Comparison of Arbitration Procedures: Models with Complete and Incomplete Information

KALYAN CHATTERJEE

***Abstract*— This paper has focused on two types of arbitration procedures that are commonly used in the United States, especially in industrial disputes involving public safety personnel who are legally not permitted to strike. The conventional arbitration (CA) procedure gives the arbiter or arbitration panel complete authority to fashion a final award. The final-offer arbitration (FOA) procedure allows the arbiter only to choose one of the two final offers made by the parties. Though some empirical work has appeared comparing the two procedures, the theoretical analysis of the procedures is still incomplete. This paper has concentrated on the effect of uncertainty about the arbiter's preferences on the bargainers, while ignoring other aspects of the comparison such as permitting the arbiter to trade-off among bargaining issues.**

I. Introduction

COMPULSORY binding interest arbitration is becoming an increasingly accepted form of third-party intervention in industrial disputes where work stoppages are socially undesirable, such as those involving public safety employees. Seventeen states now have statutory provisions for an arbitration recourse in the event of a bargaining impasse. In some cases arbitration has supplanted other modes of third-party intervention like mediation and fact-finding with nonbinding recommendations; in many others it has supplemented them as a last resort.

The arbitration procedures in different states differ in several important aspects (see [7]–[9], [13], and [14]). We shall, however, consider only two categories of procedure: conventional arbitration (CA) and final-offer arbitration (FOA).

In *conventional arbitration* the arbiter is unrestricted in his freedom to fashion what he might consider a reasonable award and to impose it on the participants. Though the statutes usually mention some criteria for making the award, the arbiter usually has a great deal of leeway in choosing a solution.

This has led to the criticism that the arbitration process allows third parties ("itinerant philosophers" as arbiters are sometimes unflatteringly described) to impose their values on the bargainers. Typically, employers have complained of a bias in favor of employees leading to unreasonably high awards on wages and other economic issues.

A second criticism of conventional arbitration has been that it leads to a "chilling" effect on the bargaining. The availability of arbitration, critics say, relieves the bargainers of the sometimes politically unpleasant task of compromising, and the collective bargaining process loses its significance.

Final-offer arbitration was developed to meet these criticisms. In this mode of arbitration the parties bargain. If they fail to reach an agreement (even after intermediate steps like mediation or fact-finding), the dispute is sent to the arbiter who asks each party to submit a final offer. The arbiter is then restricted to choosing one of the two final offers as the final settlement.

Proponents of final-offer arbitration argue that the procedure greatly reduces the chilling effect of arbitration on bargaining, since each side will moderate its offers in order to appear reasonable to the arbiter. It would also, in their view, diminish the amount of arbitrariness in arbitration, since the arbiter would have to choose a solution reflecting the values and preferences of (at least one of) the participants.

Empirical findings have not settled the question one way or the other. Earlier theoretical work [4]–[6] has arrived at the conclusion that there is no substantive difference between the results of FOA and conventional arbitration. Our theoretical models, whose rationale is described more fully in the next section, do not support this result. In our first model, which most closely resembles previous work, we show that the introduction of uncertainty about the arbiter's choice makes bargainers adopt more extreme positions under FOA than under conventional arbitration, a result that would appear to undermine the FOA argument at what appears its strongest point. In other models we examine the effects of different information patterns on the procedures and relate arbitration to our earlier models of bargaining described in [1]–[3]. As a continuation of the research described here, one direction would be to try to relate the theoretical approaches to the actual case histories of arbitration. We do not attempt this here.

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The author is with the Division of Management Science, Pennsylvania State University, 310 Business Administration Building, University Park, PA 16802.

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II. Preliminaries to Formal Modeling

In this section we discuss the various issues that need to be considered in a theoretical comparison of the two procedures and offer a rationale for the questions that we do consider in the theoretical models of the following sections.

A. Single-Issue Versus Multiple-Issue Disputes

We are going to limit ourselves to disputes along a single dimension, for example, a dispute on the wage rate. This has the disadvantage, as has been pointed out [4], that all bargaining outcomes are ex post Pareto optimal, so that issues relating to the ex post efficiency of FOA and conventional arbitration are not considered. However the assumption of single-issue bargaining serves to keep our models analytically tractable as we introduce uncertainty and differential information. We are able to obtain insights with our single-issue models on the important question of the effect of the arbitration option on collective bargaining, in particular on the magnitude of the "chilling" effect, if any.

B. How Many Stages Should Our Models Have

As mentioned earlier the process of resolution of industrial disputes could (and does) consist of many stages —beginning with bargaining and encompassing mediation, fact-finding, the "final offers" handed in at the beginning of arbitration hearings, and the amended final offers at the end of the hearings. We shall restrict ourselves to two stages: a bargaining stage and an arbitration stage. We implicitly include fact-finding with recommendations as well as those situations where we assume that the parties to the dispute know what the arbiter's ideal solution is. Though the arbiter and the fact finder are usually different persons, empirical studies [8] have noted that arbitration awards are exactly the same as the fact finder's recommendations in 70 percent of the cases studied.

In the first stage of the process, the parties bargain with each other. If they agree, the process terminates. If they fail to agree, the second stage begins: the arbitration stage. At the end of arbitration, the process terminates. (The bargaining–arbitration sequence is repeated every time a new contract is being negotiated. We consider only a single negotiation and do not discuss possible interactions between negotiations.)

C. Criteria for the Arbiter's Choice

Broadly speaking, the arbiter's choice could depend upon the offers made by the bargainers and his or her own equity judgments. We distinguish between those arbitration procedures that are independent of the bargainers' offers and those that are so dependent.

Conventional arbitration, at least of the type that is considered in this paper, depends only on the arbiter's perceptions of equity. The arbitration award is then just the arbiter's equity-maximizing solution. An alternative form of conventional arbitration, one that is not discussed here, could for example, have the arbiter choosing a solution midway between the offers of the two sides and thus make the award responsive to the offers made by the bargainers.

Final-offer arbitration would, of course, always depend on at least one of the offers submitted. We discuss two kinds of final-offer arbitration here, one that depends solely upon the closeness of the two offers to the arbiter's equity-maximizing solution, and one that seeks to reward bargainers for some compromise relative to known extreme initial positions taken by the two sides. Professional arbiters apparently often dislike the second approach because it leads to gamesmanship during the bargaining process.

A third type of FOA that we do not consider here would have the arbiter choosing each offer with probability $1/2$ as the equivalent of the conventional arbitration approach of choosing the middle point between two offers. (It is clear of course that risk-averse individuals would prefer the conventional approach, once they were in the bargaining phase.)

D. The Basic Bargaining Model and Extension to Arbitration

As mentioned earlier we are not going to be concerned with multiattribute bargaining situations in this paper, so we shall not look into the distinctions between selection of package final offers and selection of final offers issue by issue. We shall consider bargaining along one dimension, e.g., the wage rate, with the implicit understanding that all other issues have either been settled or are not up for negotiation.

We further assume that the range of possible bargaining outcomes is known to both parties and to the arbitrator. This range, for example, would be given by the current wage at the lower end and by the management's maximum ability to pay at the upper end. The latter quantity is probably not initially known but could be determined by the parties by asking management to provide evidence to back any claim it makes (such as access to financial statements). This is, of course, not a complete description of labor–management bargaining, especially in cases where strikes are permitted. Labor might be willing to strike rather than to accept the current wage, and management might prefer no agreement to one that stretches its resources to the limit. In this case the true range would be smaller than the one we have assumed to be the case. Our rationale for this is that strikes are not permitted in the environment in which our bargainers work. The arbitration recourse is our alternative to strikes, and we do model explicitly the effect of arbitration on the range of agreements that can be concluded in the bargaining phase. We do not deal, in this paper, with the situation where the end points of the bargaining range (say, the current wage and management's maximum ability to pay) are either both unknown or partially unknown. We normalize outcomes so that the bargaining range is [0, 1] where the maximum ability to pay of management is denoted by 1.

The most realistic formulation of the bargaining would be as a noncooperative multistage game. However such a game is difficult to solve analytically. We therefore consider one-stage approximations for all the analytical results. These one-stage models bear a close family resemblance to earlier work by the author and others on bargaining under uncertainty ([1]–[3], [11], and [12]). In one of these earlier bargaining models we consider a situation where each of the two bargaining parties has a *reservation price* that embodies the maximum limit of concessions the bargainer is prepared to make. The exchange of offers (in our model, a simultaneous exchange) takes place against the backdrop of the underlying reservation price structure that may be partially unknown to one or both of the players. The special feature in this paper is that the reservation price for the bargaining phase is set by the expected outcome in the arbitration phase. In FOA or in any type of arbitration procedure responsive to the offers made by the bargaining parties, the expected payoff in the arbitration phase depends on the actions of players in that phase and is thus controllable to some extent. If the actions of players in the bargaining phase also affect their payoff in the arbitration phase we have a complicated interdependence which determines both the reservation prices and the actual outcome.

It might be interesting to pause for a moment to consider the relationship of our arbitration models with the arbitration model of Raiffa and the bargaining model of Nash (see [10]). If the Nash/Raiffa solutions are interpreted as arbitration procedures the work of the arbiter is conceived as the determination of an equitable efficient outcome in the range of bargaining alternatives. In the event of conflict the arbiter has no role. In our models the arbiter essentially specifies a *conflict* payoff, which is relevant only when there is no agreement. If an agreement is possible the bargainers divide the surplus equally.

This difference is related to the point of view we adopt that bargaining is essentially a noncooperative process with players having to decide whether to hold out for individual gain or to go along with an efficient cooperative solution (see [1]–[3] for further details). Thus, in our models, it is entirely possible for an impasse to occur, even if a mutually beneficial agreement exists. This is most clearly expressed in Section IV on models with differential information. In our other models as well, impasses could occur if, for example, labor (hereafter referred to as player L) were to demand more than management (player M) were willing to accept. We therefore adopt the noncooperative solution concept of Nash equilibrium in all cases (see [10] for a discussion of this concept), including the Harsanyi extension to Bayesian Nash equilibrium in Section IV (see [11] and [12] for further discussion). A detailed discussion of various arbitration models in relation to their use of cooperative and noncooperative concepts is contained in Crawford [5].

E. Informational Assumptions

We shall assume throughout that the rules of the game, including the procedure by which an arbiter arrives at a final choice, are known to everybody. Other aspects of the process may be completely known to all parties or completely known to some and partially to others or incompletely known.

For example, we could assume *complete information*: both parties know all data of the problem, including the arbitrator's equity-maximizing solution (though each does not know what the other is going to do); each party also knows any possible costs borne by itself and its adversary as a result of continuing bargaining as well as of going to arbitration.

A less stringent assumption would be *incomplete but identical information*. That is, both parties would be uncertain about the arbitrator's ideal solution but would have the same information (embodied in a probability distribution) about what it could be. We could compare modes of behavior under this assumption to those under the complete information assumption and check whether it is better for both parties (during bargaining) for the arbitrator to reveal what an externally desirable solution might be.

Finally we could look at models with *differential information* in which each side has private information not available to the other party or to the arbitrator. One way to include differential information is to assume that bargainers have different stage costs or time costs. This would appear to give these costs a significant role in determining bargaining posture, as does happen with the last model we consider. It is not clear how large these costs could be in comparison to the settlement value. It could be substantial if a bargainer has limited resources and has to hire professional negotiators during arbitration. In addition, however, the implicit or perhaps psychic costs in going from the bargaining to the arbitration stage might explain why the industrial relations literature places so much stress on the outcome being determined in the bargaining phase. This brings us to the next issue in modeling the process.

F. Criteria for Evaluating Alternative Procedures

Most of the literature in the labor relations area seems to judge the success of an arbitration scheme by its deterrent effect. In other words, how successful has the scheme under consideration been in persuading bargainers to settle all issues without the aid of the arbitration mechanism; that is, how has a scheme affected the probability of agreement in the bargaining phase? This seems to be too narrow to use as the sole criterion to evaluate a procedure. If the arbitrator informs bargainers about his ideal solution this might well have an effect on getting the bargainers to argue in the neighborhood of the solution and to reach an agreement thereby. However the solution is imposed and may not satisfy symmetry and fairness conditions natural to the problem. It could also restrict the range of choices available to the bargainers. In the case where there is more than one issue at stake, an imposed solution is almost certainly inefficient. In our case the efficiency issue is sidestepped by the special nature of the "distributive bargaining" game we are considering.

III. One-Stage Models under Complete or Incomplete but Identical Information

Complete Information: Here we shall assume that three arbitration schemes, two of which have a possible interpretation as FOA, exist.

First, the arbitrator imposes a solution x which represents his ethical and other details. The solution x is known to both parties. We shall call this conventional arbitration (CA).

Second, the arbitrator chooses the offer (in the arbitration phase) which is nearest x in terms of absolute value. We shall refer to this as FOA 1.

Third, the arbitrator selects the offer which represents the less extreme demand (or the greater concession from one or zero, respectively, for labor and management). We call this FOA 2.[1]

We assume that the offers used by the arbiter are the same final offers the parties made to each other during bargaining. We assume that the costs of going from bargaining to arbitration and the costs incurred during bargaining are all zero. (This assumption will be relaxed later.)

Now consider the game that results. Formally, the game is as follows. Players L and M submit offers a_l and a_m, respectively to each other. If $a_m \geqslant a_l$, there is an agreement at $(a_m + a_l)/2$. If not, the arbiter is called in and he or she uses either CA, FOA 1, or FOA 2.

If x is the arbitrated outcome, the payoffs are

$$1 - x \qquad \text{to player } M \tag{1}$$

and

$$x \qquad \text{to player } L. \tag{2}$$

If $a_m \geqslant a_l$, the payoffs are

$$1 - \frac{a_m + a_l}{2} \qquad \text{to player } M \tag{3}$$

and

$$\frac{a_m + a_l}{2} \qquad \text{to player } L. \tag{4}$$

Now it is clear that the following holds.

Proposition 1: In the game as described above, a Nash equilibrium strategy pair is for each to offer

$$a_l = a_m = x \tag{5}$$

if CA or FOA 1 is used as the arbitration procedure.

[1]Mathematically, of course, FOA 2 is a special case of FOA 1 with $x = 1/2$. However, FOA 1 and FOA 2 imply different kinds of arbitration behavior: one considering only the closeness of the final offers to the arbiter's preferences, the other relating the final offers to the initial offers and noting the extent of the concessions made.

Proof: The proof is obvious (see [4] for a much more general proof).

However, if FOA 2 is used, the focal point is different. Before starting the equilibrium analysis, it might be advisable to state the procedure formally. Suppose player L offers $a_l \leqslant a_m$, there is an agreement at $(a_l + a_m)/2$. If $a_l > a_m$, the arbitrator chooses the minimum of $(1 - a_m, a_l)$. That is, he chooses a_m, if $1 - a_m < a_l$ and vice versa for the opposite strict inequality. In the case where $1 - a_m = a_l$, the arbitrator tosses a coin to choose between a_l and a_m.

Proposition 2: Under FOA 2, a pure strategy Nash equilibrium is for players to make their offers such that

$$a_l = a_m = \frac{1}{2}. \tag{6}$$

Proof: Suppose $a_m = 1/2$. Player L will not offer anything less than $1/2$, since he could gain by moving to $1/2$. However if player L offers $a_l > 1/2$, the arbitrated solution will be a_m, so that he cannot gain by offering more than $1/2$. A similar chain of reasoning holds for player M (this result would be identical, if FOA 1 is used and $x = 1/2$).

If player L makes an offer $a_l > 1/2$, say $3/4$, what is player M's best response offer? Clearly, $a_m \leqslant 3/4$, but there is no reason why player M should stop below $a_m = 1/4 + \epsilon$. If he offers this, he is certain to win the award and obtain a payoff $(3/4 - \epsilon)$ where ϵ can be made arbitrarily small. Therefore, points on the line apart from $1/2$ are not equilibria. This is different from the case of CA where (with zero costs) any offer however extreme would lead to the same solution. FOA 1 gives a similar unique equilibrium.

The arbitration games considered above set reservation prices for the bargaining phase. For example, if player L knows that CA or FOA 1 is to be used if there is no agreement, he would not settle for less than x in the bargaining. Player M, on the other hand, would not be willing to accept any compromise greater than x. This effectively limits the bargaining to one point. A similar argument holds for FOA 2.

We have assumed costs of going to the arbitration phase to be zero. If we introduce some costs, c_l and c_m to labor and management, respectively, the reservation prices for CA and FOA 1 become $x - c_l$ for player L and $x + c_m$ for player M. If the costs are both known we could predict (maybe not very successfully) the bargaining outcome by using something like the Nash scheme. The case where the costs are not known is covered later.

Incomplete Information: Now suppose that the arbitrator's ideal solution x is unknown to the players but that they share common probabilistic assessments (summarized in a distribution $F(\cdot)$) on x. Suppose further that both players are risk-neutral.

Proposition 3: Under conventional arbitration, the players will offer, in equilibrium,

$$a_l = a_m = \bar{x}. \tag{7}$$

Proof: Obvious.[2]

The situation is somewhat different under FOA 1. Assume that both players are risk-neutral again. Consider the arbitration phase as a separate game as before. Assuming the player M is going to play a_m, what would be player L's best response in pure strategies? He could either choose $a_l = a_m$ or decide to choose $a_l > a_m$ and maximize his expected value given the probability distribution on the arbitrator's award.

The arbitrator's choices are as follows.

If $x > a_l > a_m \rightarrow$ choose a_l.

If $a_l > x > a_m$

and

$a_l - x < x - a_m \rightarrow$ choose a_l.

If $a_l > x > a_m$

and

$a_l - x - a_m \rightarrow$ choose a_m.

If $a_l > a_m > x \rightarrow$ choose a_m.

Whenever the equality sign holds, randomization is used. Player L's expected value under a distribution $f(\cdot)$ is

$$\int_{(a_l+a_m)/2}^{\infty} a_l f(x)\,dx + \int_{-\infty}^{(a_l+a_m)/2} a_m f(x)\,dx. \quad (8)$$

Notice that (8) minus a_m is always positive, so that it is always to player L's advantage to state $a_l > a_m$. Differentiating this and by setting it equal to zero to obtain the optimal a_l in the interior of the region $[a_m, 1]$, we get

$$1 - F\left(\frac{a_l + a_m}{2}\right) - \frac{a_l}{2} f\left(\frac{a_l + a_m}{2}\right) + \frac{a_m}{2} f\left(\frac{a_l + a_m}{2}\right) = 0. \quad (9)$$

If $F(\cdot)$ is uniform on $[\alpha, \beta]$, this simplifies to

$$\left(\beta - \frac{a_l + a_m}{2}\right) + \frac{a_m}{2} - \frac{a_l}{2} = 0, \quad (10)$$

or

$$(\beta - a_l) = 0. \quad (11)$$

This gives $a_l = \beta$, irrespective of a_m. Thus we have the following proposition.

Proposition 4: Under FOA 1, and with $\tilde{x}$ uniform, a dominant strategy for labor is to make the most extreme demand feasible.

Therefore, under certain conditions, FOA 1 could lead to more extreme positions than CA.[3] This could happen by one player reasoning that if the other also took an extreme position, each had a good chance of winning a large amount, while if the other player took a more moderate position, the arbitration award would be lost but it would not matter because it would be more acceptable anyway.

The rather surprising result obtained for the uniform distribution does not generalize. That is, it is not true in general that player L's best strategy is to announce β irrespective of player M's choice a_m. However, even in other cases it seems a player would gain by extreme demands. We consider just one case.

Let x be distributed on $[0, \beta]$ with a density function

$$f(x) = \frac{2}{\beta} - \frac{2x}{\beta^2}.$$

Notice that this is largest when $x = 0$. We would expect therefore that player L, fearing a low value of x, would moderate his offer. However, applying (9) above we obtain

$$1 - \frac{2}{\beta}\left(\frac{a_l + a_m}{2}\right) + \frac{1}{\beta^2}\left(\frac{a_l + a_m}{2}\right) + \frac{(a_m - a_l)}{2}\left(\frac{2}{\beta} - \frac{2}{\beta^2} \cdots \frac{a_l + a_m}{2}\right) = 0,$$

or

$$1 - \frac{a + a_m}{\beta} + \frac{1}{4\beta^2}\left(a_l^2 + a_m^2 + 2a_1 a_m\right) + \frac{2}{\beta}\left(\frac{a_m - a_1}{2}\right)\cdot\left(1 - \frac{1}{2}(a_1 - a_m)\right) = 0.$$

Simplifying, we get

$$1 - \frac{2a_l}{\beta} + \frac{3}{4}\frac{a_l^2}{\beta^2} - \frac{a_m^2}{4\beta^2} + \frac{a_l a_m}{2\beta^2} = 0.$$

Remembering that $\beta \geqslant a_l > a_m$, we see that the derivative of the above expression is negative so that the second-order condition is satisfied. Further, a quick calculation shows that

$$\frac{da_l}{da_m} \geqslant 0.$$

Thus, the offers move in the same direction. Now suppose that $a_m = 0$, the most extreme management offer. Substituting above we get

$$a_l = \frac{2}{3}\beta.$$

However, as management becomes more moderate, it is to labor's advantage to become more extreme, so that the procedure appears to contain incentives for doing exactly the thing its proponents wish to avoid.

IV. A Model with Differential Information

A. A Description of FOA and Conventional Arbitration in a Simple Model with Differential Information

Conventional Arbitration: Two players, labor (player L) and management (player M), bargain on a wage bill which

[2]However, if players L and M have different utility functions, the lottery on $\tilde{x}$ will be evaluated differently by them. The arbitration game would be worth certainty equivalent (CE)(M) to player L and (CE)(L) to management. These would then be their reservation prices for the bargaining phase. If one player's CE is not known to the other, we are in the situation where players have private information about their own reservation prices and bargain noncooperatively, using their information to gain strategic advantage. This is analyzed elsewhere.

[3]As pointed out by a referee, the equilibrium of Proposition 3 is only one equilibrium, and more extreme positions are also in equilibrium (because it does not matter what the bargainers do, so long as they do not agree). This might appear to undermine the position taken here that FOA leads to more extreme positions than CA. However, the incentive to shift to extreme positions in CA is clearly a weak one (you do not lose by so doing), while in FOA, bargainers can expect to gain by extreme positions.

has to be in the range [0, 1] where the current wage bill is normalized at zero, and the known maximum profit level (or maximum ability to pay of the management) at one.

The game proceeds as follows: there are independent random drawings from distributions $F_l(\cdot)$ and $F_m(\cdot)$. Player L is told the value of c_l, the result of the first drawing, and player M the value of c_m, the second drawing. In the next move the players simultaneously make offers. Let player L's offer be denoted by a_l and player M's offer by a_m, where

$$a_l = A_l(c_l)$$

and

$$a_m = A_m(c_m)$$

where the $A_l(\cdot)$ and $A_m(\cdot)$ functions relate a player's act to his or her information. If $a_m \geqslant a_l$, the payoffs are

$$\frac{a_m + a_l}{2} \qquad \text{to player } L \tag{12}$$

and

$$1 - \frac{a_m + a_l}{2} \qquad \text{to player } M.$$

If $a_m < a_l$, the arbitrator imposes a solution x and the payoffs are

$$x - c_l \qquad \text{to player } L \tag{13}$$

and

$$1 - x - c_m \qquad \text{to player } M.$$

The returns R_l^c, R_m^c to each player can be written as

$$R_l^c(a_l, a_m; c_l) = \left(\frac{a_m + a_l}{2}\right)\delta(a_m, a_l) + (x - c_l)(1 - \delta(a_m, a_l)), \tag{14}$$

$$R_m^c(a_l, a_m; c_m) = \left(1 - \frac{a_m + a_l}{2}\right)\delta(a_m, a_l) + (1 - x - c_m)(1 - \delta(a_m, a_l)), \tag{15}$$

where

$$\delta(a_m, a_l) = 1, \qquad a_m \geqslant a_l,$$
$$= 0, \qquad a_m \leqslant a_l. \tag{16}$$

In other words, the failure of bargaining implies costs to both parties and each party's cost is unknown to the other. The arbitrator's solution is externally decided and imposed.

Final-Offer Arbitration: The informational aspects of the model are the same as in the conventional arbitration procedure. The difference is in the arbitrator's solution. The arbitrator chooses

$$a_l \text{ if } a_l < 1 - a_m, \tag{17}$$

and

$$a_m \text{ if } a_l > 1 - a_m$$

and his award. Therefore, he chooses that offer which demands a more modest payoff. (If the offers make equal demands, randomization decides which is chosen.)

The returns to players L and M are therefore

$$R_l^F(a_l, a_m; c_l) = \left(\frac{a_m + a_l}{2}\right)\delta(a_l, a_m) + (a_l - c_l)\gamma(a_l, a_m)(1 - \delta(a_l, a_m)) + (a_m - c_l)(1 - \gamma(a_l, a_m))(1 - \delta(a_l, a_m)) \tag{18}$$

and

$$R_m^F(a_l, a_m; c_m) = \left(1 - \left(\frac{a_l + a_m}{2}\right)\right)\delta(a_l, a_m) + (1 - a_l - c_m)\gamma(a_l, a_m)(1 - \delta(a_l, a_m)) + (1 - a_m - c_m)(1 - \gamma(a_l, a_m))(1 - \delta(a_l, a_m)) \tag{19}$$

where

$$\gamma(a_l, a_m) = 1 \text{ if } a_l < 1 - a_m,$$
$$= 0 \text{ if } a_\omega > 1 - a_m, \tag{20}$$

and

$$\delta(a_l, a_m) \text{ is defined as before.}$$

In the next section we calculate how the players' strategies differ under the two modes of arbitration when costs are distributed uniformly. We expect player L's offer to decline as his cost rises and player M's offer to rise with his cost. The rate of that change in offer as cost increases could be interpreted as a concession rate (though dynamics are not explicitly built into the model), since the cost to a bargaining party goes up as the time spent in bargaining increases. If we assume that these costs will be borne at the end of the bargain (if there is no agreement), we obtain a model like ours.

B. Equilibrium Analysis of the Two Procedures

Proposition 5: One equilibrium pair of strategies under conventional arbitration is

$$a_l = x = a_m, \qquad \text{for all } c_l, c_m > 0. \tag{21}$$

Proof: Suppose player M offers $a_m = x$. Player L would not then offer anything less than x, since he could always gain by moving up to x. If he offers anything more than $x, a_l > a_m$, and there is a disagreement so that his payoff reduces to $x - c_l \leqslant x$, for $c_l \geqslant 0$. The same reasoning holds for player M.

However, this equilibrium is not very interesting since it does not exploit a player's knowledge about his costs and his beliefs about the other player's costs. Intuitively one would expect such information to be used in the bargaining phase by players each seeking his individual advantage, with a low-cost player seeking to exploit his advantage by holding out for more.

In order to derive an equilibrium taking the differential cost aspect into *account*, we make the following assumptions.

1) Player L's strategy is a function $A_l(\cdot)$ which is strictly decreasing in c_l.

2) Players M's strategy is a function $A_m(\cdot)$ which is strictly increasing in c_m.
3) The probability distributions $F_m(\cdot), F_l(\cdot)$ have density functions everywhere.
4) The inverse functions $A_l^{-1}(\cdot)$ and $A_m^{-1}(\cdot)$ are differentiable everywhere.

Proposition 6: Given the assumptions about the model, an equilibrium pair of strictly monotone strategies under conventional arbitration will satisfy the following linked differential equations:

$$(x-c_l-a_l) = -\frac{1}{2}\frac{\left(1-F_m\left(A_m^{-1}(a_l)\right)\right)}{f_m\left(A_m^{-1}(a_l)\right)A_m^{-1\prime}(a_l)}, \tag{22}$$

and

$$(a_m-x-c_m) = \frac{1}{2}\frac{1-F_l\left(A_l^{-1}(a_m)\right)}{f_l\left(A_l^{-1}(a_m)\right)A_l^{-1\prime}(a_m)}. \tag{23}$$

Proof: Define Player L's conditional expected return given c_l and $F_m(\cdot)$ as

$$\bar{R}_l^c[a_l, A_m(\cdot)|F_m(\cdot), c_l] = \int_{A_m^{-1}(a_l)}^{\infty}\frac{a_l+A_m(c_m)}{2}f_m(c_m)\,dc_m + \int^{A_m^{-1}(a_l)}(x-c_l)f_m(c_m)\,dc_m. \tag{24}$$

In order to determine the maximum, we differentiate this and set it equal to zero, thus obtaining

$$\frac{1}{2}\left(1-F_m\left(A_m^{-1}(a_l)\right)\right) - a_l f_m\left(A_m^{-1}(a_l)\right)A_m^{-1\prime}(a_l) + (x-c_l)f_m\left(A_m^{-1}(a_l)\right)A_m^{-1\prime}(a_l) = 0. \tag{25}$$

Rearranging terms gives us (22). Equation (23) is derived analogously.

The quantity $(x-c_1)$ acts as a reservation price for player L, and it is partially unknown to player M, so that the framework set up here for the analysis of conventional arbitration is an extension of the distributive bargaining model analyzed elsewhere. However the extension enables us to obtain equilibria like that in Proposition 5 which is not possible in the earlier analysis.

From the point of view of conventional arbitration, the possibility of obtaining an equilibrium in monotone strategies may explain why agreements are not always reached in the bargaining phase. One would have been led to expect if Proposition 5 were the only equilibrium (other than the unappealing "no-bargain" kind) that the arbitration recourse would never be used. This is, of course, not empirically borne out.

In the next result we look at a special distribution of costs, the uniform distribution, which we shall use here for the purpose of comparing conventional arbitration and final-offer arbitration. First, we derive the equilibria under conventional arbitration.

Proposition 7: Let $F_m(\cdot)$ be uniform $[\alpha,\beta]$ and $F_l(\cdot)$ be uniform on $[\gamma,\delta]$. Then a pair of linear equilibrium interior strategies are

$$a_l = A_l(c_l) = x + \frac{\beta}{4} - \frac{\delta}{12} - \frac{2}{3}c_l, \tag{26}$$

and

$$a_m = A_m(c_m) = x - \frac{\delta}{4} + \frac{\beta}{12} + \frac{2}{3}c_m. \tag{27}$$

Proof: The proof follows from (22) and (23) on substituting in the assumptions about $F_m(\cdot)$, $F_l(\cdot)$ and the linearity of the strategies.

The factor "x" which represents the arbitrator's intervention in the bargaining process can be manipulated by the arbitrator to shift the possible distribution of the total pie in favor of either labor or management. It cannot, however, affect the slope of the strategies (which we have interpreted, somewhat artificially, as a rate of concession in the bargaining). Since it occurs in exactly the same way in both players' strategies, it cannot be used to increase the probability of agreement.

Note that, as pointed out by a referee, the strategies given by (26) and (27) do not depend on the lower bounds α, γ of the two distributions. The mathematical reason for this is the form of (22) and (23) where the distributions enter in the form of $(1-F)/f$, so that the lower bounds cancel where F is uniform. Intuitively, (26) and (27) say that each player is worried only about how high the other player's costs could go and about how high the other player thinks his or her costs could go. Thus player L would increase his demand if he knew that β was high and would decrease his demand if the upper bound of his own cost distribution, namely δ, were to increase.

We now look at final-offer arbitration, where the arbitrator selects one of the two final offers in accordance with the mechanism postulated in the previous section. Player L's expected return given c_l under FOA is written as

$$\bar{R}_l^F(a_l, A_m(\cdot)|F_m, c_l) = \int_{A_m^{-1}(a_l)}^{\infty}\frac{a_l+A_m(c_m)}{2}\,dF_m(c_m) + \int_{A_m^{-1}(1-a_l)}^{A_m^{-1}(a_l)}(A_m(c_m)-c_l)\,dF_m(c_m) + \int^{A_m^{-1}(1-a_l)}(a_l-c_l)\,dF_m(c_m). \tag{28}$$

The necessary conditions for optimality in this case do not have the same relatively neat expression as they do in the conventional arbitration case since they involve both $A_m^{-1}(a_l)$ and $A_m^{-1}(1-a_l)$ in the arguments of both $F_m(\cdot)$ and f_m. For reference, they are given in Proposition 8.

Proposition 8: The necessary conditions for equilibrium strategies $A_l(c_l)$ and $A_m(c_m)$ under FOA are as follows:

$$\frac{1}{2}\left(1-F_m\left(A_m^{-1}(a_l)\right)\right) + F_m\left(A_m^{-1}(1-a_l)\right) - c_l f_m\left(A_m^{-1}(a_l)\right)A_m^{-1\prime}(a_l) + f_m\left(A_m^{-1}(1-a_l)\right)A_m^{-1\prime}(1-a_l)(1-2a_l) = 0, \tag{29}$$

and

$$-\frac{1}{2}\left(1-F_l\left(A_l^{-1}(a_m)\right)\right)-F_l\left(A^{-1}(1-a_m)\right) -c_m f_l\left(A_l^{-1}(a_m)\right)A_l^{-1\prime}(a_m) +(2a_m-1)f_l\left(A_l^{-1}(1-a_m)\right)A_l^{-1\prime}(1-a_m)=0. \quad (30)$$

Proof: These follow by differentiating the expressions for conditional expected returns and setting the result equal to zero.

Note that under this mode of arbitration there is no fixed focal point like "x" in our version of conventional arbitration. One would therefore expect the players to use monotone strategies, and if they were players well-versed in noncooperative game theory, the equilibrium monotone strategies given by (29) and (30). We now go to the uniform distribution special case with $F_m(\cdot)$ uniform on $[\alpha, \beta]$ and $F_l(\cdot)$ uniform on $[\gamma, \delta]$.

Proposition 9: An equilibrium pair in linear strategies with the above distributional assumptions is given by

$$a_l=A_l(c_l)=\frac{1}{12}\left(\frac{\beta}{2}-\alpha\right)-\frac{1}{84}\left(\gamma-\frac{\delta}{2}\right)+\frac{1}{2}-\frac{2}{7}c_l, \quad (31)$$

and

$$a_m=A_m(c_m)=\frac{1}{12}\left(\gamma-\frac{\delta}{2}\right)-\frac{1}{84}\left(\frac{\beta}{2}-\alpha\right)+\frac{1}{2}+\frac{2}{7}c_l. \quad (32)$$

Proof: As in analogous results here, we apply (29) and (30) assuming that $A_l(c_l)$ is a strictly decreasing linear function and $A_m(c_m)$ is a strictly increasing linear function. This gives us the slope 2/7 of the linear strategies and two equations relating the two intercepts. Solving these gives us (31) and (32).

In order to simplify our comparison even further, let us consider the case where $F_m(\cdot)$ and $F_l(\cdot)$ are identical uniform distributions on $[0, \beta]$.[4] The equilibrium strategies are then as follows.

Under conventional arbitration we have

$$a_l=A_l(c_l)=x+\frac{\beta}{6}-\frac{2}{3}c_l, \quad (33)$$

and

$$a_m=A_m(c_m)=x-\frac{\beta}{6}+\frac{2}{3}c_m. \quad (34)$$

Under FOA we have

$$a_l=A_l(c_l)=\frac{\beta}{21}+\frac{1}{2}-\frac{2}{7}c_l, \quad (35)$$

and

$$a_m=A_m(c_m)=-\frac{\beta}{21}+\frac{1}{2}+\frac{2}{7}c_m. \quad (36)$$

[4] The linear strategies given hold as long as the probability of agreement is less than one. As soon as this probability becomes one (e.g., for values of $c_e \geq \beta/2$), the strategies become constant. This, of course, does not affect the probability of agreement as calculated in the text.

Equations (33) to (36) illustrate two interesting properties.

1) If the arbitrator's choice x is 1/2, we see that under FOA, players are less extreme in their offers, differing only by $2\beta/21$ at most, while under conventional arbitration, they could differ by as much as $\beta/3$.
2) However, while bargainers tend to come down at a rate 2/3 under conventional arbitration, they only do so at a 2/7 rate under FOA. So flexibility in negotiation is not a virtue of FOA in this model, contrary to the claims of its sponsors.

If the probability of agreement is the criterion used to judge the two procedures, FOA comes out better than conventional arbitration. Under FOA there will be agreement if the sum of the costs exceeds $\beta/3$. In conventional arbitration agreement will take place when the sum of the costs exceeds $\beta/2$, a greater quantity.

These conclusions have been derived from considering a particular example. Different models of FOA and conventional arbitration and fewer heroic assumptions might yield different results. We intend to follow this up in future research.

V. Conclusion

We have discussed models of FOA and conventional arbitration under different informational assumptions. Our results would not cause us to recommend clear advocacy of one form of arbitration over another. However we feel that we have pointed out the inherent instability in final-offer arbitration, especially in Proposition 4. The model on which that proposition is based is particularly simple and surely reflects an essential feature of real-life bargaining—uncertainty about the arbiter's choice. The fact that this uncertainty is capable of causing extreme stands in bargaining is, or should be, disquieting. The distributional assumption (the uniform distribution) is certainly not unchallengeable, but neither does it appear implausible since bargainers might be expected to know the range of "fair" outcomes, but not which one the arbiter prefers.

Section IV of our paper, on models with different information, is an attempt to construct a model where the probabilities of agreement (in bargaining) under different arbitration procedures can be calculated explicitly. The author believes that there are very few other models in existence, if any, that enable us to do that, though as Crawford ([5]) indicates, research in this direction is urgently called for. Once again we assume the uniform distribution (once again surely not a wild assumption) to obtain explicit results. (Typically, bargaining models with differential information tend to have equilibria that are difficult to determine explicitly unless an analytically tractable distribution is assumed.) Section IV allows us to give only half a vote for FOA, since though the probability of agreement is higher under FOA, it appears to lead to a slow concession rate during the bargaining.

Our efforts in the direction of comparing the two procedures are, of course, still preliminary. A complete model would include risk-aversion as well as the trade-off between efficiency *after* disagreement and efficiency ex ante (before the process begins).

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