THE JOINT DYNAMICS OF CAPITAL AND EMPLOYMENT AT THE PLANT LEVEL*

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Abstract

We study the joint dynamics of plant-level capital and labor adjustment using data from the Korean manufacturing sector. We document that investment and reductions in employment frequently occur together. Employment declines in 39.5 percent of plant-year observations in which the investment rate exceeds 10 percent. Moreover, these episodes in which employment falls account for 42 percent of total capital accumulation. Viewed through the lens of the standard model of costly factor adjustment, these data cannot be rationalized using standard Cobb-Douglas and constant-elasticity of substitution specifications for the plant-level production function. They are more consistent with a model in which capital directly replaces labor in tasks.

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It is by now well known that manufacturing plants adjust capital and employment only infrequently, but often by a large amount conditional on adjusting. Each of these regularities is inconsistent with the traditional quadratic adjustment cost approach, but is informative about the nature of the factor adjustment technology. Models embedding theories of capital demand consistent with plant-level adjustment dynamics have been important in understanding the determinants of aggregate investment and its dynamics; similarly, models embedding theories of labor demand motivated by plant-level data are potentially important in understanding the cyclical properties of the aggregate labor market. However, much less attention, both empirical and theoretical, has been paid to how plants adjust both capital and labor jointly and to what this implies for broader economic questions.

This paper makes two contributions. The first is empirical. We document the adjustment dynamics of capital and labor in data from the Korean census of manufacturers. We highlight a novel and striking stylized fact: even plants undergoing large investment episodes very often reduce employment. In 39.5 percent of the plant-year observations in our sample in which a plant’s investment rate exceeds 10 percent, employment falls. Such episodes are not be neglected for understanding aggregate capital accumulation: a full 42 percent of aggregate capital investment occurs in plants where employment falls. Moreover, the falls in employment among such plants are large and persistent. The average decline in employment among plants which invest and reduce employment is nearly as large as the average increase in employment among plants which invest and increase employment, and 71 percent of the decline in employment persists after one year. Finally, episodes of investment and em-

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1These facts are documented by many authors. For example, on investment, see Doms and Dunne (1998), Nilsen and Schiantarelli (2003), Cooper and Haltiwanger (2006), and Gourio and Kashyap (2007); on employment, see Davis and Haltiwanger (1990, 1992), Davis, Faberman and Haltiwanger (2006, 2010), and Cooper, Haltiwanger and Willis (2007). The literature on investment has stressed the skewness and kurtosis of the investment distribution more than exact inaction.

2For investment, see Caballero, Engel and Haltiwanger (1995, 1997); Caballero and Engel (1999); Cooper, Haltiwanger and Power (1999); Thomas (2002); Veracierto (2002); Gourio and Kashyap (2007); Khan and Thomas (2008, 2013), and Arellano, Bai and Kehoe (2010); for employment, see Cooper, Haltiwanger and Willis (2007); Veracierto (2009); Kaas and Kircher (2011); Acemoglu and Hawkins (2013), and Elsby and Michaels (2013).

3There are some exceptions. Theoretical analyses of models with multiple frictions include Bloom (2009), Reiter, Sveen and Weinke (2011), and Bloom et al. (2012). Empirical analyses of plant-level data include Sakellaris (2001, 2004), Letterie, Pfann and Polder (2004), Polder and Verick (2004), and Eslava et al. (2010). We discuss the latter papers in more detail below. There is also an older empirical literature which uses sectoral-level data and thus assumes a quadratic adjustment cost (Nadiri and Rosen, 59; Shapiro, 1986; Rossana, 1990; Hall, 2004).

4We also replicate our results in Chilean data, in which the joint dynamics of capital and labor are qualitatively similar to those for Korea. For example, nearly 30 percent of investment episodes in our Chilean sample involve a reduction in employment. However, our sample size is an order of magnitude lower for Chile than for Korea, so that we report our results here mainly for Korea.
ployment reduction are prevalent across all plant size classes, and also across all two-digit industries.

The second contribution of the paper is to show that the fact that investment and employment reduction frequently occur together is very informative for understanding the nature of the plant-level production technology. As a starting point, we consider the prototypical model of joint factor adjustment, which assumes that the production function is Cobb-Douglas, the adjustment costs on each factor are piecewise linear, and the single forcing variable is a neutral shock (for example, to demand or to Hicks-neutral technology). These strong assumptions allowed Dixit (1997) and Eberly and van Mieghem (1997) to derive a very strong implication for joint factor adjustment dynamics: whenever the more costly-to-adjust factor is adjusted, so too is the other factor, in the same direction, and by the same proportion. This implication is clearly inconsistent with the data.

There are two broad categories of explanations for why the benchmark model might fail to match the data, and we consider each in turn. The first class holds constant the production function and alters other model assumptions. For example, we allow for various kinds of aggregation bias, across production units within a plant or over time. We also consider explanations, such as required maintenance and lags in the delivery of machinery, that attempt to de-couple (reported) investment and employment growth. Finally, other modifications to the benchmark model, such as (product) price stickiness, factor price shocks, and a richer set of adjustment frictions, affect desired employment growth in states where investment is positive. None of these explanations seems likely to account for more than a small fraction of the observed frequency of investment and employment reduction.

More promising are explanations which depart from the Cobb-Douglas plant-level production function assumed by Dixit (1997). A natural alternative is to consider the standard constant elasticity of substitution production function and allow for factor-biased technical change. Here, if capital and labor are gross complements (Chirinko, 2008; Raval, 2012),

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5 We thus echo the message of Gorodnichenko (2012), who stresses the importance of explicit theories of input behavior for inference with respect to the (gross) output technology.

6 Piecewise linear adjustment costs imply inaction due to the discrete change in the marginal cost of adjusting at zero adjustment. The payoff from adjusting in response to small variations in productivity therefore may not outweigh the cost. Linear costs of control have been studied theoretically, in the context of investment, by Abel and Eberly (1996), Veracierto (2002), and Khan and Thomas (2013). Ramey and Shapiro (2001) give direct evidence on the cost of reversibility, and Cooper and Haltiwanger (2006) estimate the degree of irreversibility in a structural model. Linear costs of adjusting employment have been considered in Bentolila and Bertola (1990), Anderson (1993), and Veracierto (2008).

7 We also consider explanations based on investment-specific technical change, following (Krusell et al., 2000), which emphasize a shift of labor demand away from unskilled labor but towards skilled labor. However, the declines in employment which accompany investment in our dataset are not exclusive to unskilled labor. Fuentes and Gilchrist (2005) also report little correlation between equipment investment and the skill mix.
an improvement in capital-augmenting technology leads to an increase in the plant’s desired capital-labor ratio. However, unless either returns to scale at the plant level are strongly decreasing (contrary to Basu and Fernald, 1997) or plant-level demand is highly inelastic (contrary to Broda and Weinstein, 2006 and Foster, Haltiwanger and Syverson, 2013), the associated improvement in the plant’s TFP leads it to increase its desired scale of operations enough to ensure that labor demand does not fall. That is, a large improvement in capital-augmenting technology will lead the plant to hire (as well as invest).

The most successful explanation is one which views the plant’s production process as an aggregate over a set of tasks, each of which can be performed either by capital or labor (Zeira, 1998; Acemoglu, 2010). Improvements in the efficiency of capital lead the plant to alter the allocation of tasks, so that tasks formerly performed by workers are effected using machinery. We allow for the process of reassigning tasks between labor and capital to be costly. In effect, this framework is consistent with the elasticity of substitution between capital and labor being less than one both at the plant level (at least when task allocation is not changed) and in the aggregate (consistent with Oberfeld and Raval, 2013), while at the same time allowing for episodes where the plant shifts tasks from labor to capital and faces a much higher substitution elasticity.

The remainder of the paper proceeds as follows. Section 2 introduces the Dixit (1997) model and discusses its key testable implication on the joint dynamics of capital and employment adjustment. Section 3 compares the model’s prediction to the data. This section discusses a number of robustness tests and, in particular, analyzes the implications of time aggregation. Section 4 then takes up a number of possible extensions to the baseline model that may rationalize the empirical result. Section 5 concludes.

1 The firm’s problem

We first consider a model of capital and labor demand in which the cost of adjusting either factor is proportional to the size of the change. Formally, the costs of adjusting capital and
employment, respectively, are assumed to be

\[ C_k (k, k_{-1}) = \begin{cases} \ c^+_k (k - k_{-1}) & \text{if } k > k_{-1} \\ \ c^-_k (k_{-1} - k) & \text{if } k < k_{-1} \end{cases} \]

\[ C_n (n, n_{-1}) = \begin{cases} \ c^+_n (n - n_{-1}) & \text{if } n > n_{-1} \\ \ c^-_n (n_{-1} - n) & \text{if } n < n_{-1} \end{cases} \]  

(1)

The cost \((c^+_k)\) of expanding employment is often interpreted as the price of recruiting and/or training. The cost \((c^-_k)\) of contracting employment may represent a statutory layoff cost. Capital decisions can be costly to reverse because of trading frictions (e.g., lemons problems, illiquidity) in the secondary market for capital goods (Abel and Eberly, 1996). In that case, \(c^+_k\) is interpreted as the purchase price and \(-c^-_k\) is the resale value such that \(c^+_k > -c^-_k > 0\). For concreteness, we interpret the problem along these lines, but the analysis does accommodate any linear adjustment cost that implies costly reversibility, i.e., \(c^+_k > -c^-_k\).

For now, we omit fixed costs of adjusting from (1), but we consider the effect of these later.

The problem of a competitive firm subject to (1) is characterized by its Bellman equation,

\[ \Pi (k_{-1}, n_{-1}, x) = \max_{k,n} \left\{ x^{1-\alpha-\beta} k^\alpha n^\beta - wn - C_k (k, k_{-1}) - C_n (n, n_{-1}) + \varrho \int \Pi (k, n, x') dG (x' | x) \right\} \]  

(2)

where \(x\) is plant-specific productivity, \(w\) is the wage rate and \(\varrho\) is the discount factor. We assume revenue is given by \(x^{1-\alpha-\beta} k^\alpha n^\beta\). The Cobb-Douglas form is particularly tractable, but the analysis carries through as long as \((k, n, x)\) are \((q-)\) complements; technology is Hicks-neutral; and the production function displays constant returns jointly in the triple, \((k, n, x)\). Throughout, we assume plant-level TFP, \(x\), follows a geometric random walk,

\[ x' = xe^{\varepsilon'}, \quad \varepsilon' \sim N \left( -\frac{1}{2} \sigma^2, \sigma^2 \right) \]  

(3)

8Throughout, a prime (‘) indicates a next-period value, and the subscript, \(-1\), indicates the prior period’s value.

9In South Korea’s two-tier labor market, the termination of a permanent worker does not entail a layoff cost per se – dismissals are regulated more directly. Labor law directs managers to “make every effort to avoid dismissal” of permanent workers and, if the plant is unionized, to engage in “sincere consultation” with the workers’ representatives (Grubb, Lee and Tergeist, 2007). Temporary workers include those on fixed-term employment contracts. A fixed-term contract lasts for up to two years, but there are few restrictions on their renewal. The dismissal of fixed-term employees is largely unregulated. To retain tractability, we (and much of the literature) do not attempt to model labor market institutions in detail. Permanent and temporary workers are melded into a representative worker, and the adjustment frictions are assumed to act, in effect, as a simple tax on adjustments.
We omit depreciation and attrition only to economize on notation and simplify (slightly) the presentation of the dynamics below. They do not affect the predictions of the model, and will be included in the quantitative assessment in the next section.\footnote{The only source of variation in (2) is idiosyncratic TFP shifts. Unless otherwise noted, we abstract from aggregate uncertainty. This is consistent with our current focus on the cross section rather than aggregate fluctuations.}

Figure 1 summarizes the optimal policy and is taken from Dixit (1997).\footnote{See also Eberly and van Mieghem (1997).} The assumption of a random walk ensures that the problem is linearly homogeneous in \((k, k_{-1}, n, n_{-1}, x)\). As such, it admits a normalization.\footnote{We have numerically investigated the behavior of the model when productivity is stationary. Persistent but stationary shocks do not overturn the main implication of the analysis.} We normalize with respect to \(x\). So let us set

\[
\tilde{n} \equiv n_{-1}/x, \quad \tilde{k} \equiv k_{-1}/x.
\]

The figure then places \(\log \tilde{n}\) along the vertical axis and \(\log \tilde{k}\) along the horizontal. We summarize the policy rule with respect to employment; the capital demand rule follows by symmetry.\footnote{The form of the optimal policy follows from the concavity, supermodularity, and linear homogeneity of the value function. See Dixit (1997) and Eberly and van Mieghem (1997) for details. Our discussion in this section is based on their analysis.} Holding \(k_{-1}\) and \(x\) constant, a higher start-of-period level of employment, \(n_{-1}\), is tolerated within a range because of the cost of adjusting. But if the firm inherits a \(n_{-1}\) from last period that is sufficiently high, the marginal value of the worker, evaluated at \(n_{-1}\), is so low as to make firing optimal. That is, if we let

\[
\tilde{\Pi}_2 \left(k_{-1}, n_{-1}, x \right) \equiv x^{1-\alpha-\beta} k_{-1}^{\alpha} n_{-1}^{\beta} = wn_{-1} + \varrho \int \tilde{\Pi} \left(k_{-1}, n_{-1}, x' \right) G \left(x' | x \right) dx',
\]

then \(\tilde{\Pi}_2 \left(k_{-1}, n_{-1}, x \right) < -c_n\). At this point, the firm reduces employment to the point where \(n\) satisfies the first-order condition, \(\tilde{\Pi}_2 \left(k_{-1}, n, x \right) = -c_n\). Thus, the upper barrier (the northernmost horizontal line in the parallelogram) traces the values of \(\log \tilde{n}\) which satisfy this FOC, making the firm just indifferent between firing one more worker and “doing nothing”. Conversely, if the firm inherits an especially low value of employment, then it is optimal to hire (i.e., \(\tilde{\Pi}_2 \left(k_{-1}, n_{-1}, x \right) > c_n\)). Employment is then reset along the lower barrier (the southernmost horizontal line). This lower threshold thus traces the values of \(\log \tilde{n}\) which satisfy the FOC for hires, making the firm indifferent between inaction and hiring one more worker.

It is important to note that, if \(\log \tilde{k}\) increases, then the firm tolerates higher employment than otherwise – that is, the upper (firing) barrier is increasing in \(\log \tilde{k}\). This is because
of the complementarity between capital and labor. Complementarity also implies that the lower threshold is increasing in \( \log \tilde{k} \); the firm is willing to hire given higher values of \( \log \tilde{n} \) if its capital stock is larger.\(^{14}\)

Figure 2, also from Dixit (1997), distills the implications of the optimal policy for the joint dynamics of capital and employment. Assume a firm has initial levels of capital and labor such that \( \log \tilde{k} \) and \( \log \tilde{n} \) lie in the middle of the inaction region. Now suppose productivity, \( x \), rises, in which case \( \log \tilde{k} \) and \( \log \tilde{n} \) each begin to fall toward their lower barriers.\(^{15}\) The figure is drawn to convey that the cost of adjusting capital is relatively high – the space between the capital adjustment barriers exceeds that between the labor adjustment barriers – so as productivity increases, the hiring barrier is the first to be reached. At this point, employment, \( n \), is set such that \( \tilde{n} \) moves southwest along the lower barrier. As a result, when \( \log \tilde{k} \) eventually reaches the investment barrier, the firm is already just indifferent between hiring and inaction. Therefore, complementarity implies that the increase in capital must tip the marginal value of labor, \( \tilde{n} \), above the marginal cost, \( c_n^+ \): an increase in capital is always accompanied by hiring.

Now suppose that \( x \) begins to decline. As a result, \( \log \tilde{n} \) moves southwest along the lower barrier. As a result, when \( \log \tilde{k} \) eventually reaches the investment barrier, the firm is already just indifferent between hiring and inaction. Therefore, complementarity implies that the increase in capital must tip the marginal value of labor, \( \tilde{n} \), above the marginal cost, \( c_n^+ \): an increase in capital is always accompanied by hiring.

\(^{14}\) The thresholds are flat in regions where the firm adjusts both factors. For instance, if \( \tilde{n} \) and \( \tilde{k} \) are sufficiently low, then the firm increases both such that \( \Pi_k (k, n, x) = c_k^+ \) and \( \Pi_n (k, n, x) = c_n^+ \). For any \( x \), this system of first-order conditions yields a unique solution for \( n/x \) and \( k/x \). On the figure, this unique pair is given by the southwestern corner of the parallelogram. Regardless of the exact levels of capital and employment, the firm resets to this point as long as \( \tilde{n}, \tilde{k} \) initially lies to the southwest. Hence, in this region, the hiring threshold is independent of the initial level of capital and the investing barrier is independent of the initial level of employment.

\(^{15}\) If \( x \) rises by one log point, for instance, then \( \tilde{n} \) and \( \tilde{k} \) each fall by one log point. Hence, the pair \( \log \tilde{n}, \log \tilde{k} \) travels along the 45° degree line. This simple characterization is made possible when both factors are expressed in logs, which explains why we do so in Figures 1 and 2.

\(^{16}\) There is another, somewhat more subtle point to note, namely, the investment barrier has slope greater than one. Therefore, as \( \log \tilde{k}, \log \tilde{n} \) travels northeast along the 45° line, it does not intersect the investment
This argument has essentially traced the ergodic set of \((\log \tilde{k}, \log \tilde{n})\) induced by the model. This is shown in the shaded region of the figure. This set stretches from the 45° ray from the southwest corner to the 45° ray that extends from the northeast corner of the parallelogram. The shape of the policy rules (and the structure of the stochastic process, \(x\)) implies that, any particle \((\log \tilde{k}, \log \tilde{n})\) will eventually enter this space. Moreover, as our discussion has shown, the particle, once inside, will never leave.

### 2 Evaluation of the baseline model

#### 2.1 The basic properties of the data

To assess the implications of the model, we use two sources of plant-level data. The first is the Korean Annual Manufacturing Survey, for which we have data from 1990-2006. The second is the Chilean Manufacturing Census, for which we have annual data from 1979-96. Both surveys cover all manufacturing establishments with at least 10 workers and include observations on the size of the plant’s workforce and investment. In what follows, we focus on machinery investment specifically, since that category is most consistent with the type of capital envisioned in the baseline model.

Though we have analyzed both datasets, we regard the South Korean survey as our principal source. The reason is that the measurement of employment in the Korean data is better suited to our purposes. Since investment is measured in both datasets as the cumulation of all machinery purchases over a calendar year, the analogous measure of employment growth is the change in employment between the end of the prior year and the end of the current year. However, employment in the Chilean Census is typically reported as an annual average. There are several consecutive years in the 1990s in which employment is reported as of the end of the (calendar) year, and our use of Chilean data must be restricted to this subsample. This is, in part, why the South Korean panel is so much larger. There are about 508,000 plant-year observations in the Korean survey between 1990-2006, which is an order of magnitude larger than than the (usable) Chilean panel.

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\[17\] After 2006, plants with 5 – 10 workers were included in the Korean Census. For comparability across time, we use data only through 2006.
Accordingly, our discussion in the main text focuses on the Korean data. Moreover, the results from Korean data will also serve as the targets of our calibration when we study the baseline model quantitatively. Appendix A catalogues the results from Chilean data. As noted in the introduction, the findings from Chilean data are largely similar to what we report off the Korean sample.

To take the model to data, we recall that the model’s prediction pertains to employment growth conditional on any positive investment. However, measurement error in investment may mean that some of the smaller reported investments are in fact zeros. This is problematic insofar as negative employment growth conditional on zero investment is quite consistent with the baseline model. Hence, this form of measurement error could lead us to wrongly reject the model. Therefore, we wish to condition on investment rates in excess of 10 percent to guard against this. The investment rate is defined as \( i/k_{-1} \), where \( i \) is real investment and \( k_{-1} \) is the end of the prior year real machinery stock.

The construction of a series for \( i/k_{-1} \) is done in the usual way via the perpetual inventory method. The capital stock in the first year of a plant’s life in the sample is computed by deflating the book value of machinery by an equipment price index. Capital in the succeeding years is calculated using the law of motion, \( k = (1 - \delta) k_{-1} + i \), assuming a depreciation rate of \( \delta = 0.1 \). Real investment, \( i \), is obtained by deflating investment expenditure by the equipment price index. Note that each dataset includes information on gross purchases and sales; investment expenditure is the difference of the two.\(^{18}\)

Our exploration of the data begins with a characterization of the distributions of net investment and employment growth. This is shown in Table 1. Two features of the data are significant. First, judging by the adjustment frequencies of the individual factors, capital appears to be the more costly one to adjust. Investment is reported to be zero in South Korea manufacturing in 47 percent of all plant-year observations, whereas employment growth is zero in 13.5 percent of plant-year observations. (As shown in Appendix A, the relative frequency of investment is similar in Chile.) We will see that, when we calibrate the baseline model, these adjustment frequencies do in fact map to a relatively wide \( Ss \) band with respect to capital, which in turn makes capital the hard-to-adjust factor in the baseline model.\(^{19}\)

The pervasiveness of inaction with respect to investment contrasts with estimates from other datasets. Cooper and Haltiwanger (2006) report that less than 10 percent of plant-year

\(^{18}\)We use the price index for equipment purchases in the manufacturing sector. The price index is available from the OECD STAN database.

\(^{19}\)To the degree that employment adjustment is less costly than capital adjustment in Korea, it is likely due to the fixed-term contract (see footnote 13 for more on this contract). However, our data do not identify fixed-term workers, so we are unable to investigate this more deeply.
observations in their U.S. data show zero net investment. Letterie, Pfann, and Polder (2004) also report a small inaction rate in Dutch data. There are two points to bear in mind here. It is important to bear in mind that the results based on U.S. and Dutch data are derived from a balanced sample of relatively large plants. For instance, Cooper and Haltiwanger’s panel is related to that used in Caballero, Engel, and Haltiwanger (1997), and mean employment in the latter’s sample was nearly 600. Since large plants tend to be aggregations over at least somewhat heterogeneous production units, we anticipate that inaction rates are lower in these establishments. At plants with more than 100 workers, the share of plant-year observations in Korean data that involve no investment is 10.6 percent, and the share with zero employment change is 4 percent. These estimates are in the neighborhood of the numbers reported in the related papers. Note that, for large plants, investment is still the relatively less frequently adjusted factor.

For most of the paper, we use our full sample. But, in an attempt to strike a compromise between our approach and that of others, we have re-run the main analysis—the distribution of employment growth conditional on positive investment—with a sample that excludes the first and last years of any plant’s lifespan if that establishment enters or exits in our sample. The results hardly change. This should help address concerns that entry and exit drive the difference between the empirical moments and the model’s predictions.

The second feature of the data that is worth notice has to do with the re-sale of machinery. Our data allows us to measure the gross sales of machinery by manufacturers. The frequency of re-sale in Korea (6.3 percent of sample observations) is roughly in line with that in U.S. (Cooper and Haltiwanger, 2006). More intriguingly, if a plant sells machinery, it is very likely to purchase equipment in the same year. The probability of an investment purchase conditional on positive sales in the same year is 74.6 percent. To the best of our knowledge, this fact has not been documented in the recent literature on factor adjustment. Notably, it is a fact that is at odds with the baseline model; the plant in this model would never simultaneously sell (at a discount) and then purchase the same machinery. We suspect one

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20Letterie, Pfann, and Polder (2004) also use a balanced sample, so their sample is likely to consist of relatively large units. In related work by Polder and Verick (2004) on Dutch and German data, mean plant size is three (Dutch) and 9.5 (German) times that in Korean data.

21Of course, the adjustments at these larger plants contribute greatly to the change in aggregate employment. However, the approach in the literature has been to study the problem of a production unit, and the dynamics among small and mid-sized plants can inform our assessment of this model. To the extent there are heterogeneous units within a single plant, this aggregation problem ought to be addressed explicitly, but there is no consensus in the literature in this regard.

22Although it is possible in the baseline model that a plant may purchase machinery in one month after a positive TFP realization but sell later in the year when a new productivity level is revealed, our work below suggests that this is unlikely given the size of the Ss bands on investment.
may need a model with heterogeneous equipment in order to speak to this result. In such a model, a plant may scrap or sell one machine and upgrade to a new vintage (see Cooper, Haltiwanger, and Power (1999)).

This paper does not pursue this issue at greater length. This partly reflects our sense that it does not drive the extent of negative co-movement between investment and employment. When we look at episodes where plants invest and reduce employment, we do see that these episodes sometimes (23 percent of the time) coincide with the sale of equipment. But this pattern of sale-and-purchase does not appear to go hand-in-hand with the incidence of negative co-movement. We therefore leave a more thorough examination of this issue for later work.

2.2 The principal result and its robustness

Given the relative frequency of investment, the baseline model predicts that investment (or disinvestment) is always accompanied by employment adjustment. The left panel of Figure 1 shows that it is not (we will discuss the right panel shortly). The figure shows the unconditional distribution of the net change in log employment across calendar years and the distribution conditional on a plant-level investment rate \( i/k_{-1} \) greater than 10 percent. The distribution of net employment growth, conditional on positive investment, is slightly shifted to the right – plants do typically raise employment more if they also invest – but the similarity across these distributions is inconsistent with the model. The model predicts that investment should perfectly predict an expansion in labor demand; employment growth should lie everywhere to the right of zero.\(^{23}\)

To summarize the features of Figure 1, there are two moments that are particularly helpful and are reported in Table 1. First, among those plant-year observations that involve an investment rate greater than 10 percent, we compute the share in which employment declines. This is 39.5 percent (in Korea). Second, we calculate the average contraction in employment among the set of plants that both invest and “fire”. This is 19 percent (the median is 12.9 percent).

The employment losses in these investment episodes are both large and persistent. To put their size in context, consider the average expansion in employment among plants that both invest and hire. This is 21.4 percent (the median is 15.4 percent). Hence, among investing

\(^{23}\)Results in this section are based on episodes of (sufficiently) positive net investment. But since a plant may sell and purchase machinery in the same year, there will be more instances of positive gross investment than net investment. However, because re-sale is relatively infrequent, there are not that many instances in which gross investment exceeds 10 percent when net investment does not. Therefore, when we condition instead on gross investment rates greater than 10 percent, our results are largely unaffected.
plants, the declines in employment at contracting establishments are nearly as large as the increases in employment at expanding plants. This indicates that the failure of the baseline model is not due merely to the occasional coincidence of small declines in employment with positive investment. To gauge the persistence of these employment losses, we follow Davis and Haltiwanger (1992). For each plant which sheds workers in year $t$, Davis and Haltiwanger compute the share of that decline that is present one year later. Aggregating across plants (and weighting by that plant’s share of total year-$t$ job destruction), we obtain a measure of the persistence of employment loss. We do this for all plants which report job destruction and for the sub-sample of plants which both reduce employment and invest. Our calculations show that, for all contracting plants and for the sub-sample of investing plants, about 70 percent of the employment decline persists. Thus, the employment declines at investing plants are not (relatively) quickly reversed.

In addition, the amount of investment undertaken at times when employment falls is substantial. It is not the case, in other words, that the extent of negative co-movement between capital and labor is due to periods of small investments. There are two ways to illustrate this point. First, we have verified that the results are largely unaffected if we instead condition on investment rates greater than 20 percent, which is the threshold used to define an investment spike in Cooper and Haltiwanger (2006). The probability of negative employment growth, conditional on this 20 percent threshold, remains high at 37.6 percent. Second, when we cumulate investment in these episodes, it amounts to 42 share of all investment done. Thus, these periods account for a significant portion of aggregate capital accumulation.

We stress that this result does not contradict research which finds that, on average, plant-level employment growth is higher conditional on investment (Letterie, et al, 2004; Eslava, et al, 2010). We stress, however, that the class of theoretical models which often provides the conceptual underpinnings of earlier empirical analysis implies much stronger restrictions on the joint dynamics of capital and labor demand. In this sense, we have sought to provide a more rigorous test of this class of models.

We now consider the robustness of these results. We are particularly concerned about the consequences for inference of various forms of aggregation. For instance, sectors and plants likely differ to some degree in their production technologies and adjustment costs. As a result, capital and labor may co-move negatively in some industries or plants, and still act

$^{24}$Eslava, Haltiwanger, Kugler, and Kugler appear to have in mind a model where there are (at least) fixed costs of adjusting each factor. The addition of fixed costs complicates the analytics, but, as we show later, the quantitative implications are unaffected.
in accordance with the standard model in others. We do not find any clear evidence of this. We re-run the analysis by industry and find that the probability of negative employment growth conditional on positive investment is contained in a narrow range between 37 and 46 percent (see Table 2). Of course, there remains considerable heterogeneity within industry. But it is hard to see how heterogeneity alone, particularly with regard to adjustment costs, could account for what we find. To see this, we note that nearly 80 percent of plants in our sample adjust employment more often then machinery. The baseline model’s prediction clearly applies to this majority. Hence, our aggregate result is not driven by plants with adjustment cost structures very different from the median. Furthermore, even if a plant faces relatively costly employment adjustment, the baseline model implies that this plant should always invest (the relatively inexpensive adjustment in this example) whenever it hires. This prediction also fails when taken to data.

Second, the result may be partly due to aggregation over heterogeneous production units within large plants. Perhaps one division of the establishment undertakes investment and hires. Another division contracts employment substantially, though does not disinvest. The net establishment-wide employment change may well be negative, even though establishment-wide investment is positive. This issue is difficult to address conclusively. We do note, however, that the scope for heterogeneous operations is presumably limited at smaller plants. Thus, within-plant aggregation were largely responsible for the result, the probability that employment declines conditional on positive investment should vanish at smaller plants. We do not find this. The bottom panel of Table 2 presents results by size class. The probability of employment contraction, conditional on positive investment, is lower at smaller plants: it is 35.8 percent at plants with 10 – 24 workers and 47.6 percent at plants with more than 100 workers. But the frequency with which employment falls in years of positive investment still appears economically significant.

Third, it is possible that the result reflects time aggregation. Establishments may invest and hire in one quarter but later reduce employment more significantly. As a result, the data show positive investment over the year but a net employment decline. This concern is more difficult to address, since we do not have higher-frequency data. In lieu of that, we can address this concern only via simulation analysis. We now detail our approach.

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25 That we are able to repeat the analysis on relatively smaller plants is an important advantage of these data. This sort of robustness test would not be possible if we performed this analysis on Compustat data, which includes only (relatively large) U.S. publicly traded corporations.

26 Even within production units, plants’ equipment stocks are aggregates over heterogeneous machines. This raises the question of whether episodes of negative employment growth and positive investment involve predominantly “small” investments, such as the replacement of hand tools. But we have already shown that this does not appear to be the case.
The objective is to simulate quarterly plant-level decisions from the model of section 1 and determine whether time-aggregating these observations to an annual frequency yields the joint adjustment dynamics observed in the annual data. There are five parameters introduced in section 1 whose values must be pinned down: the adjustment costs, \( c_k^+, c_k^-, c_n^+, c_n^- \), and the standard deviation, \( \sigma \), of the innovation to productivity. In addition, since we want to allow for depreciation in the simulation, we must select the rate of decay, \( \delta_k \).

These structural parameters are chosen as follows. The price of the investment good, \( c_k^+ \), is normalized to one. The resale price, \( -c_k^- \), is calibrated to target the frequency of investment. The idea behind this strategy is straightforward: a large wedge between the purchase and resale prices raises the cost of reversing investment decisions, and so induces a greater degree of inaction. Next, we impose symmetry on the costs of adjusting labor, \( c_n^+ = c_n^- \) and choose this cost to target the frequency of net employment adjustment.\(^{27}\) The variance of the innovation to productivity is used to target the average expansion in \( \log \) employment among plants which increase employment and undertake investment. Given this, we ask whether the model can also generate the average contraction in employment among those plants which decrease employment and invest. Lastly, we fix \( \delta_k = 0.02 \).

The other parameters are selected partly based on external information and partly to be consistent with choices in the related literature. The full list of parameters is given in Table 3. The discount factor, \( \varphi = 0.9875 \), is set to be consistent with the average real interest rate in South Korea over the sample for which we have data.\(^{28}\) The elasticity of output with respect to labor is assumed to be \( \beta = 0.50 \), which is consistent with the labor share of value-added in South Korean manufacturing.\(^{29}\) To ensure a well-defined notion of plant size, the coefficient, \( \alpha \), attached to capital must then be set below 1. We fix \( \alpha = 0.40 \), so \( \alpha/\beta \) is broadly consistent with the ratio of capital income to labor income in the sectoral accounts.\(^{30}\) For simplicity, worker attrition is set to zero, but we have verified that this is of

\[^{27}\text{The assumption } c_n^+ = c_n^- = c_n \text{ may seem at odds with the sense that layo¤ restrictions are nontrivial in Korea (see footnote 13). However, without data on the gross ‡ ows of workers into and out of the plant, we can only target the distribution of net employment growth, and this is roughly symmetric about zero. The symmetry between } c_n^+ \text{ and } c_n^- \text{ allows us to replicate this.}\]

\[^{28}\text{The real interest rate is calculated from OECD data on the short-term nominal interest and (realized) CPI inflation. Labor share can be computed directly from our census data; it is also publicly available through the EU KLEMS dataset. Note that in what follows, if we refer to sectoral accounts data, we mean EU KLEMS.}\]

\[^{29}\text{The use of value-added to compute labor share implicitly assumes a “true” production function of gross output of the form } x^{1+\alpha-\beta-\psi} k^\alpha n^\beta m^\psi, \text{ where } m \text{ is materials. If materials are costless to adjust and if the materials price is fixed, then one may concentrate out } m \text{ and obtain } y = x^{1-\alpha-\beta-\psi} k^{1-\psi} n^{1-\psi}. \text{ The exponent attached to labor, } \beta \equiv \frac{\bar{\beta}}{1-\psi}, \text{ is now interpretable as labor’s share in value-added.}\]

\[^{30}\text{To be precise, in the sectoral accounts, labor share’s of value-added is measured to be 0.56 and capital’s share is therefore 0.44. The difference between } \alpha, \beta, \text{ and these estimates implicitly represents the returns to}\]
little consequence for the analysis.\footnote{In principle, attrition may interact with time aggregation. Specifically, the plant may hire and invest in one quarter, but subsequent attrition then leads to a net decline in employment over the year. There are two challenges to this. First, attrition appears very modest in Korea (Chang, Nam, and Rhee, 2003). Second, attrition makes hiring less costly to reverse, so firms hire more often. This blunts the ability of the model to generate years in which the plant invests but then permits attrition to deplete its workforce on net.}

Once the model is calibrated, it is solved via value function iteration. The homogeneity of the value function with respect to $x$ is helpful at this point, as it allows us to re-cast the model in terms of the normalized variables, $\tilde{n}$ and $\tilde{k}$. This eliminates a state variable. Once the policy functions are obtained, we simulate 20,000 plants for 250 quarters. Results are reported based on the final 20 years of data. (Appendix B reports the exact normalized model used in the simulations.)

Table 4 summarizes the simulation results. The table reports a few sets of statistics, each computed off the simulated panel. The top panel allows one to gauge how well the model matches the marginal distributions of each factor change. We report, for instance, the inaction rates with respect to employment and investment. (More precisely, these are the shares of establishment-year observations in the simulated panel for which there is no net change in employment and no investment, respectively.) These were moments targeted in the calibration, so the model (nearly) reproduces the empirical estimates. In the top panel, we also report the unconditional standard deviation of the year-over-year change in log employment. This moment was not targeted, and the model does understate this measure of dispersion. An increase in $\sigma$ would ameliorate this, but the presence of larger shocks would also imply larger average employment changes conditional on positive investment, which was targeted in the calibration. Note that the model does very nearly replicate this latter moment.

The next few rows summarize the joint adjustment of capital and employment at an annual frequency.\footnote{At a quarterly frequency, positive investment and negative employment growth should not co-exist. In fact, we find that they occur together in only 0.02 percent of sample observations. We view this as a useful check on the accuracy of the simulations.} The results indicate that time aggregation cannot, in the context of the baseline model, account for the empirical findings. Just a little over two percent of plant-year observations in the simulated data involve positive investment and employment contraction, and the average decline in employment among those plants which are contracting is slightly under two percent. These results are illustrated in the right panel of Figure 3, which plots the model-implied distribution of the log change in employment given positive investment,
that is, given a net investment rate in excess of 10 percent. For reference, we show the
distribution from the South Korean data next to it.\textsuperscript{33}

It is instructive to re-run the annual simulations with a lower threshold for positive
investment. Rather than condition on a net investment rate greater than 10 percent, we set
the threshold at 0.01. In that case, 18 percent of observations in the simulated data show a
year-over-year net decline in employment, given positive investment. However, the average
decline in employment among contracting plants is 4 percent, which is less than one-fourth
of the average in Korean data.

Hence, in the model, contraction does occur, but it is limited. This reason for this is
intuitive. Since the model is quarterly, the theory indicates that positive investment – of
any magnitude – and negative employment growth do not co-exist at a quarterly frequency.
Thus, in the annual panel, we see both occur only if firing later in the year undoes the
hiring that accompanied investment earlier in the year. This happens, though, only if the
number of hires was small. That in turn means that the increase in investment must have
been relatively small, too. So episodes of firing and positive investment are ones where the
investment was quite limited, e.g., less than 10 percent. This is a revealing property of the
model because it contrasts so clearly with the plant-level data. As we discussed, the share
of observations which involve investment and employment contraction in the actual data is
much more robust to the choice of the investment threshold.

We conclude this section by noting that our findings have been observed in some other
industrialized economies. In Sakellaris’ (2001) analysis of the U.S. Annual Survey of Manu-
facturers, for instance, he finds that in years of investment spikes (when the net investment
rate exceeds 20 percent), large declines in employment (declines in excess of 10 percent)
occur with the same frequency as large increases in employment.\textsuperscript{34} Polder and Verick (2004)
observe in German and Dutch data that employment often declines when investment is pos-
itive.\textsuperscript{35} Our contribution, relative to these papers, is to relate the results to the specific
predictions of the baseline factor demand model of section 2 and to explore a greater variety
of extensions of that model that may reconcile theory and evidence. We now turn to these

\textsuperscript{33}The reader will notice that the model-implied distribution is tilted to the left. When (capital) deprec-
iation is set to zero, the distribution becomes more symmetric. Intuitively, since depreciation erases some of
the undesired capital but the firm has the same number of workers, a fall in $x$ leaves the firm with relatively
more “excess” employment than in the no-depreciation baseline. This makes it more likely that the firm
reaches the firing barrier in the model with depreciation.

\textsuperscript{34}Sakellaris mentions the result only in the working paper version (2001) of his paper. The published
paper (Sakellaris, 2004) omits any discussion of this finding.

\textsuperscript{35}We note that in Polder and Verick (2004), capital appears to be adjusted more often than employment.
In this case, the canonical model of section 2 suggests that, conditional on positive investment, plants ought
to always hire. This prediction is clearly violated in their data.
3 Extensions to the baseline model

In this section, we discuss a few modifications of the baseline model that may plausibly induce more realistic co-movement of investment and employment growth at the plant level. The theories discussed in this section fall into two broad classes. The first preserves factor-neutral technical change and seeks to account for the data along other lines. The second class introduces factor-biased technology.

3.1 Non-technological explanations

Sticky product prices. It has been noted that if the firm’s product price is sticky, factor-neutral technical change can be contractionary. If the firm does not wish to lower its price in order to sell additional output, it does not require its current factors to maintain its output given the higher level of TFP. So its factor demand declines. However, in these models, technology is contractionary with respect to both capital and labor (see Basu, Fernald, and Kimball, 2006). \(^{36}\) It is hard to see, then, how this mechanism accounts for the extent of negative co-movement of these two factors. \(^{37}\)

Replacement investment. When machinery is subject to stochastic breakdown, it is possible to partially de-couple investment from employment adjustment. For instance, suppose that the failure of a machine requires immediate replacement. This reflects the notion that a distinct machine may be required to perform each task of a given production process and that (in the short run at least) each task is required for the production of final output. Assume these failures are orthogonal to productivity and/or demand (and their histories). This means that there will be quarters in which a firm may invest merely to produce at all, even if productivity is so low as to warrant a reduction in employment.

This account of plant-level dynamics has some evidence in its favor. Consider, for example, the co-movement of employment with transportation equipment, such as trucks used for delivery. We also see that plants sometimes contract employment even as they increase invest in these vehicles. This seems consistent with the notion that the failure of a truck

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\(^{36}\) This literature has also explored the effects of investment-specific shocks. But in this case, the shocks are typically expansionary despite price stickiness (see Smets and Wouters, 2007).

\(^{37}\) Changes in factor prices, as opposed to rigid product prices, may appear to the reader to be a more promising explanation of our results. But an increase in a single factor price will function like factor-biased technical change, an issue we take up in detail below.
would create such a disruption that replacement is necessary even if productivity is relatively low.

The challenge for this explanation is that failure rates for industrial machinery appear to be relatively low. Erumban (2008) summarizes estimates of survival rates from a number of countries. The median estimate implies a service life of about 20 years, which corresponds to a failure rate of around 3 percent per year. Even if we assume all breakdowns require replacement, this data disciplines the calibration of the frequency and size of machine failures.

To pursue this point, we have incorporated into the model a simple process for machine failure: a fixed fraction $\Phi > 0$ of the (start-of-period) machinery stock fails with probability $P$ per quarter. In the event of failure, the plant must replace. (In this sense, required replacement takes the form of a stochastic tax on capital.) The key to the calibration is that the pair $(\Phi, P)$ should be chosen so that $\Phi P$, the implied average failure rate, is in the neighborhood of 3 percent per year. We have found that this significantly limits the explanatory power of the model. If $P$ is large, for instance, it is possible to reproduce a relatively high incidence of investment and contraction. But the data on survival rates then requires that $\Phi$ is small, so the average employment decline in these episodes is much smaller than in the data.

Lastly, we stress that an explanation of the factor adjustment patterns centered on replacement investment would grow only more difficult if we drop the assumption of required, immediate replacement. If other machines or labor can substitute to some degree for the failed equipment, then the plant will defer at least some replacement until productivity recovers. Indeed, Eisner (1978) finds evidence consistent with this: in his data, firms report that they expect to undertake more replacement investment after years of high profits. This behavior will tend to re-establish the positive co-movement between capital and labor predicted by the baseline model. Hence, if a version of the model with required, immediate replacement is unable to engage the plant-level data, it is unlikely that any account based on replacement investment will succeed.

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38 In fact, they are much lower than the failure rates of transportation equipment, such as the trucks cited above. The failure rates of transport equipment appear to be 4-5 times greater.

39 This estimate of the failure rate is noticeably less than the typical estimate of equipment depreciation rates. However, the latter are derived from used-equipment prices in secondary capital markets rather than more direct measures of the deterioration of installed machinery. Erumban (2008) estimates survival rates of machinery from Dutch plant-level data on the retirement of machinery. This is not ideal, insofar as retirements include scrappage as well as the re-sale of (still functional) equipment. Thus, his estimate of the failure rate may, to an extent, be upwardly biased. Still, the results are instructive: they imply a failure rate of about 3 percent per year.
The lag between order and delivery. The baseline model assumes that new machinery is installed and operated in the period in which it is ordered. But in reality there may be a delay between the order and delivery of new capital goods. If the plant reports investment in the year in which the order is placed, then this delay does not disrupt the implications of the baseline model: orders, and thus recorded investment, should still perfectly predict positive employment growth.

The implications of a delay are more significant if plants record investment in the year in which the machinery is delivered. Suppose machinery is ordered in August 2001 and received in February 2002. If plant productivity deteriorates over the course of 2002, employment may be reduced that year even though positive investment is observed. The delivery lag, in other words, amplifies time aggregation. If there were no delay, the delivery would coincide with the order. Thus, productivity would have to decline sufficiently between August 2001 and December 2001 in order to generate a year in which the plant undertakes investment and reduces its labor demand (on net). But if there is a delay, we could observe a decline in employment along with investment if productivity falls (sufficiently) at any point in 2002.

To investigate this, we generalize the baseline model in a simple way in order to allow for delivery lags. We adopt a Calvo-like mechanism: an order for machinery, regardless of when it was made, is filled with probability $\lambda$ in any one period. As a result, if an order goes unfilled this period, it takes its place in the prior backlog of unfilled orders, and whole stock is filled with probability $\lambda$ next period. This assumption greatly simplifies the problem, albeit at the expense of a certain degree of realism.

We now sketch the plant’s problem in the presence of a delay. Let $\kappa_{-1}$ denotes the initial backlog of unfilled orders at the start of the present period. The plant’s productivity, $x$, is revealed, and it chooses its capital and labor demands. With respect to capital, the plant may choose to sell a portion of its installed machinery, $k_{-1}$. We let $\hat{k}$ denote its machinery stock after any sales have taken place but before orders are placed. If the plant chooses to order machinery, its stock of unfilled orders rises to $\kappa$. If these orders arrive, they are delivered in the final instant of the period. At that point, production ensues.

The plant’s Bellman equation then becomes

$$
\Pi (\kappa_{-1}, \hat{k}_{-1}, n_{-1}, x) = \max_{\kappa, k, n} \left\{ \begin{align*}
-wn - C_n (n, n_{-1}) - C_k \left( \hat{k} + \kappa, k_{-1}, \kappa_{-1} \right) \\
+ \mathbb{E} \left[ x^{1-\alpha-\beta} \left( \hat{k} + \kappa \right)^{\alpha} n^{\beta} + \Pi \left( 0, \hat{k} + \kappa, n, x' \right) \right] \\
+ (1 - \lambda) \cdot \left[ x^{1-\alpha-\beta} \hat{k}^{\alpha} n^{\beta} + \Pi \left( \kappa, \hat{k}, n, x' \right) \right]
\end{align*} \right\}.
$$

This says that if orders arrive, the plant produces with a capital stock, $\hat{k} + \kappa$. In that case,
the backlog of unfilled orders taken into the next period is 0. Otherwise, the plant produces with \( \hat{k} \), and takes an unfilled order stock \( \kappa \) into the next period.\(^{40}\) The cost of adjusting capital, \( \mathcal{C}_k(\hat{k}, \kappa, k_{-1}, k_{-1}) \), is notationally more cumbersome but substantively the same as before. It is given by

\[
\mathcal{C}_k(\hat{k}, \kappa, k_{-1}, k_{-1}) = c_k^+ (k - k_{-1})_{1[k > k_{-1}]} + c_k^- (k_{-1} - \hat{k})_{1[k_{-1} > \hat{k}]}
\]

The difference, \( \kappa - k_{-1} \), represents the new orders placed this period, and \( k_{-1} - \hat{k} \) is the number of units sold.

We solve the model numerically. The calibration of the size of the adjustment costs, the returns to scale, and the stochastic process of productivity \( (x) \) does not differ materially from that presented in Table 3. The only free parameter to be pinned down, then, is \( \lambda \) – the probability per quarter that a delivery is made. There is a not great deal of evidence on this, and what we have found pertains to the U.S. rather than Korea. Abel and Blanchard (1988) estimate a delivery lag of 2 to 3 quarters, whereas 2 quarters is on the high end of the range of estimates reported in Carlton (1983). In Table 5, we therefore present results for \( \lambda = 1/3 \). Note that if \( \lambda = 1/3 \), then \((2/3)^3 \equiv 30\) percent of firms wait longer than 3 quarters for delivery.\(^{41}\)

The results in Table 5 indicate that delivery lags do not appear to be so large as to account for the majority of the coincidence of positive investment and negative employment growth. As shown in the bottom panel, the share of plant-year observations that involve opposite movements in investment and employment is now 12.5 percent. This is far higher than in the baseline model but still less than one-third of the empirical estimate. The mean decline in log employment, conditional on positive investment, is 3.5 points, which is less than one-fifth what we see in the data. This reflects the difficulty in generating sizable reductions in employment purely from time aggregation. As shown in the middle panel, orders of machinery (at the quarterly frequency) are virtually never accompanied by contractions in labor demand – desired employment perfectly co-moves with desired capital. Arguably, a model might have more potential if increases in investment triggered a reduction in desired labor demand. We will take up such a theory in the next subsection.

**The disruption of factor adjustment.** Cooper and Haltiwanger (1993) provided

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\(^{40}\)Note that we assume the plant pays for the orders when they are placed. This is innocuous in a model with perfect credit markets.

\(^{41}\)To be clear, when the model is simulated, investment is recorded at the time of delivery. This captures the mis-match between the dates at which employment is adjusted and investment is recorded in the data.
a model in which machine installation disrupts production and reduces current labor demand. This can be captured in our context by the assumption that revenue takes the form 
\((1 - \tau \cdot 1_{\Delta k > 0}) \cdot x^{1-\alpha-\beta} k^\alpha n^\beta\), where \(\tau \in (0, 1)\). This has two important features.\(^{42}\) First, for any given \((k, n, x)\), this means that any amount of investment \((\Delta k > 0)\) involves a discrete cost of adjusting equal to \(\tau x^{1-\alpha-\beta} k^\alpha n^\beta\). Second, conditional on adjusting, the marginal product of each factor is degraded by \(\tau\). The latter is especially critical to Cooper and Haltiwanger’s mechanism. If we retain their assumption that labor is a frictionless factor, the choice of employment conditional on investment is now governed by the static first-order condition, \((1 - \tau) \beta x^{1-\alpha-\beta} k^\alpha n^{\beta-1} = w\). Here, \(k\) is interpreted as the start-of-period capital available for use in production this period; the newly installed machinery is assumed (as in Cooper and Haltiwanger) to be inoperative until next period. It follows that if the change in \(x\) required to induce investment is not too large relative to \(\tau\), the disruption cost can dominate: labor demand will decline in the period of installation.

It is not obvious that this result survives if labor is subject to a cost of adjusting, however. Once the capital is installed and available for use, the marginal value of labor will be high. If layoffs are costly to reverse, the firm has an incentive to retain its workforce.\(^{43}\)

We also note that the timing convention in this rendering of the model is significant. If the new machinery came on-line in the period of installation (as in section 1), the large adjustment to \(k\) would likely offset the effect of \(\tau\) on the marginal product of labor, triggering an increase in labor demand. In what follows, we will maintain the assumption of a one-period delay between the date of installation and the date at which the new capital becomes operative. This allows us to focus on the importance of labor adjustment costs.

We now solve the extended model numerically. To this end, we need to calibrate \(\tau\), which functions as a fixed (discrete) cost of adjusting in this formalization.\(^{44}\) It seems natural, then,

\(^{42}\)Cooper and Haltiwanger’s 1993 paper did not include stochastic productivity, but a specification like this has been used often in the more recent dynamic factor demand literature. See, among others, Caballero and Engel (1999), Cooper, Haltiwanger, and Willis (2005), Cooper and Haltiwanger (2006), and Bloom (2009).

\(^{43}\)In a richer model, one might differentiate between a temporary furlough and a permanent severance of the firm-worker match. If the former is allowed, then temporary layoffs would likely coincide with machine installation. It is unlikely that this is the source of the co-movement we see in the data, though. First, since we measure year-over-year changes in our data, this kind of short-term (intra-year) variation in the workforce is unlikely to account for our results. Moreover, as discussed in section 2, there is no evidence that employment declines, conditional on investment, are reversed at a faster rate than otherwise.

\(^{44}\)This is easy to see if new capital does not become operative until the period after installation. In that case, current revenue – and thus, the disruption cost – is independent of the size of the change in the capital stock. But even if capital becomes operative immediately, the disruption cost has a discontinuity at the origin, as in \(\lim_{\Delta k \to 0} \tau \cdot x^{1-\alpha-\beta} k^\alpha n^\beta \neq 0\). This means that it rises at an infinitely fast rate when any investment is undertaken. This discontinuity is the distinguishing feature of fixed costs and gives rise to the economies of scale in adjusting, which make lumpy changes optimal.
to select this to target the frequency of investment, and then use the resale price of capital to target the frequency of negative investment. However, we found that it takes only an exceptionally small value of $\tau$ to induce a substantial amount of inaction. Yet if $\tau$ is set too low, then the model has little chance of engaging the data – the marginal product of labor would not fall by enough to induce declines in employment at the time of installation. To strike a compromise, we fix $\tau = 0.025$, which implies that investment is zero around 70 percent of the time. This inaction rate about 1.5 times that in the data.\

Table 5 displays the results. The middle panel of the table shows that the disruption cost does not trigger declines in employment in the quarter in which the installation is undertaken. However, since capital is not operative until the next period, the growth in employment in the installation period is subdued relative to the baseline (see Table 4). This may account for why, at an annual frequency, we do see some instances of employment declines and investment, although the coincidence of the two is still far below that in the data. Suppose the plant fires in quarter one but $x$ then reverses course and by the fourth quarter, it is optimal to hire and invest. Since the new machinery is not operative, the increase in employment is muted in quarter four. This shows through as a larger net reduction in employment for the year.

3.2 Factor-biased technical change

The coincidence of investment and (net) separations brings to mind the notion of factor-biased technical change. However, we argue that relatively standard implementations of factor-biased technical change do not induce the incidence of investment and separations that we observe in the data. These production functions do induce, conditional on output, a shift away from labor after a technology improvement. However, we find that the scale effect of an increase in technology dominates unless demand is highly inelastic. Thus, in the context of these standard models, one may view the co-movement of capital and labor in one of two ways. It may be treated as a moment which informs the estimation of the demand schedule and the mark-up, in which case typical estimates of the mark-up used in macroeconomic analysis appear very low. Alternatively, if the implied mark-up clashes

\[\text{Such a low value of } \tau \text{ is in conflict with Cooper and Haltiwanger (Table 5, 2006). But they did not target the adjustment frequency.}\]

\[\text{On the other hand, if the plant installs machinery in quarter 2, there will be a large increase in employment in the next quarter. This reflects the lumpy change in the marginal product due to (discrete) machinery investment. As a result, Table 5 shows that the average annual change in employment, conditional on positive investment, is around 30 log points. We could ratchet down the size of TFP shocks to bring this more closely in line with the data. However, this comes at a price: we would further understate the unconditional dispersion in employment growth.}\]
with reliable external evidence, it may be interpreted as a challenge to canonical models of technical change.

As one arbitrates between these two views, it is important to note that the product demand elasticity to which we refer (throughout this section) is the one that applies to idiosyncratic changes in the plant’s output. It may be the case that the elasticity of (product) substitution across industries is very low (and this does not have to imply any market power at the plant level). However, in our data, the vast majority of variation in factor inputs does not appear to be due to industry-wide shocks; all of our empirical results hold within narrowly defined sectors. Thus, we interpret the incidence of negative co-movement between machinery and employment as largely driven by idiosyncratic shocks to plants. In other words, it does not seem that this pattern is due to a common shock that drives all plants up a steeply sloped industry demand curve. As a result, the relevant demand elasticity is that which operates at the plant level. It follows that a very low elasticity implies very high mark-ups.

We are skeptical of such high mark-ups and so consider an alternative model of factor-biased technology. In this framework, the final good is produced by aggregating over the outputs of many tasks. The firm allocates machinery to a subset of tasks and labor to the remainder. If there an increase in the productivity of machinery in this model, labor is directly replaced in the marginal task by capital. As we will see, this form of factor bias will enable the model to replicate the coincidence of investment and (net) separations.

**Factor-augmenting technology in CES production.** We first consider a model of CES production in which there is labor-augmenting technical change. The latter can, in theory, be labor-saving and thus capable of inducing a simultaneous increase in capital and reduction in employment. However, for a wide range of parameter values, labor-augmenting innovations are not contractionary with respect to labor.47

To introduce labor-augmenting technology in a meaningful way, we must depart from the Cobb-Douglas production function used above. Instead, we specify that production is given by the CES function,

\[ y = \left( \varphi k^{\frac{\varphi-1}{\varphi}} + (1 - \varphi) (\xi n)^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}}, \tag{4} \]

where \( \varphi \) is the elasticity of substitution across the factors, \( \xi \) is labor-augmenting technical change, and \( \varphi \) is related to capital’s share of output. This function displays constant returns, under which plant size is indeterminate. To have a notion of plant size, we assume the firm

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47Capital-augmenting technical change is arguably even less promising as a means of inducing investment and employment contraction, as we discuss below.
faces an isoelastic product demand schedule given by

\[ y = \zeta p^{-\epsilon}. \]  

(5)

where \( \zeta \) is the demand shifter and \( p \) is the price of the plant’s product. It follows that the value of the firm becomes

\[
= \max_{k,n} \left\{ \frac{\Pi(k_{-1},n_{-1},\xi,\zeta)}{\zeta^{1/\epsilon} y^{1-\epsilon} - wn - C_k(k_{-1}) - C_n(n_{-1})} \right. \\
\left. + \rho \int \int \Pi(k,n,\xi',\zeta') f_{\xi}(\xi'|\xi) f_{\zeta}(\zeta'|\zeta) \, d\xi d\zeta, \right. 
\]  

(6)

where, for any \( \chi \), \( f_\chi \) is the conditional p.d.f. of \( \chi \). It is useful to assume, as we did above with regard to the Hicks-neutral technology, that the demand shifter follows a geometric random walk. Then the problem is homogeneous of degree one in \((k,n,\xi,\zeta)\). This allows us to normalize the factors with respect to \( \zeta \), giving \( \bar{n} \equiv n/\zeta \) and \( \bar{k} \equiv k/\zeta \), and recast the problem in terms of \((\bar{k},\bar{n},\bar{\xi})\). Thus, the policy rules given in section 1 (and plotted in Figure 1) hold here, conditional on \( \zeta \).

To gauge how well this model matches the plant-level co-movement of capital and labor, it is instructive to consider a simple exercise. Suppose a plant is now investing and firing, so its first-order conditions corresponding to these choices are in effect. Then if \( \xi \) is labor-saving, these FOCs must remain in effect after an increase in \( \xi \), implying an increase in \( k \) and a decrease in \( n \). To pursue this point, consider the version of (6) with static expectations, i.e., the firm does not anticipate that \( \zeta \) or \( \xi \) will evolve from current levels. In that case, one can differentiate the FOCs for investing and firing to obtain the comparative static,

\[
\frac{d \ln n}{d \ln \xi} = (\epsilon - 1) - (\epsilon - \varphi) \frac{s}{s + 1},
\]

where \( s \) is the ratio of capital income to labor-related payments (the wage bill plus the amortized cost of firing). This is negative only if

\[
\epsilon - 1 < (1 - \varphi) s.
\]  

(7)

This says, in part, that the elasticities of demand and substitution must be sufficiently small. When \( \varphi \) is near zero, the factors are highly complementary, so an increase in the effective supply of labor induces the plant to increase capital relative to labor. This can involve an absolute reduction in labor. However, at the same time, an increase in \( \xi \) reduces the marginal cost of production and the product price. This yields an increase in sales. If
demand is sufficiently inelastic ($\epsilon$ is small), though, this scale effect is muted. As a result, (7) is more likely to hold, and labor declines.

Equation (7) provides a hint as to the difficulty that (4) has in generating contractionary technology improvements. The range of $\varphi s$ that satisfy (7) is increasing in $s$ and declining in $\epsilon$. But even for polar values of $s$ and $\epsilon$, it is difficult to make $\frac{d \ln n}{d \ln N}$ turn negative. Table 6 illustrates this. To interpret the table, note that equation (7) implies that the elasticity of substitution, $\varphi$, must satisfy $\varphi < \hat{\varphi} \equiv 1 - \frac{1}{s}$. The table shows the values of the threshold, $\hat{\varphi}$, that correspond to values of $\epsilon$ listed along rows and $s$ listed along columns. Since $\varphi$ is bounded below by zero, only the shaded values of $\hat{\varphi}$ are feasible. Hence, if $s = 0.8$, which is the mean among Korean manufacturers, the product elasticity, $\epsilon$, would have to be less than 2 to even be consistent with (7). This is well below estimates in the literature.\footnote{According to the model, we would like to measure the denominator in $s$ by summing the wage bill and the amortized cost of firing. However, the data do not include the latter. Thus, the empirical estimate of $s = 0.8$ likely overstates the value of $s$ that is relevant for evaluating (7).}

Of course, the ratio $s$ is likely to vary across plants. Perhaps the coincidence of positive investment and (net) separations in the data reflects the presents of plants that are relatively capital intensive. Unfortunately, it is hard to investigate this in our data since we do not have detailed information on capital income.\footnote{For the U.S., the Bureau of Labor Statistics does provide industry-level data on rental prices. These data reveal that the average value of $s$ is less than one in 14 of 16 three-digit NAICS manufacturing industries. This limits the degree to which plausible variation in $s$ could account for the degree of negative co-movement between capital and labor documented by Sakelleris (2000) on U.S. data.} What we can say is that, if $s$ were higher, one must still invoke what seem like low values of $(\epsilon, \varphi)$ to make (7) hold. For example, suppose $s = 1.6$, or twice its empirical average. We are not aware of plant-level estimates of $\varphi$, but a value of 0.1 brings us close to Leontief and arguably serves as a reasonable lower bound. In that case, one needs values of $\epsilon$ below 2.44 in order to make (7) hold. This is less than the estimates in Broda and Weinstein (2006, Table IV, “Median” row, “TSUSA/HTS” column). Moreover, in this model, an $\epsilon$ of 2.44 implies a markup of almost 70 percent. This conflicts with recent evidence from Feenstra and Weinstein (2010), who estimate a richer translog demand system and uncover a median mark-up of around 30 percent.\footnote{Both of these papers use detailed data on U.S. imports. In particular, the Broda and Weinstein estimate is based on 10-digit Harmonized Tariff System data. We highlight these estimates, since the more disaggregated data are arguably more indicative of the elasticities faced by individual plants.}

We now demonstrate that our intuition gleaned from (7) is highly indicative of the behavior of the dynamic factor demand model. To do this, we set $\epsilon = 2.44$, $\varphi = 0.1$, and $s = 1.6$ and simulate plant-level data on investment and employment growth.\footnote{For this exercise, we omit capital disruption costs. Thus, we re-calibrate the resale price of capital and the cost of labor adjustment to target the individual frequencies of adjusting.}
ports the results. Positive investment and negative employment growth hardly ever coexist in the simulated data.

Before we leave this subsection, we note that a model of capital-augmenting technical change is even less effective at engaging the data. To see this, suppose production is now given by

\[ y = \left( \vartheta (\xi k)^{\frac{\varphi - 1}{\varphi}} + (1 - \vartheta) n^{\frac{\varphi - 1}{\varphi}} \right)^{\frac{\varphi}{\varphi - 1}}. \]  

(8)

Rearranging this, we may write

\[ y = n^{\left( \vartheta (\Gamma)^{\frac{\varphi - 1}{\varphi}} + (1 - \vartheta) \right)^\varphi}, \]

where \( \Gamma \equiv \frac{\xi k}{n} \) is the effective capital-labor ratio. This indicates that the effect of an increase in \( \xi \) on labor demand will hinge on the reaction of the marginal revenue product of labor, \( MRPL \), to a change in \( \Gamma \). Specifically, the firm’s first-order conditions imply\(^{52}\)

\[ \frac{d\ln n}{d\ln \xi} = (\epsilon - \varphi) \frac{s}{s + 1} \propto \frac{d\ln MRPL}{d\ln \Gamma}. \]  

(9)

If \( \epsilon > \varphi \), \( \frac{d\ln n}{d\ln \xi} \) is unambiguously positive.

The condition on \( \epsilon - \varphi \) is intuitive, since \( \epsilon > \varphi \) implies the factors are q-complements. This accounts for why, under \( \epsilon > \varphi \), an increase in the effective supply of capital raises labor demand. This restriction is in fact upheld if one views the data through the lens of this model. Raval (2012) estimates this CES model off U.S. micro data for each two-digit manufacturing sector. Virtually all of his estimates of \( \varphi \) are less than 1. Therefore, acknowledging the obvious caveats, we view the restriction \( \epsilon - \varphi > 0 \) as a reasonably safe one to make.

Investment-specific technical change & skill bias. Thus far, we have assumed only two factors. Within the skill-biased technical change, literature, though, it is common to allow for skilled and unskilled labor, such that the capital and skilled labor are gross complements whereas capital and unskilled labor are gross substitutes. Krusell, Ohanian, Rios-Rull, and Violante (2000) consider this kind of production function. Technical change in their model takes the form of advances in the rate at which investment is transformed into installed capital.

It is difficult to find support for this explanation in our data. The model predicts that employment declines should be concentrated among low-skilled workers. But the evidence for this is far from clear. In the Korean census, plants record the number of production and non-production workers. In the literature on skill-biased technical change, non-production

\(^{52}\)We again derive the comparative static under the assumption of static expectations.
status is often treated as a proxy for skill.\textsuperscript{53} Therefore, we re-compute the share of plant-year observations that involve both positive investment and declines in non-production workers. This is 35.5 percent, which is only slightly less than the baseline estimate reported in Table 1. Moreover, conditional on positive investment and negative non-production employment growth, the average contraction in non-production workers is not any smaller than the decline among the general workforce reported in Table 1.\textsuperscript{54}

These results are not too surprising in light of what the literature has learned about the impact of technology on the workplace. Tasks performed by clerical and administrative workers, who would be classified as non-production employees in our data, have become increasingly codified and performed by machines (Acemoglu and Autor, 2010). This suggests that it is not only workers on the factory floor for whom machines may substitute.\textsuperscript{55} Accordingly, we adopt a different formalization of the notion that machines substitute for workers. We now discuss this in detail.

**Machinery replaces labor in tasks.** There has been growing interest in models in which machinery directly replaces labor in tasks. As we will show, this model has a greater capability of generating the incidence of investment and separations observed in the data. Our analysis here is based on the framework of Acemoglu (2010) (who in turn adapts the model of Zeira (1998)).\textsuperscript{56}

The plant produces a single good that is subject to the demand schedule (5). The good is produced by aggregating the outputs of a continuum of tasks. A task is indexed by $v$ on the unit interval. We assume plant output is then given by the CES aggregate,

$$y = \left[ \int_0^1 y(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

\textsuperscript{53}See, e.g., Berman, Bound, and Machin (1998) and Berman and Machin (2000).

\textsuperscript{54}Nonetheless, there has been an upward trend in the non-production worker share in our data. Fuentes and Gilchrist (2005) also find such an upward trend in Chile even though the non-production worker share in their dataset does not tend to increase in periods of investment spikes. They argue that the source of the secular trend was a shift in favor of R&D-related personnel after trade liberalization.

\textsuperscript{55}The evidence for the prevalence of computing equipment is typically taken from the U.S. and European countries. But South Korea is a relatively advanced economy by this measure. If we take the volume of exports and imports of computer equipment (per capita) as indicative of pervasiveness of this technology, then Korea ranks on par with several Western European and North Atlantic economies (see OECD, 2008).

\textsuperscript{56}The analysis in this section describes a version of this model in which the plant’s problem of allocating labor and capital to tasks is static (that is, it is costless to reallocate labor and capital across tasks). This is not the most promising version of the model which we have implemented. As described in the introduction, a more promising version of this model allows for factor reallocation to be costly. This version will be written up shortly.
where $y(v)$ is the output of task $v$. A task may be performed by labor or machinery. Specifically, one unit of output in task $v$ can be produced with $\lambda(v)$ units of labor or $\kappa(v)$ units of machinery:

$$y(v) = \begin{cases} \frac{n(v)}{\lambda(v)}, & \text{or} \\ \frac{k(v)}{\kappa(v)}, & \end{cases}$$

We assume tasks may be ordered from 0 to 1 such that tasks at which machines are relatively productive are ordered first. Therefore, the relative productivity of labor, $\theta(v) := \frac{1/\lambda(v)}{1/\kappa(v)} = \frac{\kappa(v)}{\lambda(v)}$, is increasing in $v$.

The plant’s optimization problem can be solved in two steps. First, given total workers $n$ and machinery $k$, the firm chooses which tasks to perform with capital and which to perform with labor. It also decides the quantities of each factor to allocate to its respective tasks. These are static problems. Second, the firm solves a dynamic problem to select $n$ and $k$, subject to the adjustment costs described in equation (1). We now briefly summarize the first step.

Formally, the allocation of factors across tasks solves the following problem. (The details of the solution are provided in Appendix B.) The monotonicity of $\theta(v)$ suggests the existence of a single crossing, $v^*$, such that all tasks $v \leq v^*$ are automated and all other tasks are performed by labor. Taking this as given for the time being, the firm’s problem is to decide the quantities of each factor to allocate per task in order to maximize revenue. Hence, given (5) and (10), the firm solves

$$\max_{\{y(v)\}_{v=0}^1} \int_0^1 y(v) \frac{x^v - 1}{x^v - 1} dv$$

subject to the resource constraints,

$$k = \int_0^{v^*} k(v) dv = \int_0^{v^*} \kappa(v) y(v) dv$$

$$n = \int_0^{v^*} n(v) dv = \int_0^{v^*} \lambda(v) y(v) dv.$$  

Here, $n(v)$ denotes the quantity of labor allocated to task $v$, and $k(v)$ represents the number of machines allocated to task $v$.

The solution to this problem gives the optimal quantity $y(v)$ of each intermediate in

\footnote{That is, we imagine a single type of worker and a single machine. This same machine is used in each task, but, as we will see, it is relatively more productive in some tasks.}
terms of \( k, n, \) and \( v^* \). We are then able to rewrite the plant-wide production function (10) exclusively in terms of the total stocks, \( k \) and \( n \), and the threshold, \( v^* \). Specifically, one can show that

\[
y = \left[ A(v^*)^{1/\phi} k^{\phi-1} + B(v^*)^{1/\phi} n^{\phi-1} \right]^{\phi^{-1}},
\]

where the terms, \( A(v^*) \), and \( B(v^*) \), are defined by

\[
A(v^*) \equiv \int_0^{v^*} \left( \frac{1}{x(v)} \right)^{\phi-1} dv
\]

\[
B(v^*) \equiv \int_{v^*}^1 \left( \frac{1}{x(v)} \right)^{\phi-1} dv.
\]

To understand these terms, recall that \( \frac{1}{x(v)} \) is output per machine in task \( v \). Hence, \( A(v^*) \) is an aggregate over the average products of machinery on the tasks to which machinery is applied. In this sense, it may be interpreted as an index of capital productivity. The same idea applies, of course, to \( B(v^*) \).

The indexes, \( A \) and \( B \), differentiate (12) from the standard models of factor-augmenting technical change. For instance, in the canonical model of capital-augmenting technology (see (8)), an increase in the productivity of capital does not imply a degradation, in absolute terms, in the contribution of labor to output. In other words, there is no decline in the analogue of \( B \) in that model. However, when production is given by (12), an increase in the threshold, \( v^* \), leads to a replacement of labor in tasks, which implies a greater contribution per unit of (plant-wide) capital, \( k \), and a smaller contribution per worker. This deterioration in labor productivity amplifies the effective bias of technical change.

Thus far, we have taken \( v^* \) as given, but it is straightforward to determine the threshold. Cost minimization implies that the firm uses machinery for task \( v \) if the marginal cost of machinery is less. The marginal cost is evaluated using the shadow price of capital to the firm, conditional on the total stock \( k \). Since this shadow price is declining in \( k \), it is not surprising that the threshold task is increasing in the capital-labor ratio. Formally, as demonstrated in Appendix B, \( v^* \) satisfies

\[
\frac{k}{n} = \theta(v^*)^{\phi} \frac{A(v^*)}{B(v^*)} \equiv \Theta(v^*)
\]

The right side of (13) is increasing in \( v^* \), implying a single-crossing. Equation (13) is intuitive, as it says that the optimal allocation of factors across tasks requires that the firm executes a greater share of tasks with the more abundant factor.

It should be noted at this point that the solution (13) assumes a frictionless reallocation of factors within plant across tasks. This abstracts from a number of issues, but we do not believe this omission invalidates the application of (12) to problems of plant-level dynamics.
For instance, machinery is likely specific, to some degree, to the task. This raises the question of whether we ought to require the plant to purchase new machinery when capital is deployed to new tasks. This is arguably realistic, but it is not clear to us that it would significantly affect the model’s principal predictions. In addition, the model allows the manager to reallocate workers from tasks below $v^*$ to tasks above the threshold. Of course, in reality, a worker on one task may not have the skills to perform others. But this would likely amplify the depressive effects of capital-specific technology on labor demand.\footnote{We have considered a version of the model that exploits the fact that equation (12) is valid for any $v^*$. This version interprets $v^*$ – the allocation of tasks – as a separate control variable subject to adjustment costs. The idea here is that a shift in $v^*$ represents a change in organizational capital. To illustrate, imagine the firm can “transform” the task content of machinery if it makes complementary investments in plant organization and techniques. For instance, it may have to train workers who use the intermediate input produced by a new machine to de-bug the new equipment. This model seems descriptively more realistic, but its implications for the co-movement between capital and labor are similar. However, the model of organizational capital might avoid some of the unpleasant implications of the tasks framework for the volatility of the labor income share, as long as the workers (who remain following layoffs) can appropriate some of the return from the organizational capital.}

To study the short-run dynamics of factor demand, we now introduce exogenous variation in the (relative) productivity of machines. We do this in a way that preserves continuity with the standard model of capital-augmenting technology, (8). This allows us to more easily compare results across the two specifications. Assume $\nu(v)$ takes the form

$$\nu(v) = \xi^{-1} \tilde{\nu}(v),$$

(14)

where $\tilde{\nu}$ is a deterministic, increasing function of $v$, and $\xi$ is an innovation to machinery productivity. Note that an increase in $\xi$ raises the productivity of machinery on all tasks.

Equation (14) implies that $A(v^*)$ in becomes $\xi^{\phi-1} \tilde{A}(v^*)$, where $\tilde{A}(v^*) \equiv \int_0^{v^*} \tilde{\nu}(v) \frac{1}{1-\phi}$. Likewise, we may write $\Theta(v^*)$ in (13) as $\xi^{-1} \tilde{\Theta}(v^*)$, where $\tilde{\Theta}(v^*) \equiv \left( \frac{\tilde{\nu}(v^*)}{\tilde{\lambda}(v^*)} \right)^{\phi} \frac{\lambda(v^*)}{B(v^*)}$. It then follows that

$$v^* = \tilde{\Theta}^{-1} \left( \frac{\xi k}{n} \right).$$

(15)

We see that a rise in $\xi$ motivates the substitution of machines for labor in a greater share of tasks.\footnote{This discussion omits shifts in labor-augmenting productivity. But if both factor-augmenting shocks take the form in (14), then one may normalize with respect to the labor productivity shock. To be precise, suppose $\lambda(v) = \Xi^{-1} \tilde{\lambda}(v)$, where $\Xi$ is labor-augmenting productivity. Then $B$ becomes $B(v^*) \equiv \Xi^{\phi-1} B(v^*)$ and (13) implies that the threshold task, $v^*$, is given implicitly by $\frac{(\xi/\Xi)k}{n} = \tilde{\Theta}(v^*)$. Hence, it is only the ratio of $\xi$ to $\Xi$ that affects the selection of the threshold task.}
Equations (12) and (15) imply that the production function for plant-wide output is now

\[ y = \left[ a \left( \frac{\xi k}{n} \right)^{\frac{1}{\varphi}} (\xi k)^{\frac{\varphi-1}{\varphi}} + b \left( \frac{\xi k}{n} \right)^{\frac{1}{\varphi}} n^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}. \] (16)

where \( a \left( \frac{\xi k}{n} \right) \equiv \tilde{A} \left( \tilde{\Theta}^{-1} \left( \frac{\xi k}{n} \right) \right) \) and \( b \left( \frac{\xi k}{n} \right) \equiv B \left( \tilde{\Theta}^{-1} \left( \frac{\xi k}{n} \right) \right) \) are increasing and decreasing functions, respectively. With (16) in hand, it is straightforward, in principle, to solve the dynamic factor demand problem. We merely replace the standard CES production function in (6) with (16).

To implement the model quantitatively, we need to specify, of course, functional forms for the factor-augmenting terms in (16). We are aided here by a normalization: one may show that the solution to the static task allocation problem hinges, perhaps not surprisingly, only on the relative productivity of the factors. Thus, we may fix \( \lambda(v) = 1 \) and reinterpret \( \kappa(v) \) as the productivity of machinery on task \( v \) relative to that of labor. It then remains to parameterize \( \kappa \). Unfortunately, we simply do not have rich enough data to identify a flexible form of this. To illustrate the model’s potential, we therefore assume a simple power function, \( \tilde{\kappa}(v) = v^\psi \).\(^{60}\) It follows that \( \tilde{A}(v^*) = \frac{\kappa(v^*)^\psi(1-\psi) + 1}{\psi(1-\psi) + 1} \) and \( B(v^*) = 1 - v^* \).

The implications of the parameter, \( \psi \), for joint factor adjustment are intuitive. In particular, if this is small (\( \kappa \) is relatively “flat” over much of its domain), then the productivity of capital falls off slowly as it is deployed to additional tasks. This implies that a given impulse \( \xi \) triggers a more substantial substitution of machinery for labor. We have found that (conditional on the stochastic process for \( \xi \), discussed below) \( \psi = 0.225 \) enables the model to largely replicate both the frequency of investment and separations as well as the average employment decline in these episodes.

A few other structural parameters merit comment. They are \( \varphi \), the parameters that govern the stochastic process, \( \xi \), and the innovation variance of the demand (neutral) shock. Given the functional form for \( \kappa \) and interaction between \( \kappa \) and \( \varphi \) in the production technology (16), it is difficult to separately identify \( \varphi \). We have solved the model under a number of values of \( \varphi \) between one and three and found that variations of this order of magnitude have little effect on the co-movement of the factors given \( \psi \) in the neighborhood of 0.225. The results presented in Table 7 assume \( \varphi = 2.5 \).

In order to retain comparability with the treatment of technical change in the standard models of section 3.1, we assume that \( \xi \) follows a geometric AR(1) process. We select

\(^{60}\)The absence of a constant in front of \( v^\psi \) is unimportant: we may select the real wage in order to target the empirical capital cost share. Thus, the intercept in \( \kappa \) can be normalized to 1.
the persistence so that the employment declines, conditional on investment, are broadly consistent with the empirical persistence reported in section 2. An AR(1) coefficient of 0.9 achieves this. The size of the innovation to $\xi$, relative to the size of factor-neutral innovation, has clear implications for both the frequency of negative co-movement and the size of employment losses in these episodes. Together with $\psi = 0.225$, we find that the data call for a standard deviation of the factor-bias innovation equal to 0.0725 and a standard deviation of the demand (factor-neutral) innovation equal to 0.125.$^{61}$

Table 7 reports the results. The share of observations that involve a decline in employment, conditional on positive investment, now nearly replicates that observed in the data. The model also reproduces the size of the decline in these episodes. At the same time, the presence of a factor-neutral (demand) shock, $\zeta$, allows the model to induce, at other instances, realistically large expansions in employment conditional on positive investment. Figure 5 illustrates the results graphically. It plots the distribution of employment growth conditional on positive investment in both the data and model. This represents a stark improvement relative to the baseline model (see Figure 3).

4 Conclusion

This paper has studied the joint dynamics of capital and employment at the plant level. We find that employment often declines when investment is undertaken. Moreover, these episodes account for a significant fraction of aggregate capital accumulation. Clearly, it is important to understand the plant-level dynamics behind this negative co-movement in order to obtain a complete accounting of the driving forces and mechanisms behind aggregate investment.

To this end, we explore a number of possible explanations for our findings. In the end, we conclude that the data on joint factor adjustment suggest a relatively new model of the plant’s production technology, in which machinery directly replaces labor in certain tasks. Put another way, just a few moments relating to the joint factor dynamics help substantially in identifying the environment in which manufacturers operate. This argues, more generally, in favor of using explicit, dynamic theories of input demands to uncover information about technology (Gorodnichenko, 2012).

$^{61}$It seems awkward to think of negative shocks to machinery productivity in this context, since the trend, in reality, has been toward the steady replacement of labor by machinery. But remember that $\xi$ is more accurately interpreted as the productivity of machinery relative to labor. We assume that there may be innovations to how labor is used in production that imply at least short run declines in $\xi$. 

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5 References


Krusell, Per, Lee Ohanian, José-Víctor Ríos-Rull and Giovanni Violante (2000). “Capital-


TO BE ADDED:
Appendix A: Results from Chilean data
Appendix B.1: Baseline simulation model
Appendix B.2: Details of derivations
Appendix C: Robustness analysis
Figure 1: The optimal policy

Figure 2: The joint dynamics of capital and employment
Figure 3: The distribution of employment growth in the data and baseline model

A. Data

B. Baseline model

NOTE: The left panel shows the distribution of employment growth in Korean data. The solid (green) line is the kernel-density estimate of the unconditional distribution, and the dashed (orange) line is the distribution conditional on an investment rate in excess of 10 percent. The right panel shows the distributions in the baseline model of section 1.
Figure 4: The distribution of employment growth in the data and task-based model

A. Data

B. Model

NOTE: The left panel shows the distribution of employment growth in Korean data. The solid (green) line is the kernel-density estimate of the unconditional distribution, and the dashed (orange) line is the distribution conditional on an investment rate in excess of 10 percent. The right panel shows the distributions in the task-based model discussed in section 3.2.
Table 1: Factor adjustment in Korea

**Panel A: Frequency and magnitude of individual factor adjustments**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Share of plant-year obs. for which $\frac{I}{K-1} = 0$</td>
<td>0.47</td>
</tr>
<tr>
<td>Share of plant-year obs. for which $\Delta \ln n = 0$</td>
<td>0.135</td>
</tr>
<tr>
<td>Standard deviation of $\frac{I}{K-1}$ (pooled across plants and time)</td>
<td>0.326</td>
</tr>
<tr>
<td>Standard deviation of $\Delta \ln n$ (pooled across plants and time)</td>
<td>0.266</td>
</tr>
<tr>
<td>Share of plant-year obs. with positive gross sales of machinery</td>
<td>0.143</td>
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<tr>
<td>Share of plant-year obs. with positive gross purchases given gross sales</td>
<td>0.746</td>
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**Panel B: Joint factor adjustment**

<table>
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<tr>
<td>Share of plant-year obs. in which $\Delta \ln n &lt; 0$, given $\frac{I}{K-1} &gt; 0$</td>
<td>0.395</td>
</tr>
<tr>
<td>Average decrease in $\ln n$, given $\Delta \ln n &lt; 0$ and $\frac{I}{K-1} &gt; 0$</td>
<td>0.19</td>
</tr>
<tr>
<td>Average increase in $\ln n$, given $\Delta \ln n &gt; 0$ and $\frac{I}{K-1} &gt; 0$</td>
<td>0.214</td>
</tr>
</tbody>
</table>
Table 2: The coincidence of positive investment and (net) negative employment growth

**Panel A: By two-digit industry**

\[ \text{Share of plant-year obs. in which } \Delta \ln n < 0, \text{ given } \frac{I}{K_{t-1}} > 0 \]

Minimum among industries: 0.374 (“Other machinery”)
Median among industries: 0.419 (“Computer and office equipment”)
Maximum among industries: 0.458 (“Apparel”)

**Panel B: By plant size**

\[ \text{Share of plant-year obs. in which } \Delta \ln n < 0, \text{ given } \frac{I}{K_{t-1}} > 0 \]

Plants w/ 10-24 workers: 0.358
Plants w/ 25-50 workers: 0.393
Plants w/ 51-99 workers: 0.412
Plants w/ 100+ workers: 0.476
Table 3: Baseline calibration

<table>
<thead>
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<th>Meaning</th>
<th>Value</th>
<th>Reason</th>
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<td>$\beta$</td>
<td>Elasticity of output wrt $n$</td>
<td>0.50</td>
<td>Labor share</td>
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<tr>
<td>$\alpha$</td>
<td>Elasticity of output wrt $k$</td>
<td>0.40</td>
<td>Capital share</td>
</tr>
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<td>$\rho$</td>
<td>Discount factor</td>
<td>0.9875</td>
<td>Annual real interest rate = 5.2%</td>
</tr>
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<td>$c_{k}^{+}$</td>
<td>Purchase price of capital</td>
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<td>Normalization</td>
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<td>$c_{k}^{-}$</td>
<td>Resale price of capital</td>
<td>0.923</td>
<td>Frequency of investment</td>
</tr>
<tr>
<td>$c_{n}^{+, -}$</td>
<td>Cost to hire &amp; fire</td>
<td>33% of monthly wage</td>
<td>Frequency of net employment adj.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Std. dev. of innovation to $x$</td>
<td>0.15</td>
<td>Avg. increase in $\ln n$, given $I/K_{-1} &gt; 10%$</td>
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</tbody>
</table>
Table 4: Empirical and simulated moments: The baseline model

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob($\Delta n = 0$):</td>
<td>$\delta_n = 0$</td>
<td>$\delta_n &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>0.148</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>0.135</td>
<td></td>
</tr>
<tr>
<td>Prob($\frac{I}{K_{-1}} = 0$):</td>
<td>0.468</td>
<td>0.438</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Std dev of $\Delta \ln n$ :</td>
<td>0.16</td>
<td>0.176</td>
</tr>
<tr>
<td></td>
<td>0.266</td>
<td></td>
</tr>
<tr>
<td>Prob($\frac{I}{K_{-1}} &lt; 0$):</td>
<td>0.04</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td>Share of qtly. obs. w/ $\Delta n &lt; 0$ given $\frac{I}{K_{-1}} &gt; 0$:</td>
<td>0</td>
<td>0.026</td>
</tr>
<tr>
<td>Avg. qtly. decrease in $\ln n$ given $\frac{I}{K_{-1}} &gt; 0$:</td>
<td>n.a.</td>
<td>-0.005</td>
</tr>
<tr>
<td>Avg. qtly. increase in $\ln n$ given $\frac{I}{K_{-1}} &gt; 0$:</td>
<td>0.114</td>
<td>0.108</td>
</tr>
<tr>
<td>Share of yrly. obs. w/ $\Delta n &lt; 0$ given $\frac{I}{K_{-1}} &gt; 0.1$:</td>
<td>0.022</td>
<td>0.012</td>
</tr>
<tr>
<td>Avg. yrly. decrease in $\ln n$ given $\frac{I}{K_{-1}} &gt; 0.1$:</td>
<td>-0.019</td>
<td>-0.016</td>
</tr>
<tr>
<td>Avg. yrly. increase in $\ln n$ given $\frac{I}{K_{-1}} &gt; 0.1$:</td>
<td>0.212</td>
<td>0.213</td>
</tr>
</tbody>
</table>

NOTE: The top panel presents moments related to the annual distribution of investment and employment growth across plants. The moments (in this order) are the probability of not adjusting employment; the probability of not adjusting capital; the standard deviation of the distribution of the annual change in log employment; and the share of observations that involve disinvestment. The second panel presents three moments related to the quarterly distribution of employment growth, conditional on positive investment. The third panel presents those same three moments, computed this time from the annual (yearly) distribution of employment growth. Of the two columns related to the model, the first pertains to the case with no worker attrition. The second assumes a quarterly attrition rate of 4%.
Table 5: Sensitivity analysis – non-technological theories

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delivery lag</td>
<td>Disruption cost</td>
</tr>
<tr>
<td>Prob(Δn = 0):</td>
<td>0.06</td>
<td>0.063</td>
</tr>
<tr>
<td>Prob(1/K−1 = 0):</td>
<td>0.503</td>
<td>0.729</td>
</tr>
<tr>
<td>Std dev of Δ log n:</td>
<td>0.134</td>
<td>0.185</td>
</tr>
<tr>
<td>Prob(1/K−1 &lt; 0):</td>
<td>0.032</td>
<td>0.048</td>
</tr>
</tbody>
</table>

| Share of qtly. obs. w/ Δn < 0 given 1/K−1 > 0: | 0.02 [orders] | 0 | n.a. |
| Avg. qtly. decrease in ln n given 1/K−1 > 0: | -0.003 [orders] | n.a. | n.a. |
| Avg. qtly. increase in ln n given 1/K−1 > 0: | 0.059 [orders] | 0.03 | n.a. |

| Share of yrly. obs. w/ Δn < 0 given 1/K−1 > 0.1: | 0.125 | 0.108 | 0.395 |
| Avg. yrly. decrease in ln n given 1/K−1 > 0.1: | -0.035 | -0.029 | -0.19 |
| Avg. yrly. increase in ln n given 1/K−1 > 0.1: | 0.168 | 0.306 | 0.214 |

NOTE: In regards to the definitions of the moments, see the Note to Table 4. The column labeled “Delivery lag” reports results for the model in which there is a quarterly probability of \( \frac{1}{3} \) that a delivery of machinery is made. Note that, in the middle panel, we reported changes in labor conditional on positive orders. This illustrates that, at a quarterly frequency, positive orders and negative employment growth virtually never coincide. In the third panel, we report changes in labor conditional on positive deliveries, which corresponds to annual measured investment in the data. The column “Disruption cost” pertains to the model in which capital adjustment reduces revenue by \( \tau \) percent. Both models are discussed in section 3.1.
Table 6: The maximum feasible values of $\varphi$ in the model of labor-augmenting TFP

<table>
<thead>
<tr>
<th>$c$</th>
<th>0.4</th>
<th>0.8</th>
<th>1.2</th>
<th>1.6</th>
<th>2</th>
<th>2.4</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>-0.25</td>
<td>0.375</td>
<td>0.5833</td>
<td>0.6875</td>
<td>0.75</td>
<td>0.7917</td>
<td>0.8214</td>
</tr>
<tr>
<td>2</td>
<td>-1.5</td>
<td>-0.25</td>
<td>0.1667</td>
<td>0.375</td>
<td>0.5</td>
<td>0.5833</td>
<td>0.6429</td>
</tr>
<tr>
<td>2.5</td>
<td>-2.75</td>
<td>-0.875</td>
<td>-0.25</td>
<td>0.0625</td>
<td>0.25</td>
<td>0.375</td>
<td>0.4643</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>-1.5</td>
<td>-0.667</td>
<td>-0.25</td>
<td>0</td>
<td>0.1667</td>
<td>0.2857</td>
</tr>
<tr>
<td>3.5</td>
<td>-5.25</td>
<td>-2.125</td>
<td>-1.083</td>
<td>-0.563</td>
<td>-0.25</td>
<td>-0.042</td>
<td>0.1071</td>
</tr>
<tr>
<td>4</td>
<td>-6.5</td>
<td>-2.75</td>
<td>-1.5</td>
<td>-0.875</td>
<td>-0.5</td>
<td>-0.25</td>
<td>-0.071</td>
</tr>
<tr>
<td>4.5</td>
<td>-7.75</td>
<td>-3.375</td>
<td>-1.917</td>
<td>-1.188</td>
<td>-0.75</td>
<td>-0.458</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

NOTE: For each pair $(c, s)$, the table reports values of $\varphi = 1 - \frac{c-1}{s}$. In the static version of the model with labor-augmenting technology, the elasticity of substitution between capital and labor, $\varphi$, must be less than $\hat{\varphi}$ in order for technology to be labor-saving. It follows that, since $0 \leq \varphi$, only the cells shaded in gray are feasible.
Table 7: Sensitivity analysis – factor-biased technical change

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob(Δn = 0):</td>
<td>0.155</td>
<td>0.159</td>
</tr>
<tr>
<td>Prob((\frac{I}{K_{-1}} = 0)):</td>
<td>0.50</td>
<td>0.459</td>
</tr>
<tr>
<td>Std dev of Δ log n:</td>
<td>0.139</td>
<td>0.235</td>
</tr>
<tr>
<td>Prob((\frac{I}{K_{-1}} &lt; 0)):</td>
<td>0.0045</td>
<td>0.154</td>
</tr>
<tr>
<td>Share of qtly. obs. w/ Δn &lt; 0 given (\frac{I}{K_{-1}} &gt; 0):</td>
<td>0</td>
<td>0.127</td>
</tr>
<tr>
<td>Avg. qtly. decrease in ln n given (\frac{I}{K_{-1}} &gt; 0):</td>
<td>n.a.</td>
<td>-0.113</td>
</tr>
<tr>
<td>Avg. qtly. increase in ln n given (\frac{I}{K_{-1}} &gt; 0):</td>
<td>0.078</td>
<td>0.101</td>
</tr>
<tr>
<td>Share of yrly. obs. w/ Δn &lt; 0 given (\frac{I}{K_{-1}} &gt; 0.1):</td>
<td>0.032</td>
<td>0.341</td>
</tr>
<tr>
<td>Avg. yrly. decrease in ln n given (\frac{I}{K_{-1}} &gt; 0.1):</td>
<td>-0.033</td>
<td>-0.187</td>
</tr>
<tr>
<td>Avg. yrly. increase in ln n given (\frac{I}{K_{-1}} &gt; 0.1):</td>
<td>0.163</td>
<td>0.2</td>
</tr>
</tbody>
</table>

NOTE: In regards to the definitions of the moments, see the Note to Table 4. The column labeled “Labor-aug. TFP” reports results for the model in which production is given by a CES function with labor-augmenting technology. The column “Task-based” pertains to the model in which plant-level output is the aggregate over intermediate tasks. Both models are discussed in section 3.2.