THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Fall 2015

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

I.1 Suppose there are L commodities and J firms, each of which has a production set $Y_j \subset \mathbb{R}^L$, $j = 1, \ldots, J$, so the aggregate production set is $Y = \sum_j Y_j$. Assume that each Y_j is convex and $0 \in Y_j$ (these assumptions apply to both parts (a) and (b) below). Let $y_j^* \in Y_j$ for each j and let $y^* = \sum y_j^*$.

a) If y^* is efficient in Y, does there exist a price $p^* \in \mathbb{R}_+^L \backslash \{0\}$ (the same price for all j) such that for each j,

$$y_j^* \text{ maximizes } p^* y_j \text{ on } Y_j?$$

If your answer is YES, give a proof. If your answer is NO give a counterexample.

b) Now suppose that $J = 2$, and there is a severe externality between firms 1 and 2 such that if either firm operates ($y_j \neq 0$), the other must shut down. This means that the *attainable* aggregate production set is $A = Y_1 \cup Y_2$. Is it possible to "price the externality" in the following sense: Suppose $y_1^* \in Y_1$ is efficient in A. Does there exist a price $p^* \in \mathbb{R}_+^L \backslash \{0\}$ and an "externality price" $e \in \mathbb{R}$ (the same p^* and e for both firms) such that

$$y_1^* \text{ maximizes } p^* y_1 - e 1_{\{y_1 \neq 0\}} \text{ on } Y_1$$

and

$$y_2^* = 0 \text{ maximizes } p^* y_2 - e 1_{\{y_2 \neq 0\}} \text{ on } Y_2,$$

where 1_S denotes the indicator function of the set S? If your answer is YES, give a proof. If your answer is NO give a counterexample.

I.2 Let $\succ$ be a binary relation on $\mathbb{R}^n$ that is acyclic and upper semi-continuous (for each $x \in \mathbb{R}^n$ the set $\{x' : x \succ x'\}$ is open), and let K be a compact subset of $\mathbb{R}^n$. Prove that $\succ$ has a maximal element in K.

I.3 Consider an exchange environment with $L = I = 2$; the endowments

$$w_1 = (6, 0) \quad \text{and} \quad w_2 = (0, 6);$$

and preferences represented by the utility functions $u_i : \mathbb{R}_+^2 \to \mathbb{R}$ given by

$$u_1(x_{11}, x_{21}) = \max\left\{\min\{x_{11} + 1, x_{21}\}, \min\{x_{11}, x_{21} + 1\}\right\},$$

$$u_2(x_{12}, x_{22}) = x_{12} x_{22}.$$

a) Find the set of Pareto efficient allocations.

b) Find the set of competitive equilibrium prices and allocations.

c) For parts (a) and (b) above, if the set is nonempty, show it clearly on an Edgeworth Box diagram on the graph paper provided. If the set is empty, explain what feature of the example violates the hypothesis of the relevant existence theorem.

Section II

II.1 Consider the following two-person, two-period game:

	C	D
C	$2,2$	$-1,3$
D	$3,-1$	$0,0$

$+$

	G	B
G	$4,4$	$0,0$
B	$0,0$	$1,1$

where the first (second) period payoffs are given in the left-hand (resp. right-hand) bimatrix above. Players wish to maximize the sum of the payoffs in the two periods.

(a) First, suppose that players can observe each other's first-period actions, C or D before choosing their second-period actions, G or B. Show that there is a subgame perfect equilibrium of resulting game in which both players play C in the first period.

(b) Now suppose that players cannot observe each other's first-period actions directly. Rather, both players see the same *signal*, either H ("high") or L ("low") prior to choosing their second-period actions, G or B. Thus, players' second period actions depend only on the signal that they both see. Signals are generated as follows. If both players choose the same action in period 1, then both players get the signal H with probability $q \in [0, 1]$ and signal L with probability $1-q$. If the players choose different actions in period 1, then both players get the signal H with probability $1-q$ and signal L with probability q. Find the values of q for which there is a perfect Bayesian equilibrium in which the players play C in the first period.

II.2 There are two players and a public good is to be provided. The value of this public good is 1 to both the players. The good is provided if one of the two individuals provides it. Player i's cost of providing the good is c_i. The c_i's are i.i.d draws from the uniform distribution on $[0, 1]$.

The game is as follows: Nature chooses the cost realization for each player. Each player knows her own cost exactly but not the cost realization of the other player. Once the costs are realized, players i ($i = 1, 2$) simultaneously choose times $t_i \geq 0$ such that if the good has not been provided until time t_i, player i would provide it. Time is continuous here. The rules of the game are common knowledge.

If the public good is provided at time $t \geq 0$ by player i, then i obtains a payoff of $e^{-rt}(1 - c_i)$ and player $j \neq i$ obtains a payoff of e^{-rt}. Here r is the discount *rate* and e^{-rt} is the discount factor.

(a) Does a player with cost 0 have a (weakly) dominant strategy? What is it?

(b) Suppose there is a symmetric equilibrium in which player i uses a continuosly differentiable, strictly increasing strategy $t_i = \tau(c_i)$. Construct an incentive compatible direct mechanism equivalent to the game above. Then derive the symmetric Bayes-Nash equilibrium strategy $\tau(.)$.

II.3 Consider a game with two players, A and B. The game consists of two rounds and a player will have to win both rounds to win the game and obtain a prize of $v > 1$ units of utility. If a player wins only one round, she will get 0. The effort choices and outcome of the first round is publicly observable before players start the second.

Each player has one indivisible unit of effort to allocate between the two rounds—that is, she can choose either 0 or 1 in the first round and either 0 or 1 in the second round, but obviously, only 1 in total. Effort, if employed, costs the player 0.5 units of utility. If Player i puts in effort e_i and Player j exerts e_j, then the probability that i wins in that round is $\frac{1}{2} + \frac{1}{2}(e_i - e_j)$. (Only one player can win a round.)

The timeline is as follows: players simultaneously and independently choose effort levels for the first round and chance chooses the winner subsequently according to the probabilities above. The outcome is revealed and now players simultaneously choose effort levels for the second round. *Note that if a player has spent a unit of effort in the first round, she cannot put in any effort in the second round. If she has conserved the unit of effort by not spending it initially, she can spend it in the second round.*

Again outcomes for each round are determined by the probabilities as given above. (The cost of effort also has to be subtracted.)

Derive a strategy profile that is a subgame perfect equilibrium in this game. (State any additional assumptions you make on the value of v.)