What do Exporters Know?

Michael J. Dickstein
Eduardo Morales

Stanford University
Princeton University

November 7, 2014
In 2003, Mr. Siegel and a friend started in Sherman Oaks, CA, Orb Audio, a maker of high-end home theater systems.

When the value of the dollar dropped in 2008, Mr. Siegel noticed a boom in the calls and web visits from abroad.

He began marketing with Internet ads and country-specific Web pages aimed at consumers in Britain, Australia, Finland, and Canada.

In 2010, 10 percent of Orb Audio’s business was in Finland.

According to Mr. Siegel, the big volume of exports to Finland can be explained because a couple years ago someone got their speakers and wrote a nice review in an Internet forum.

(information taken from NYT 04/22/2010)
According to U.S. Census, in 2012 there are approximately 300,000 firms that sell to foreign markets.

Firms’ entry decision into export markets explains approximately 70% of the cross-country variation in aggregate US exports (Bernard et al., 2009).

When taking such decision, potential exporters face considerable uncertainty.

Firms enter a market if the expected value of exporting is positive.

Firms’ choice will depend on expectations about the future evolution of variables such as their own productivity, exchange rates, trade policy, political stability in foreign countries, etc.

The impact of currency devaluations and policy changes on exports will depend on firms’ expectations.

Launch-aid subsidies that reduce entry costs into export markets will have no effect on firms’ decisions if they expect to obtain negative profits in any case.
Mr. Siegel’s Problem is Common

- Besides entry into export markets, there are many other firms’ decisions that depend on their expectations about future payoffs:
  - developing a new product (Bernard et al., 2010; Bilbiie et al., 2012).
    - Depends on expectations about future demand.
  - investing in R&D (Aw et al., 2011).
    - Depends on expectations about success of research activity.
  - importing (Blaum et al., 2014, Antràs et al., 2014)

- Consumers also make many decisions that depend on expectations:
  - Enlistment of soldiers in the US Army (Greenstone et al., 2014).
    - Depends on expectations about riskiness of tasks assigned.
  - Retirees’ decisions to purchase a private annuity (Ameriks et al., 2014).
    - Depends on expectations about life expectancy.
  - Buying a durable good (Gowrisankaran and Rysman, 2012).
    - Depends on expectations about timing of future update.

- We can think of all these decisions as single-agent binary choices in which the payoffs are uncertain at the time at which choices are made.
In this paper

- In this paper, we focus on exporters’ entry decision into foreign markets as an example of a single-agent binary decision.

- Using a standard partial equilibrium two-period model of export participation, (CES demand, constant marginal cost, and monopolistic competition) and firm-level data, we
  - estimate export entry costs for 22 countries;
  - compute the predicted effect on export participation and aggregate exports of a subsidy that decreases export entry costs in 40%.


- This previous literature assumes that the researcher has full or perfect knowledge of the content of firms’ information sets at the time of deciding whether to enter a foreign market.
First, we show that the estimates of export entry costs and the model-based predicted effect on firms’ entry and aggregate exports of a reduction in these costs are sensitive to the assumptions imposed on agents’ information sets.

Estimates of export entry costs (in 2000 USD):

<table>
<thead>
<tr>
<th>Exporters’ Information</th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Foresight</td>
<td>894,031</td>
<td>1,736,934</td>
<td>2,796,218</td>
</tr>
<tr>
<td>Dist. + Lag. Sales + Lag. Exports</td>
<td>593,845</td>
<td>1,174,016</td>
<td>1,903,122</td>
</tr>
</tbody>
</table>

Effect on aggregate exports of decreasing export entry costs in 40%:

<table>
<thead>
<tr>
<th>Exporters’ Information</th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Foresight</td>
<td>40.50%</td>
<td>53.07%</td>
<td>163.36%</td>
</tr>
<tr>
<td>Dist. + Lag. Sales + Lag. Exports</td>
<td>42.91%</td>
<td>40.50%</td>
<td>192.31%</td>
</tr>
</tbody>
</table>
In this paper

- **Second**, we use two new types of moment inequalities,
  - (1) generalized revealed-preference and (2) odds-based inequalities,
  to identify the value of export entry costs and the predicted effect on firms’ entry and aggregate exports of a reduction in these costs, while allowing for **partial knowledge** of exporters’ information sets by the researcher.

- Estimates of export entry costs:

<table>
<thead>
<tr>
<th>Exporters’ Information</th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least</td>
<td>(269,992, 298,153)</td>
<td>(592,552, 632,030)</td>
<td>(977,631, 1,061,959)</td>
</tr>
<tr>
<td>Dist. + Lag. Sales + Lag. Exports</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Effect on aggregate exports to U.S. of decreasing export entry costs in 40%:

<table>
<thead>
<tr>
<th>Exporters’ Information</th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least</td>
<td>(77 M, 86 M)</td>
<td>(100 M, 107 M)</td>
</tr>
<tr>
<td>Dist. + Lag. Sales + Lag. Exports</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this paper

- **Third**, we use our moment inequalities to learn about exporters’ true information sets.

- We can test the null hypothesis that a set of variables observed by the researcher is a subset of the information set of the exporters.

  **Test 1:**  \( H_0 : \left\{ \text{exporters at least know dist.} \right. \) 
  \[ + \text{lag. sales} + \text{lag. exports} \] 
  \( \left. \text{exporters at least know dist.} \right\} \) 
  \( \text{p-value: } = 0.66 \)

  **Test 2:**  \( H_0 : \left\{ \text{exporters at least know dist.} \right. \) 
  \[ + \text{lag. sales} + \text{lag. exports} + \] 
  \[ \text{avg. prod. other exporters} \] 
  \( \left. \text{exporters at least know dist.} \right\} \) 
  \( \text{p-value: } = 0.001 \)

- At 5% significance level, we cannot reject that exporters know distance and lagged aggregate exports to each destination country, and their own lagged domestic sales.

- However, we can reject that exporters know the relative productivity level (or domestic sales) of the other exporters.
Manski (2002) shows that preference parameters and unobserved expectations are not separately identified from the distribution of choices alone.

Solution 1: data on ex-post realizations to proxy for unobserved expectations.

Rosen and Willis (1979), Manski (1991), and Ahn and Manski (1993) propose a solution that relies on assuming that agents

- are rational;
- form expectations conditioning on a set of variables that are **fully observed by the researcher**.

We introduce a moment inequality approach that partially identifies the preference parameters of agents’ that

- make binary decisions (restriction with respect to RW, 1979);
- are rational;
- form expectations conditioning on a set of variables that might be **partially unobserved by the researcher** (generalization with respect to RW, 1979).
Manski (2002) shows that preference parameters and unobserved expectations are not separately identified from the distribution of choices alone.

Solution 2: model the process through which exporters acquire information.

Examples:
- Rational inattention: Dasgupta and Mondria (2014);
- Search models: Allen (2014);
- Learning from previous export experience: Albornoz et al. (2012);

We show that one may use standard statistical tests of moment inequality models to test the validity of the information sets predicted by these different theories of information acquisition.

We also show that one can compute predicted probabilities without the need to fully specify firms’ information sets.

- Useful to perform counterfactual exercises that are robust to different assumptions on firms’ information sets.
The revealed-preference inequalities in Pakes (2010) allow firms’ expectations to depend on variables that are unobserved to the researcher.

The moment inequality approach in Pakes (2010) and PPHI (2014) assumes that agents’ preferences are not affected by structural errors.

The standard approach in binary choice models (e.g. logit, probit) is to assume that structural errors follow a known parametric distribution.

Our moment inequality approach allows:
- agents’ preferences to depend on a nonzero structural error; → weaker assumptions than in Pakes (2010) and PPHI (2014).
- agents’ beliefs to depend on variables that are not observed by the researcher. → weaker assumptions than RW (1979), Manski (1991), and AM (1993).

Specifically, we introduce two new types of moment inequalities:
- odds-based inequalities;
- generalized revealed-preference inequalities.
Estimates of international trade models based on wrong assumptions on agents’ information sets are generally biased.

Correctly specifying firms’ information sets is important
  - for any model in which firms’ make binary choices,
  - even when one estimates static trade models.

In the case of entry models, under specific distributional assumptions on firms’ expectational errors, wrongly assuming that firms’ have perfect foresight will bias the estimates of entry costs upwards.

Assuming that exporters are able to predict “too well” their returns to foreign market entry might be the reason why standard trade models predict export entry costs that are “too high”.
Overview

- Theoretical Model
- Data and Measurement
- Estimation Conditional on Full Knowledge of Exporters’ Information Sets.
- Counterfactuals under Partial Knowledge of Exporters’ Information Sets.
- What do Exporters Know?
- Conclusions
THEORETICAL MODEL
Theoretical Model: Timing

- Single-agent partial equilibrium model.
- Two-period model.
- First period:
  - firms decide whether to export to each possible destination market;
  - if they decide to export, they must pay export entry costs;
  - entry decision depends on expectations about potential export profits;
  - expectations are rational;
  - exporters’ information set is left unspecified.
- Second period:
  - firms observe the realized demand in each destination and their realized productivity, determine their optimal price in each market in which they have decided to enter, and obtain export profits.
- No discounting between periods.
- We index firms by $i$ and countries by $j$.
Every firm faces an isoelastic demand in every country,

\[ x_{ij} = \frac{p_{ij}^{-\eta}(e_j Y_j)}{(e_j P_j)^{1-\eta}}, \]

where \( p_{ij} \) is the price set by \( i \) in \( j \), \( e_j \) is the nominal exchange rate, \( Y_j \) is the total expenditure, and \( P_j \) is the ideal price index

\[ P_j = \left[ \int_{i \in A_j} p_{ij}^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \]

where \( A_j \) denotes the set of all firms in the world selling in \( j \).

Elasticity of demand assumed constant across firms and countries: \( \eta \).
Theoretical Model: Supply

- Each destination country defines a market.
- Every firm is the single world producer of a variety.
- Market structure: monopolistic competition in every market.
- Constant marginal production cost: $a_i c$.
- If a firm exports a positive amount to $j$, it must pay two additional costs:
  - iceberg trade costs: $\tau_j$, so total the marginal costs of exporting to $j$ is $\tau_j a_i c$.
  - fixed entry costs: 
    \[
    f_{ij} = \beta_0 + \beta_1 \text{dist}_j + \nu_{ij},
    \]
    where $\text{dist}_j$ denotes distance from home country to destination $j$. 
Theoretical Model: Profits from Exporting

- Conditional on entering destination market \( j \), firm \( i \) obtains revenue

\[
rij = \left[ \frac{\eta}{\eta - 1} \frac{\tau_{jaic}}{P_j} \right]^{1-\eta} Y_j e_j^\eta,
\]

and gross profits

\[
\eta^{-1} r_{ij}.
\]

- Net of export entry costs, the export profits that \( i \) will obtain in \( j \) are

\[
\pi_{ij} = \eta^{-1} r_{ij} - f_{ij}.
\]
Firm $i$’s expectations are rational.

Firm $i$ will export to $j$ if and only if

$$\mathbb{E}[\pi_{ij}|J_{ij}, \nu_{ij}] \geq 0,$$

where $(J_{ij}, \nu_{ij})$ denotes the information set of firm $i$ in period 1.

Assuming that $\text{dist}_j \in J_{ij}$, we can define an export dummy as

$$d_{ij} = 1\{\eta^{-1}\mathbb{E}[r_{ij}|J_{ij}, \nu_{ij}] - f_{ij} \geq 0\}.$$

Assuming that

$$\nu_{ij}|(r_{ij}, J_{ij}) \sim \mathcal{N}(0, \sigma^2_{\nu}),$$

we can write the probability that $i$ exports to $j$ conditional on $J_{ij}$ as

$$P_{ij} = P(d_{ij} = 1|J_{ij}) = \Phi(\sigma^{-1}_{\nu}(\eta^{-1}\mathbb{E}[r_{ij}|J_{ij}] - \beta_0 - \beta_1\text{dist}_j)).$$
Theoretical Model: Effect of Change in Entry Costs

- Effect of a 40% reduction in systematic part of entry costs:

\[ f_{ij}^1 = 0.6(\beta_0 + \beta_1 \text{dist}_j) + \nu_{ij}. \]

- As an e.g., this change could be due to a launch-aid subsidy.
- Assume that \( \tau_j, c, e_j, P_j, Y_j, \) and \( \{a_i, i = 1, \ldots, N\} \) remain invariant.
  - Exporters from home are small in destination market \( j \).
- Therefore, \( r_{ij} \) is invariant to the change in \( f_{ij} \). Conditional on exporting, the change in export entry costs does not affect gross export profits.
- The change in \( f_{ij} \) affects the export decision of firms: \( \{d_{ij}, i = 1, \ldots, N\} \).
- The growth rate in aggregate exports generated by the reduction in \( f_{ij} \):

\[ g^R_j = \frac{R^1_j}{R_j} - 1 = \frac{\int_{i\in N} d_{ij}^1 r_{ij} di}{\int_{i\in N} d_{ij} r_{ij} di} - 1 = \frac{\int_{i\in N} P_{ij}^1 r_{ij} di}{\int_{i\in N} P_{ij} r_{ij} di} - 1 \]

where the change in \( f_{ij} \) is assumed to be known in period 1:

\[ P_{ij}^1 = P^1(d_{ij} = 1|J_{ij}) = \Phi(\sigma_\nu^{-1}(\eta^{-1} \mathbb{E}[r_{ij}|J_{ij}] - 0.6(\beta_0 - \beta_1 \text{dist}_j))). \]
DATA AND MEASUREMENT
Data sources and variables used in the analysis:

- Chilean customs database:
  - dummy for positive exports: $d_{ijt}$,
  - aggregate exports: $R_{jt} = \sum_i r_{ijt}$,

- Chilean industrial survey:
  - domestic sales: $r_{iht} = r_{it} - \sum_j r_{ijt}$

- CEPII:
  - geographical distance to Chile: $dist_j$ (in thousands of kilometers)


Sector: manufacture of chemicals and chemical products.

Restrict to 22 destinations where at least 5 firms export in every year.
Measurement

- Export revenue is unobserved for firms that do not export.
- We need to find a proxy for the potential revenue from exporting.
- Using the model described above,
  \[ r_{ij} = \frac{a_i^{(1-\eta)}}{V_j} R_j \quad \text{with} \quad V_j = \int_{i \in N} d_{ij} a_i^{(1-\eta)} di, \]
  or, equivalently,
  \[ r_{ij} = \frac{r_{ih}}{R_{hj}} R_j \quad \text{with} \quad R_{hj} = \int_{s \in N} d_{sj} r_{sh} ds. \]
- We define a proxy for \( i \)'s potential export revenue in \( j \) using information on
  - domestic sales of firm \( i \): \( r_{ih} \).
  - total domestic sales of all firms exporting to \( j \) at \( t \): \( R_{hj} \).
  - total aggregate exports from \( h \) to \( j \) at \( t \): \( R_j \).
ESTIMATION CONDITIONAL ON FULL KNOWLEDGE OF EXPORTERS’ INFORMATION SETS
Assuming an Information Set

- The model above does not impose any assumption on $\mathcal{J}_{ij}$.
- These information sets are generally unobservable in standard trade datasets.
- Standard assumption in empirical work: $\mathcal{J}_{ij}$ is identical to a set of variables observed by the researcher:
  \[ \mathcal{J}_{ij} = Z_{ij}. \]

- Under this assumption, the export probability conditional on $Z_j$ is:
  \[ P(d_{ij} = 1|Z_{ij}) = \Phi(\sigma_{\nu}^{-1}(\eta^{-1}E[r_{ij}|Z_{ij}] - \beta_0 - \beta_1 \text{dist}_j)). \]

  and we can estimate $E[r_{ij}|Z_{ij}]$ non-parametrically (approach in RW, 1979).

- Normalization by scale: fix the elasticity of substitution to a known constant
  \[ \eta^{-1} = k. \]

- In our case, we fix the elasticity of substitution to 5.
Bias Due to Misspecified Information Sets

- If firms’ true information sets differ from observed vector of covariates and

  \[ \mathbb{E}[r_{ij}|Z_{ij}] = \mathbb{E}[r_{ij}|J_{ij}] + \xi_{ij}, \quad \xi_{ij} \neq 0, \]

  then the true export probability conditional on \( Z_j \) is

  \[ P(d_{ij} = 1|Z_{ij}) = \int_{k\xi+\nu} \mathbb{1}\{k\mathbb{E}[r_{ij}|Z_{ij}] - \beta_0 - \beta_1 \text{dist}_j - \nu - k\xi \geq 0\} f(k\xi + \nu|Z_j) d(k\xi + \nu). \]

- The estimates of \( \beta_0 \) and \( \beta_1 \) will be biased unless

  \[ f(k\xi + \nu|Z_j) = \phi(\nu). \]

- This equality holds if and only if \( \mathbb{E}[r_{ij}|Z_{ij}] = \mathbb{E}[r_{ij}|J_{ij}] \).

- Therefore, if firms’ true information sets differ from the information sets assumed by the researcher, parameter vector estimates are always biased.
Bias Due to Misspecified Information Sets

- In the specific case in which perfect foresight is assumed
  \[ E[r_{ij} | Z_{ij}] = r_{ij}, \]
  agent’s true expectations are normally distributed
  \[ E[r_{ij} | J_{ij}] \sim \mathcal{N}(0, \sigma^2_e), \]
  and the expectational error is normal conditional on agents’ information sets
  \[ \xi_{ij} | (J_{ij}, \nu_{ij}) \sim \mathcal{N}(0, \sigma^2_\xi), \]
  there is an upward bias in the estimates of \( \beta_0 \) and \( \beta_1 \). Specifically, the ML estimates of these entry costs parameters converge to
  \[ \beta_0 \frac{\sigma^2_e + \sigma^2_\xi}{\sigma^2_e} \quad \text{and} \quad \beta_1 \frac{\sigma^2_e + \sigma^2_\xi}{\sigma^2_\xi}. \]
- Deviations from normality in the distribution of expectations or errors will alter the exact formula for the bias. However, the correlation between \( r_{ij} \) and \( \xi_{ij} \) will always push the bias in the direction of overestimating entry costs.
Empirical Application: Perfect Foresight

- Parameters (estimate and standard deviation):

  \[
  \begin{pmatrix}
  \beta_0 \\ \beta_1 \\ \sigma_{\nu}
  \end{pmatrix} =
  \begin{pmatrix}
  760,880 \\ 118,008 \\ 1,074,030
  \\ (36,671) \\ (53,212) \\ (46,737)
  \end{pmatrix}
  \]

- Country-specific entry costs (estimate and 95% confidence interval):

  \[
  \begin{pmatrix}
  \text{Argentina} \\ \text{U.S.} \\ \text{Japan}
  \end{pmatrix} =
  \begin{pmatrix}
  894,031 \\ 1,736,934 \\ 2,796,218
  \\ (799,048, 989,014) \\ (1,564,976, 1,908,891) \\ (2,518,448, 3,073,988)
  \end{pmatrix}
  \]
Empirical Application: Perfect Foresight

- Effect of counterfactual changes in the environment in 2005.
- Effect on total number of exporters:

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ 40% Entry Costs</td>
<td>50.35%</td>
<td>157.54%</td>
<td>578.72%</td>
</tr>
</tbody>
</table>

- Effect on aggregate exports:

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ 40% Entry Costs</td>
<td>40.50%</td>
<td>53.07%</td>
<td>163.36%</td>
</tr>
</tbody>
</table>
Empirical Application

- Alternatives to perfect foresight assumption.
- In our model, export revenues may be written as a function of:
  - domestic sales of firm $i$: $r_{ih}$,
  - proxy for firms' productivity;
  - total domestic sales of all firms exporting to $j$: $R_{hj}$,
  - proxy for variables determining firms' selection into exporting;
  - total aggregate exports from $h$ to $j$: $R_j$,
  - proxy for market characteristics affecting export revenue for all firms: aggregate demand, price index, transport costs, and exchange rate;
- Information set that exporters might have at the time of deciding on entry at $t$:
  - lagged own domestic sales: $r_{ih,t-1}$,
  - lagged total aggregate exports from $h$ to $j$: $R_{jt-1}$,
  - distance from $h$ to $j$ (as proxy for $R_{hj}$): $\text{dist}_j$.
- Therefore, as an alternative to the perfect foresight assumption, we may assume that $J_{ijt} = Z_{ijt} = (r_{ih,t-1}, R_{jt-1}, \text{dist}_j)$. 
Empirical Application: $J_{ijt} = (r_{iht-1}, R_{jt-1}, \text{dist}_j)$

- Parameters (estimate and standard deviation):

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\sigma_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>502,197</td>
<td>81,225</td>
<td>701,913</td>
</tr>
<tr>
<td></td>
<td>(20,153)</td>
<td>(30,005)</td>
<td>(24,334)</td>
</tr>
</tbody>
</table>

- Country-specific entry costs (estimate and 95% confidence interval):

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimate</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>593,845</td>
<td>(551,041, 636,649)</td>
</tr>
<tr>
<td>U.S.</td>
<td>1,174,016</td>
<td>(1,099,201, 1,248,830)</td>
</tr>
<tr>
<td>Japan</td>
<td>1,903,122</td>
<td>(1,779,295, 2,026,949)</td>
</tr>
</tbody>
</table>

- Estimates of entry costs are approximately 33% smaller than in the model that assumes that exporters have perfect foresight.
Empirical Application: \( J_{ijt} = (r_{iht-1}, R_{jt-1}, \text{dist}_j) \)

- Effect of counterfactual changes in the environment in 2005.
- Effect on total number of exporters:

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ 40% Entry Costs</td>
<td>51.92%</td>
<td>138.98%</td>
<td>692.29%</td>
</tr>
</tbody>
</table>

- Effect on aggregate exports:

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ 40% Entry Costs</td>
<td>42.91%</td>
<td>40.50%</td>
<td>192.31%</td>
</tr>
</tbody>
</table>
ESTIMATION CONDITIONAL ON PARTIAL KNOWLEDGE OF EXPORTERS’ INFORMATION SETS
In our model, firms’ export revenues may be perfectly proxied by
- domestic sales of firm $i$: $r_{ih}$,
- total domestic sales of all firms exporting to $j$: $R_{hj}$,
- aggregate exports from $h$ to $j$: $R_j$.

For each individual firm, predicting future export profits is equivalent to predicting future values of these three variables.

In order to predict future values of these variables, firms are likely to use information that is not observed by researchers.

E.g. in order to predict future domestic sales, firms’ will use private information on R&D investments, future re-organization plans, etc.

E.g. in order to predict future export demand, firms’ will use information on future trade policy changes, future currency devaluations, etc.
No matter which information sets firms have, they are likely to include:
- lagged own domestic sales: \( r_{iht-1} \),
- lagged aggregate exports from home country to each destination: \( R_{jt-1} \),
- distance from home country: \( dist_j \).

Under the assumption that firms' information sets are only partially observed, the model parameters are only partially identified.

We introduce two new types of moment inequalities,
- odds-based moment inequalities, and
- generalized revealed-preference moment inequalities,
that allow to estimate parameters under the assumption that the researcher only observes part of firms' information sets.

Furthermore, we show how to perform counterfactuals under same set of assumptions needed for inference.

Finally, we show how to test whether a vector of observed covariates belongs to the agents' true information sets; test \( H_0 : Z_{ij} \in J_{ij} \).
Odds-Based Moment Inequalities

- If $Z_{ij} \subset J_{ij}$, then

$$\mathcal{M}(Z_{ij}; (\beta_0, \beta_1, \sigma_\nu)) = \mathbb{E} \left[ \begin{array}{c} m_l(d_{ij}, r_{ij}, dist_j; (\beta_0, \beta_1, \sigma_\nu)) \\ m_u(d_{ij}, r_{ij}, dist_j; (\beta_0, \beta_1, \sigma_\nu)) \end{array} \right | Z_{ij}] \geq 0,$$

with

$$m_l(\cdot) = d_{ij} \frac{1 - \Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j))}{\Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j))} - (1 - d_{ij}),$$

$$m_u(\cdot) = (1 - d_{ij}) \frac{\Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j))}{1 - \Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j))} - d_{ij},$$

where $(\beta_0, \beta_1, \sigma_\nu)$ denotes the true value of the parameter vector.

- We denote $\mathcal{M}(\cdot)$ as the conditional odds-based moment inequalities.
Generalized Revealed-Preference Moment Inequalities

- If $Z_{ij} \subset J_{ij}$, then

$$M^r(Z_{ij}; (\beta_0, \beta_1, \sigma_\nu)) = \mathbb{E} \left[ \begin{array}{c} m^r_l(d_{ij}, r_{ij}, dist_j; (\beta_0, \beta_1, \sigma_\nu)) \\ m^r_u(d_{ij}, r_{ij}, dist_j; (\beta_0, \beta_1, \sigma_\nu)) \end{array} \right| Z_{ij} \right] \geq 0,$$

with

$$m^r_l(\cdot) = -(1 - d_{ij})(kr_{ij} - \beta_0 - \beta_1 dist_j) + d_{ij}\sigma_\nu \frac{\phi\left(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j)\right)}{\Phi\left(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j)\right)},$$

$$m^r_u(\cdot) = d_{ij}(kr_{ij} - \beta_0 - \beta_1 dist_j) + (1 - d_{ij})\sigma_\nu \frac{\phi\left(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j)\right)}{1 - \Phi\left(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 dist_j)\right)},$$

where $(\beta_0, \beta_1, \sigma_\nu)$ denotes the true value of the parameter vector.

- We denote $M^r(\cdot)$ as the conditional generalized revealed-preference moment inequalities.
Both the odds-based and the generalized revealed-preference moment inequalities identify the true value of the parameter vector in any binary choice model that has the following properties:

\[ d = 1 \{ \beta X^* + \nu \geq 0 \}, \quad \nu | (X^*, Z) \sim \nu \text{ and } F_\nu \text{ is log-concave,} \]

\[ X = X^* + \varepsilon, \quad \mathbb{E}[\varepsilon | X^*, Z] = 0, \]

\[ X^* = m(Z, W), \quad \text{no restriction on } m(\cdot) \text{ or } W, \]

with only \((d, X, Z)\) observed to the econometrician.

- In this model, \(\beta\) is partially identified.

- Literature: alternative restrictions on the relationship between \(X^*\) and \(Z\):
  - WR (1979), Manski (1991) and AM (1993): \(X^* = m(Z)\).
  - Schennach (2007): \(X^* = m(Z) + W\) and \(W \perp Z, \mathbb{E}(W) = 0\).
From Conditional to Unconditional Moments

In order to apply our moment inequalities, we need to derive unconditional moments that are consistent with our conditional moments:

$$\mathbb{E} \left\{ \begin{array}{l}
  m_l(d_{ij}, r_{ij}, \text{dist}_j; (\beta_0, \beta_1, \sigma_\nu)) \\
  m_u(d_{ij}, r_{ij}, \text{dist}_j; (\beta_0, \beta_1, \sigma_\nu)) \\
  m^r_l(d_{ij}, r_{ij}, \text{dist}_j; (\beta_0, \beta_1, \sigma_\nu)) \\
  m^r_u(d_{ij}, r_{ij}, \text{dist}_j; (\beta_0, \beta_1, \sigma_\nu))
\end{array} \right\} \times g(Z_{ij}) \geq 0,$$

where $g(Z_{ij})$ is the instrument function.

Suggested $g(\cdot)$ functions in Andrews and Shi (2013) or Armstrong (2014) are computationally infeasible in our setting.

For each $a \in \{0.5, 1, 1.5, 2\}$, we estimate an identified set with

$$g_a(Z_{kjt}) = \left\{ \begin{array}{l}
  \mathbb{1}\{Z_{kjt} > \text{med}(Z_{kjt})\} \\
  \mathbb{1}\{Z_{kjt} \leq \text{med}(Z_{kjt})\}
\end{array} \right\} \times (|Z_{kjt} - \text{med}(Z_{kjt})|)^a,$$

and $g_a(Z_{jt}) = (g_a(Z_{1jt}); g_a(Z_{2jt}); \ldots; g_a(Z_{Kjt}))$. 
Empirical Application: \( (r_{iht-1}, R_{jt-1}, \text{dist}_j) \in \mathcal{J}_{ijt} \)

- Benchmark specification: we use \( Z_{ijt} = (r_{iht-1}, R_{jt-1}, \text{dist}_j) \) and \( a = 1.5 \).
- Parameters (estimate and 95% confidence interval)

<table>
<thead>
<tr>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \sigma_\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(218,341, 245,817)</td>
<td>(42,832, 47,896)</td>
<td>(311,740, 340,959)</td>
</tr>
<tr>
<td>(123,255, 364,285)</td>
<td>(23,590, 78,571)</td>
<td>(177,777, 571,428)</td>
</tr>
</tbody>
</table>

- Country-specific entry costs (estimate and 95% confidence interval):

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{estimate} )</td>
<td>(269,992, 298,153)</td>
<td>(592,552, 632,030)</td>
<td>(977,631, 1,061,959)</td>
</tr>
<tr>
<td></td>
<td>(165,239, 451,327)</td>
<td>(411,287, 1,007,010)</td>
<td>(624,809, 1,712,294)</td>
</tr>
</tbody>
</table>

- The upper bound of the estimates of entry costs in Argentina are approx. 50% of those estimated under the assumption that \( Z_{ijt} = \mathcal{J}_{it} \), and 33% of those estimated under perfect foresight. Similar ratios for U.S. and Japan.
COUNTERFACTUALS UNDER PARTIAL KNOWLEDGE OF EXPORTERS’ INFORMATION SETS
The model presented above is incomplete, in the sense that it does not fully specify firms' information sets.

However, the restrictions embedded in it, combined with the assumption that we observe a vector $Z_{ij}$ such that

$$Z_{ij} \in J_{ij},$$

are enough to generate bounds on the export probability

$$P(d_{ij} = 1|J_{ij}) = \Phi\left(\sigma^{-1}(k\mathbb{E}[r_{ij}|J_{ij}] - \beta_0 - \beta_1 dist_j)\right),$$

for any given value of the parameter vector $(\beta_0, \beta_1, \sigma_\nu)$.

This allows to:
- evaluate the fit of the model; and,
- compute counterfactual probabilities that correspond to alternative values of the parameter vector.
Bounds on Predicted Probabilities

- Denoting $\beta = (\beta_0, \beta_1, \sigma_\nu)$ and $\Theta$ the previously computed identified set of $\beta$, we can conclude that

$$\frac{1}{1 + B_L(Z_{ij})} \leq P(d_{ij} = 1|J_{ij}) \leq \frac{B_u(Z_{ij})}{1 + B_u(Z_{ij})},$$

where

$$B_L(Z_{ij}) = \max_{\beta \in \Theta} \mathbb{E} \left[ \frac{1 - \Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 \text{dist}_j))}{\Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 \text{dist}_j))} \left| Z_{ij} \right. \right],$$

$$B_u(Z_{ij}) = \max_{\beta \in \Theta} \mathbb{E} \left[ \frac{\Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 \text{dist}_j))}{1 - \Phi(\sigma_\nu^{-1}(kr_{ij} - \beta_0 - \beta_1 \text{dist}_j))} \left| Z_{ij} \right. \right].$$

- Similarly, for any counterfactual value of the parameter vector, $\beta'$, we can compute the counterfactual predicted probability of exports for every firm $i$ in every country $j$ using exactly the same bounds described above but with $\Theta$ being a singleton equal to $\beta'$. 
Empirical Application: \((r_{iht-1}, R_{jt-1}, \text{dist}_j) \in \mathcal{J}_{ijt}\)

- Predicted number of exporters, given actual data and estimated parameters (estimates and 95% confidence interval in 2005):

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>(42.00, 46.89)</td>
<td>(19.26, 24.17)</td>
<td>(4.50, 7.19)</td>
</tr>
<tr>
<td>Lower CI</td>
<td>(30.33, 52.83)</td>
<td>(11.01, 34.27)</td>
<td>(1.48, 13.07)</td>
</tr>
<tr>
<td>Predicted</td>
<td>46</td>
<td>24</td>
<td>5</td>
</tr>
</tbody>
</table>

- Predicted total exports, given actual data and estimated parameters (estimates and 95% confidence interval in 2005; in millions of 2000 dollars):

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>(17.78, 21.74)</td>
<td>(77.34, 86.32)</td>
<td>(33.12, 51.35)</td>
</tr>
<tr>
<td>Lower CI</td>
<td>(14.13, 25.23)</td>
<td>(55.36, 108.21)</td>
<td>(11.34, 66.51)</td>
</tr>
<tr>
<td>Predicted</td>
<td>25.02</td>
<td>48.80</td>
<td>9.90</td>
</tr>
</tbody>
</table>
Empirical Application: \((r_{iht-1}, R_{jt-1}, \text{dist}_j) \in \mathcal{J}_{ijt}\)

- Effect of a 40% reduction in export entry costs.
- Predicted number of exporters, given actual data and estimated parameters (estimates and 95% confidence interval in 2005):

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(64.35, 68.18)</td>
<td>(44.91, 52.25)</td>
</tr>
<tr>
<td>(64.35, 68.18)</td>
<td>54.66, 73.38</td>
<td>33.47, 61.57</td>
<td>8.90, 33.08</td>
</tr>
<tr>
<td>(54.66, 73.38)</td>
<td></td>
<td>(64.35, 68.18)</td>
<td>(44.91, 52.25)</td>
</tr>
<tr>
<td>(54.66, 73.38)</td>
<td>(64.35, 68.18)</td>
<td>(44.91, 52.25)</td>
<td>(17.44, 21.89)</td>
</tr>
</tbody>
</table>

- Predicted total exports, given actual data and estimated parameters (estimates and 95% confidence interval in 2005; in millions of 2000 dollars):

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>U.S.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>(23.07, 25.98)</td>
<td>(19.52, 28.91)</td>
<td>(100.01, 107.93)</td>
<td>(62.90, 71.46)</td>
</tr>
<tr>
<td>(19.52, 28.91)</td>
<td>(23.07, 25.98)</td>
<td>(100.01, 107.93)</td>
<td>(62.90, 71.46)</td>
</tr>
<tr>
<td>(23.07, 25.98)</td>
<td>(19.52, 28.91)</td>
<td>(100.01, 107.93)</td>
<td>(62.90, 71.46)</td>
</tr>
</tbody>
</table>
WHAT DO EXPORTERS KNOW?
Our moment inequality estimator allows us to learn about exporters’ true information sets.

We can test the null hypothesis that a set of variables observed by the econometrician is a subset of the information set of exporters.

Under the null that a particular set of covariates is included in exporters’ information sets, the set identified by our moment inequalities is nonempty.

- It contains, at least, the true value of the parameter vector.

Given a set of moment inequalities, statistical tests like those in Romano and Shaikh (2008), Andrews and Guggenberger (2009), and Andrews and Soares (2010) allow to test whether the identified set is non-empty.

We perform this test for different covariate vectors that might potentially be included in exporters’ information sets.
Testing a Set of Variables is in the Information Set

Examples:

Test 1: \( H_0: \left\{ \text{exporters at least know dist.} \right. \) 
\( + \) lag. sales + lag. exports \) 
\( \left. \right\} \) 
\( \text{p-value: } = 0.66 \)

Test 2: \( H_0: \left\{ \text{exporters at least know dist.} \right. \) 
\( + \) lag. sales + lag. exports + 
\( \left. \right\} \) 
\( \text{avg. prod. other exporters} \) 
\( \text{p-value: } = 0.001 \)

At 5% significance level, we cannot reject that exporters know distance and lagged aggregate exports to each destination and their own lagged domestic sales. However, we can reject that they know the productivity level of the other exporters (i.e. that they know the total domestic sales of other exporters).
Conclusion

- Many binary choice decisions depend on agents’ expectations of variables entering their payoff functions.
- Agents’ expectations are generally unobservable to the econometrician.
- In many empirical settings, the econometrician observes the ex-post realization of the variables on which agents form expectations.
- In order to use this data, standard estimation approaches require strong assumptions on the content of agents’ information sets.
- We introduce a new estimation procedure that allows both for:
  - partial observability by the researcher of agents’ information sets;
  - structural error (i.e. payoff-relevant variables that are observed by the agent and not by the econometrician).
- We show that the misspecification of agents’ information sets may have large consequences for the estimates of structural models and, therefore, also for the model-predictions for any counterfactual exercise of interest.
EXTRA SLIDES
Odds-Based Moment Inequalities: Proof

- Let $L(d|J)$ denote the log-likelihood conditional on $J$ and
  \[ \theta X^* = \sigma^{-1}_\nu(k\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist}), \]

- Then
  \[ L(d|J, Z; \theta) = \mathbb{E}\left[ d \log(1 - \Phi(-\theta X^*)) + (1 - d) \log(\Phi(-\theta X^*)) | J \right]. \]

- The corresponding score function is:
  \[ \frac{\partial L(d|J, Z; \theta)}{\partial \theta} = \mathbb{E}\left[ d \frac{1}{1 - \Phi(-\theta X^*)} \frac{\partial(1 - \Phi(-\theta X^*))}{\partial \theta} + (1 - d) \frac{1}{\Phi(-\theta X^*)} \frac{\partial(\Phi(-\theta X^*))}{\partial \theta} | J \right] = 0. \]

- Reordering terms:
  \[ \frac{\partial L(d|J, Z; \theta)}{\partial \theta} = \mathbb{E}\left[ \frac{\partial(\Phi(-\theta X^*))}{\partial \theta} \Phi(-\theta X^*) \left[ d \frac{\Phi(-\theta X^*)}{1 - \Phi(-\theta X^*)} \frac{\partial(1 - \Phi(-\theta X^*))}{\partial \theta} + (1 - d) \right] | J \right] = 0. \]
Odds-Based Moment Inequalities: Proof

- Given that

\[
\frac{\partial (\Phi(-\theta X^*))}{\partial \theta} \quad \frac{\partial (1-\Phi(-\theta X^*))}{\partial \theta} \quad \Phi(-\theta X^*)
\]

is a function of \(J\) and different from 0 for any value of \(\theta X^*\), we can simplify:

\[
\frac{\partial L(d|J, Z; \theta)}{\partial \theta} = \mathbb{E} \left[ d \frac{\Phi(-\theta X^*)}{1 - \Phi(-\theta X^*)} \frac{\partial (1-\Phi(-\theta X^*))}{\partial \theta} + (1 - d) \bigg| J \right] = 0.
\]

- Given that, for any value of \(\theta X^*\),

\[
\frac{\partial (1-\Phi(-\theta X^*))}{\partial \theta} \quad \frac{\partial \Phi(-\theta X^*)}{\partial \theta} = -1,
\]

we can simplify

\[
\frac{\partial L(d|J, Z; \theta)}{\partial \theta} = \mathbb{E} \left[ d \frac{\Phi(-\theta X^*)}{1 - \Phi(-\theta X^*)} - (1 - d) \bigg| J \right] = 0.
\]
Odds-Based Moment Inequalities: Proof

- Let’s denote

\[ \theta X = \sigma^{-1}_\nu (kr - \beta_0 - \beta_1 \text{dist}_j) = \sigma^{-1}_\nu (k\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist} + k\xi). \]

- Rational expectations implies that \( \mathbb{E}[\xi|J] = 0. \)

- Given that \( \mathbb{E}[\xi|J] = 0 \) and

\[ \frac{\Phi(y)}{1 - \Phi(y)} \]

is convex for any value of \( y \), it holds that

\[ \mathbb{E}\left[ \frac{\Phi(-\theta X)}{1 - \Phi(-\theta X)} \left| J \right. \right] \geq \mathbb{E}\left[ \frac{\Phi(-\theta X^*)}{1 - \Phi(-\theta X^*)} \left| J \right. \right]. \]

- Therefore, we can conclude that

\[ \mathbb{E}\left[ d \frac{\Phi(-\theta X)}{1 - \Phi(-\theta X)} - (1 - d) \left| J \right. \right] = \mathbb{E}\left[ d \frac{1 - \Phi(\theta X)}{\Phi(\theta X)} - (1 - d) \left| J \right. \right] \geq 0. \]
If $Z \in \mathcal{J}$, using the Law of Iterated Expectations,

$$\mathbb{E}\left[d \frac{1 - \Phi(\theta X)}{\Phi(\theta X)} - (1 - d) \Big| \mathcal{J} \right] = \mathbb{E}\left[\mathbb{E}\left[d \frac{1 - \Phi(\theta X)}{\Phi(\theta X)} - (1 - d) \Big| Z \right] \Big| \mathcal{J} \right].$$

Therefore,

$$\mathbb{E}\left[d \frac{1 - \Phi(\theta X)}{\Phi(\theta X)} - (1 - d) \Big| \mathcal{J} \right] \geq 0.$$

Given that

$$\frac{1 - \Phi(y)}{\Phi(y)}$$

is also convex for any value of $y$, following analogous steps, we can prove that

$$\mathbb{E}\left[(1 - d) \frac{\Phi(\theta X)}{1 - \Phi(\theta X)} - d \Big| Z \right] \geq 0.$$
The assumption that
\[ d = \mathbb{1}\{\eta^{-1}\mathbb{E}_\nu[r|J] - \beta_0 - \beta_1 \text{dist} - \nu \geq 0\}, \]
implies the following revealed-preference inequality
\[ d(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist} - d\nu) \geq 0. \]

Given that this inequality holds for every firm \( i \) and country \( j \), it will also hold in expectation conditional on \( J_{ij} \),
\[ \mathbb{E}[d(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist} - \nu)|J] \geq 0, \]
and, given that \( d \) is a function of \( J \),
\[ \mathbb{E}[d(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist})|J] - \mathbb{E}[d\nu|J] \geq 0. \]
Using the Law of Iterated Expectations, we can rewrite
\[ \mathbb{E}[d \nu | \mathcal{J}] = \mathbb{E}[d \mathbb{E}[^1_0 d | \mathcal{J}] | \mathcal{J}] = \]
\[ P(d = 1 | \mathcal{J}) \times 1 \times \mathbb{E}[\nu | d = 1, \mathcal{J}] + P(d = 0 | \mathcal{J}) \times 0 \times \mathbb{E}[\nu | d = 0, \mathcal{J}] = \]
\[ P(d = 1 | \mathcal{J}) \mathbb{E}[\nu | d = 1, \mathcal{J}] = \mathbb{E}[d | \mathcal{J}] \mathbb{E}[\nu | d = 1, \mathcal{J}] = \]
\[ \mathbb{E}[d \mathbb{E}[\nu | d = 1, \mathcal{J}] | \mathcal{J}] . \]

Analogously,
\[ \mathbb{E}[(1 - d) \nu | \mathcal{J}] = \mathbb{E}[(1 - d) \mathbb{E}[\nu | (1 - d) = 1, \mathcal{J}] | \mathcal{J}] \]
\[ = \mathbb{E}[(1 - d) \mathbb{E}[\nu | d = 0, \mathcal{J}] | \mathcal{J}] . \]

From the assumption \( \mathbb{E}[\nu \mathcal{J}] = 0 \) we can conclude that
\[ \mathbb{E}[\nu | \mathcal{J}] = \mathbb{E}[d \nu + (1 - d) \nu | \mathcal{J}] = \mathbb{E}[d \nu | \mathcal{J}] + \mathbb{E}[(1 - d) \nu | \mathcal{J}] = 0 , \]
and
\[ \mathbb{E}[d \mathbb{E}[\nu | d = 1, \mathcal{J}] | \mathcal{J}] = -\mathbb{E}[(1 - d) \mathbb{E}[\nu | d = 0, \mathcal{J}] | \mathcal{J}] . \]
Therefore, we can rewrite our revealed preference inequality as:

\[ E[d(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist})|J] + E[(1 - d)E[\nu|d = 0, J]|J] \geq 0. \]

Using the definition of \( d \), we may write

\[ E[\nu|d = 0, J] = E[\nu|\nu \geq \eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}, J] \]

and, given that \( \nu|J \sim \mathcal{N}(0, \sigma_\nu^2) \),

\[ E[\nu|d = 0, J] = \sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}))}. \]

Therefore, we can rewrite our revealed preference preference inequality as:

\[ E[d(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}) + (1 - d)\sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1}E[r|J] - \beta_0 - \beta_1 \text{dist}))}|J] \geq 0. \]
Let’s denote \( r = \mathbb{E}[r|J] + \xi_{ij} \). Rational expectations implies \( \mathbb{E}[\xi_{ij}|J] = 0 \).

Therefore, we can rewrite our revealed-preference inequality as

\[
\mathbb{E}[d(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}) + (1 - d)\sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist}))}|J] \geq 0.
\]

Given that \( \mathbb{E}[\xi|J, d] = 0 \) and

\[
\frac{\phi(y)}{1 - \Phi(y)}
\]

is convex for any value of \( y \), it holds that

\[
\mathbb{E}
\left[
\frac{\phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))}
\right]
\geq
\mathbb{E}
\left[
\frac{\phi(\sigma_\nu^{-1}(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1}\mathbb{E}[r|J] - \beta_0 - \beta_1 \text{dist}))}
\right].
\]
Therefore, using the Law of Iterated Expectations, we can obtain a weaker inequality as

\[ E[d(\eta^{-1} r - \beta_0 - \beta_1 \text{dist})] + (1 - d)\sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))} | J \] \geq 0.

Finally, given that \( Z \in J \), we can use again the Law of Iterated Expectations to conclude:

\[ E[d(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}) + (1 - d)\sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))}{1 - \Phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))} | Z_{ij}] \geq 0. \]

Using a completely analogous procedure, we can prove that

\[ E[(d - 1)(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}) + d\sigma_\nu \frac{\phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))}{\Phi(\sigma_\nu^{-1}(\eta^{-1} r - \beta_0 - \beta_1 \text{dist}))} | Z_{ij}] \geq 0. \]