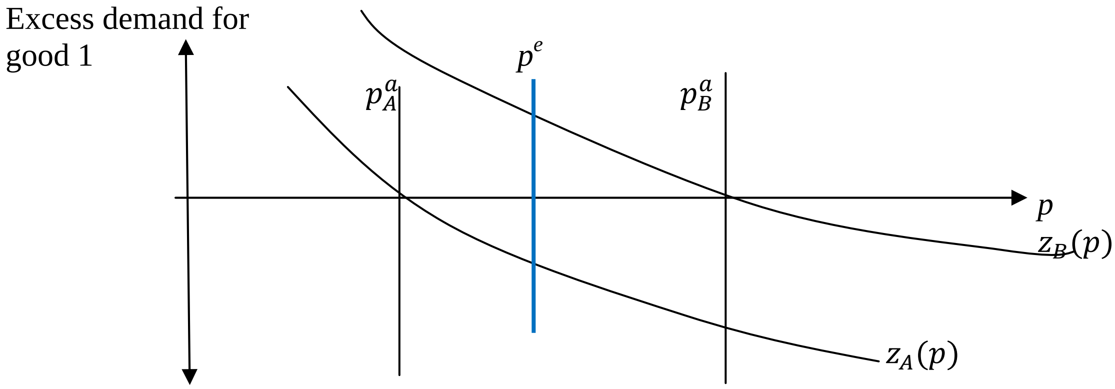


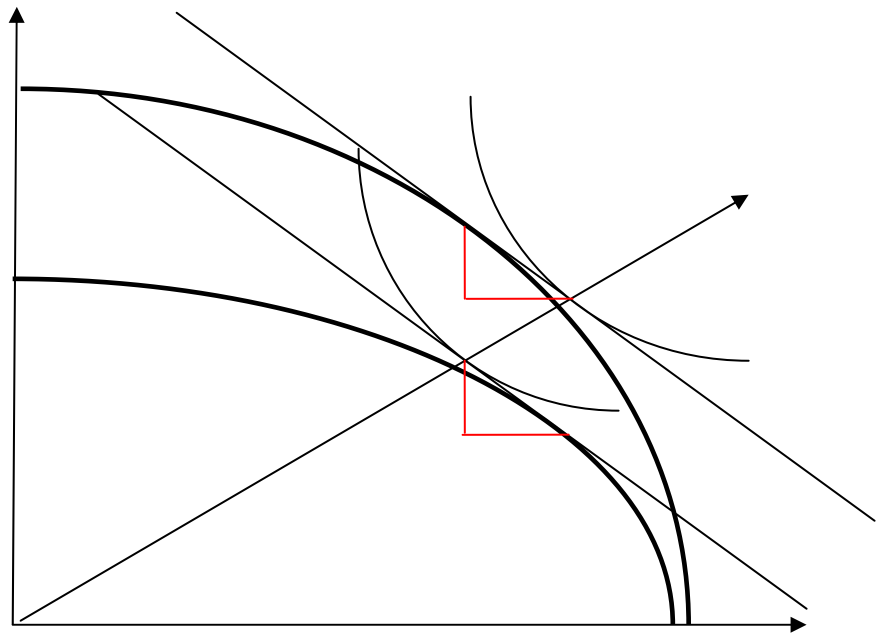
At country A's prices, production in B takes place at point *a'* and consumption at *b*. Thus there would be excess demand for good 1, and excess supply good 2. It follows that the autarky relative price of good 1 would be higher in B than in A.

Prices with free trade

In a trading equilibrium, we don't expect domestic supply to match domestic demand for each product. Rather, we expect global supplies to match global demands, or equivalently, excess supply of good 1 in country A to match excess demand for good 1 in country B. At what price ratio does this occur?

Let $z_A(p)$ denote excess demand for good 1 in country A at price p, and let $z_B(p)$ denote the same for country B. Then $z_A(p) + z_B(p) = 0$ must hold in the trading equilibrium. We've seen that there is excess demand for good 1 in country B at A's autarky prices. Similarly, it is easy to see that there is excess supply of good 1 in A at B's autarky prices. Finally, by the continuity and concavity of the PPF and the continuity and convexity of indifference curves, excess demand functions are continuous. Accordingly there must be an equilibrium world price ratio, p^e, between the two autarky price ratios at which global markets clear.





If the global market for good 1 clears, then the global market for good 2 must also clear. (Why?)

We are now ready to state the Hecksher-Ohlin theorem.

Hecksher-Ohlin Theorem (law of comparative advantage)

Version 1: ***A country will export that good which is intensive in its abundant factor, where abundance is determined relative to global endowment proportions.***

Version 2: ***A country will export that good which is relatively cheap in autarky, compared to world prices.***

Version 2 is nearly tautological since one would expect that firms will always sell their goods in the markets where they bring the highest prices. The "relatively" part of this is non-trivial. Version 2 is nearly impossible to test because we don't have many experiments where a country goes from autarky to free trade. In any case, in the context of the model we have laid out, they amount to the same prediction.

Generalizing to many goods and factors.

Suppose i =1,… N goods and j = 1, …M factors. Let the production function for the i^{th} good be $X_i = f_i(\underline{v}_i)$ where $\underline{v}_i = (v_{i1}, v_{i2}, \ldots, v_{iM})$ is the vector of factor inputs. (Underscores denote vectors.) Assume $f(\ )$ is positive, increasing, concave and homogeneous of degree 1 for all $\underline{v}_i \geq 0$. Let the associated unit cost function be $g_i(\underline{w}) = \min_{v_i \geq 0}\{\underline{w}'\underline{v}_i \mid f_i(\underline{v}_i) \geq 1\}$, where $\underline{w}$ is the vector of factor prices. $g_i(\underline{w})$ will have the same properties as the production function. Perfect competition implies

$$p_i \leq g_i(\underline{w}) \quad i = 1, \cdots, N \tag{1}$$

with equality if good i is produced.

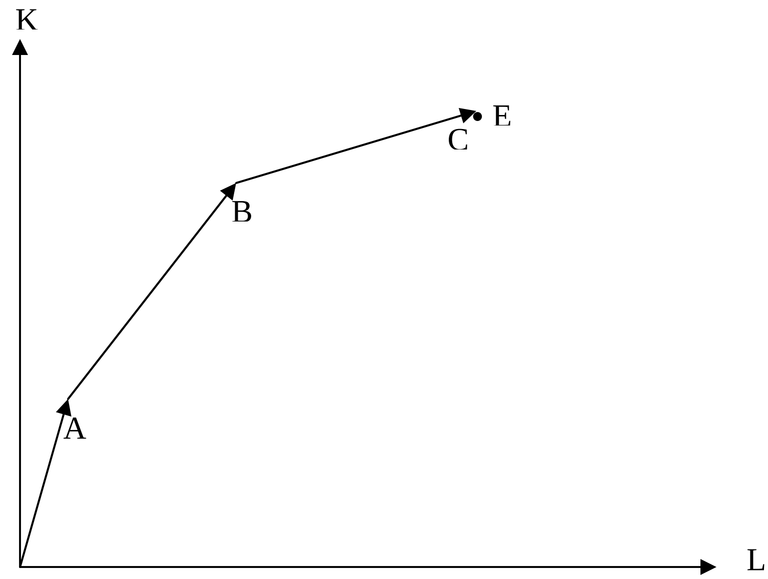
Perfect competition also implies full employment of resources. More precisely, by the envelope theorem, the cost-minimizing vector of input requirements for a unit of output is $\frac{\partial g_i}{\partial \underline{w}} = \underline{a}_i(\underline{w})$, where $\underline{a}_i(\underline{w}) = (a_{i1}(\underline{w}), a_{i2}(\underline{w}), \ldots, a_{iM}(\underline{w}))'$. Thus the full employment condition is:

$$\sum_i a_{ij}(\underline{w}) X_i = V_j \quad j = 1, \ldots, M\ , \tag{2}$$

where V_j is the economy-wide stock of factor j and y_j is the amount of good j produced. Using matrix notation these M equations can be written compactly as:

$$A\underline{X} = \underline{V} \tag{2'}$$

In the case of two factors the full employment condition can depicted graphically as the requirement that the sum of the factor employment rays yields the endowment point. Note that with more goods than factors, there is an infinite number of possible ways the full employment condition might be satisfied at a *given* set of factor prices:



Finally, given that consumers everywhere have identical homothetic tastes, so the share of their income they spend on good *i* depends only on the price vector, $\alpha_i(p)$, *i* = 1, … *n*.

We are now prepared to say something about the way that production vectors, Y_i, vary across countries with factor endowments, V_i. This will set the stage for a statement about the relationship between trade flows and factor endowments.

The key observation is that equation 2' describes the link between output vectors and endowment vectors for all countries, so long as all countries share the same factor prices. We thus want to know when this will occur.

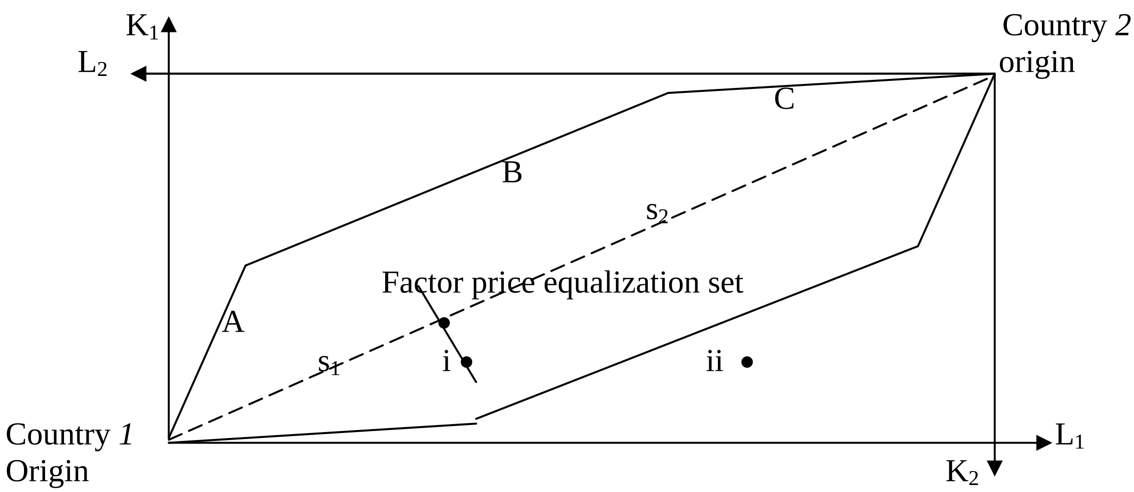
Suppose the world is comprised of countries indexed by $c = 1, \ldots, C$. Also, assume that free world trade ensures that the same product prices prevail everywhere. Then if the set of *n* equations (1) holds with equality for every country, and if these equations imply a one-to-one mapping from factor price vectors to output price vectors, the same set of factor prices will prevail everywhere (factor price equalization). Further, these factor prices will induce the same factor intensities everywhere. That is, all countries will share the same technology matrix, *A*.

When does this occur? Think of an *integrated* equilibrium (IE) in which we pool global resources into one giant economy. (Equivalently, think of all factors as perfectly mobile across borders.) There is some vector of market clearing product prices in this equilibrium, an associated vector of factor prices, and a matrix of factor intensities, *A*.

Now, starting from this IE, imagine that we divide up the world's factor endowment among countries and we prohibit international factor movements. If the division is done in such a way that all countries' factor endowment vectors are identical up to a scalar multiple, then the IE need not be disturbed. Because production technologies are homogeneous of degree 1, a country getting some fraction λ_c of the global endowment, $\underline{V}_c = \lambda_c \underline{V}_W$, could simply produce the same fraction of the global output vector. There would be no effect on global supplies of goods, so no adjustment in prices would be necessary. Thus, so long as the equilibrium vector of output prices implied a unique vector of factor prices, the integrated equilibrium (IE) would be replicated. (Would there be any need for trade?)

More interestingly, when dividing up the world's endowment, it is possible to preserve the IE even if we deviate from proportionate endowments. But if endowments get *too* far from proportionality, it will be necessary for countries to deviate from the IE factor intensities in order to use all of their factor stocks, causing factor prices to deviate too.

Consider the example depicted below with 3 goods (A,B,C), 2 countries (1,2), and 2 factors (K,L). Here the line segments A, B and C represent factor usage in each good when the IE obtains. Any point enclosed by the convex hull represents an endowment division that preserves the IE, because this set of points allows the two countries to replicate the production vectors A,B and C between them.



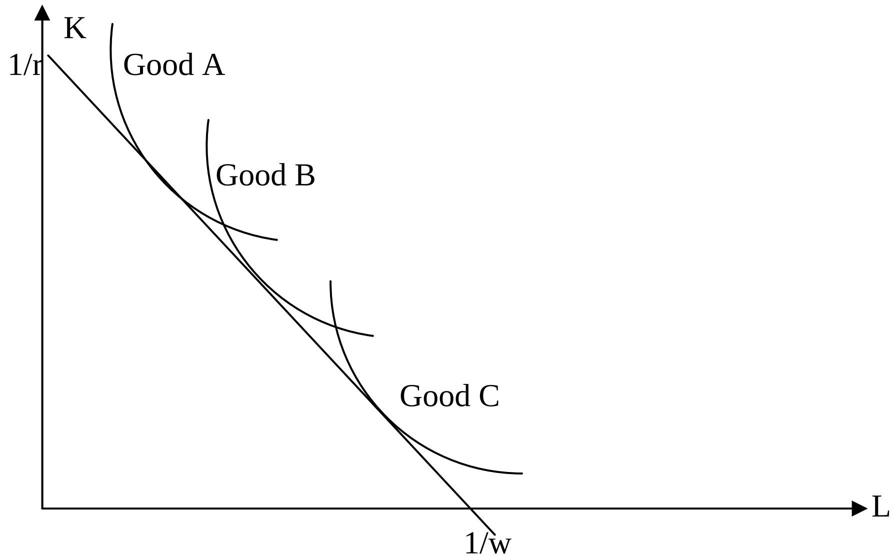
At endowment point (i), is the production mix determinant?

Moving toward (ii), what happens to A's production mix?

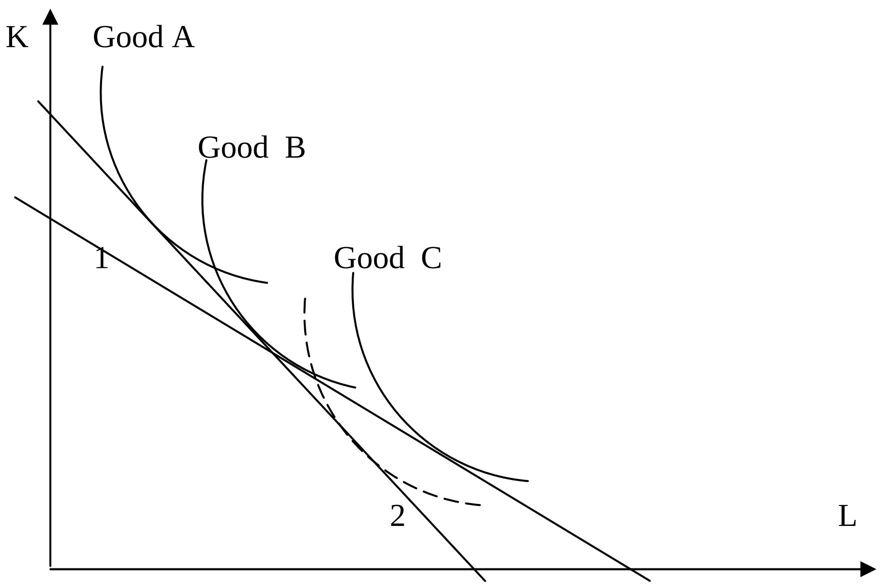
<u>Answer</u>: even if 1 supplies *all* of the most labor-intensive goods demanded, it can't absorb its factor stocks at (ii) using the factor intensities of the integrated equilibrium. Factor prices and output prices will have to adjust, so the integrated equilibrium won't be replicated. Some countries won't be producing the entire set of goods, and the same good will be produced using different factor intensities in different countries.

In short, some major simplifications emerge so long as if countries don't differ too much in their relative factor abundance.

<u>A producers' eye view</u>: In the integrated equilibrium (IE), the same factor prices prevail everywhere, all goods are producible at zero profit in each country, and each uses the same factor intensities:



But suppose that at IE prices, country 1 wants to supply too much of good C to clear the market. Then the relative price of C will adjust until a new equilibrium obtains, and the unit value isoquant will shift, perhaps as below. Now country 1 produces only B and C while 2 produces only A and B, and FPE fails. Note that different countries use different factor intensities to produce the same good (B).



Formally, Helpman and Krugman (1985, chapter 1) describe the set of factor endowments that lead to FPE (with no non-traded goods) as:

$$FPE = \left\{ (\underline{V}_1, \underline{V}_2, \ldots, \underline{V}_C) \mid \exists \lambda_{ic} \geq 0, \sum_{c \in C} \lambda_{ic} = 1 \; \forall i \in I \ni \underline{V}_c = \sum_{i \in I} \lambda_{ic} \underline{V}_{iW} \;\; \forall c \in C \right\}$$

Any factor allocation across *C* countries that leads to FPE must be consistent with non-negative outputs of each good in each country, when (1) global output levels of each good match those of the integrated equilibrium, and (2) each country exhausts its endowments when using production techniques that reflect the integrated equilibrium factor intensities. (V_{iW} is the vector of global factor usage for production of good *i* in the IE, and λ_{ic} is the share of global *i* production done in country *c*.)

Sufficient conditions for FPE are:

i) all *C* countries share the same technology (production functions);
ii) there are at least as many goods as factors;
iii) there are no factor intensity reversals;
iv) at equilibrium prices, *all* goods can be produced in *each* country at zero profit.

The HOS (factor content) theorem with many goods and factors

By assuming identical homothetic tastes everywhere, the HOS model ensures that when identical prices prevail everywhere, all consumers consume goods in the same proportion. And since global markets must clear, this implies that each country consumes goods in the proportions that they are produced globally, with its consumption level determined by its share in global income.

Define

F_{jc} = net export of factor *j* embodied in trade by country *c*

V_{jc} = country *c*'s endowment of factor *j*.

s_c = country *c*'s share in global expenditures.

If factor price equalization obtains, the same factor intensities prevail everywhere, and

Production satisfies: $A\underline{X}_c = \underline{V}_c$
Consumption satisifies $\underline{Y}_c = s_c X_W = s_c A^{-1} \underline{V}_W$
Net exports are $\underline{T}_c = \underline{X}_c - \underline{Y}_c = A^{-1}[\underline{V}_c - s_c \underline{V}_W]$

where *W* subscripts refer to the entire world. The HOS theorem simply states that when factor price equalization obtains, the factor content of trade, $\underline{F}_c = A\underline{T}_c$, is:

$$F_{jc} = V_{jc} - s_c V_{jW} \qquad (1)$$