

Simultaneous Vs. Sequential Auctions, Intensity of Competition and Uncertainty

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Abstract

Under what circumstances will an auctioneer be better off selling his inventory sequentially, rather than selling them all in a single auction? If bidders come sequentially, there is an obvious reason to sell items sequentially. However, even when all bidders present at the beginning of the auction, and both the auctioneer and bidders are impatient, we show that a sequential auction can still be more profitable for the auctioneer, depending on the “intensity of competition”, which is characterized by the number of items available relative to the number of bidders. Moreover, a sequential auction without a reserve price can perform better than an auction with one, if the auctioneer cannot commit not to (eventually) sell items below any reserve price announced at the beginning of the auction. A sequential auction can also be better when bidders do not know exactly how many items are available for sale. The auctioneer can then take advantage of this uncertainty by pretending to have low inventory, to stimulate competition among the bidders. This result is then extended to a more general T period setting.

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1 Introduction

On April 25, 2003, the BBC Nightly News reported an upcoming auction of 600 Picasso lithographs at Christie's in New York. There was apparently some concern at Christie's that auctioning off so many similar items would lead to some of them being sold for only a few hundred dollars apiece. In an article on Christie's web site ¹, Jonathan Rendell, the auction house's Deputy Chairman, mentions that the consignor had decided to sell with no reserve price, and that estimated prices were as low as five hundred dollars.

This brings up one important question in auction design: when several similar items (in our model we simplify to identical items) are being sold, and the price might be driven low by lack of competition because of the relatively low ratio of number of bidders to number of items for sale, should the auctioneer still sell all the items together in one lot or split it into several lots to be sold off sequentially?

Intuitively, if potential buyers arrive sequentially over time, there is an obvious reason for holding some items back (as airlines often do) so as to take advantage of high-value buyers who arrive late ². However, most of the voluminous literature in economics on auctions has assumed a fixed, known number of bidders and this clearly corresponds to some real-life auction situations. In the example mentioned above, there is no evidence that a substantial number of potential buyers was not present at the time of the proposed auction. In bidding for contracts, the buyer of the services often has a list of qualified bidders and it is a non-trivial task to get into this list from outside.

Moreover, when selling multiple items, will it be beneficial for the auctioneer to reveal the information about the total number of items available for sale? At some Internet auction

¹<http://www.christies.com/promos/apr03/1322/specialist.asp>

²Analogously, a university might have five positions but might wish to fill only some of them in a given year, so as to have access to future entrants into the academic market.

sites, many consumer products like electronics are usually sold in multiple units, for example, *eBay.com*, *samsclub.com*, *Onsale.com*, *uBid.com*, etc; not to mention the transaction of production supplies in B2B procurement markets such as *Freemarkets.com* . In many of these internet auctions, it is not clear if the seller has more items of the same kind that he or she has decided to hold back for a future auction; interested buyers have to take this uncertainty into account in planning their bidding strategies. Does this uncertainty help the auctioneer, or should the auctioneer reveal the inventory information?

This paper addresses the questions posed in the preceding paragraphs. We first consider the issue of simultaneous vs. sequential auctions in a framework where the number of items for sale is commonly known and when players are impatient (so that the expected price declines from auction to auction). We then discuss a case where the total number of items for sale is known only to the seller and generalize this to a multi-period setting where a monopolistic seller has a privately known capacity constraint per period, and needs to decide when to reveal his capacity information to fully rational bidders.

Before giving a synopsis of our results, we should acknowledge that for many people, the first problem (whether to split the items or not, under certainty) has been solved. Given that each buyer has a maximum demand of one unit, and that the buyer valuations are independent, private and from identical distributions, the optimal auction is known and is a simple one: set an optimal reserve price and sell all the items at the highest rejected bid ([11], [6]). However, this assumes, as does the entire mechanism design literature, that the auctioneer is able to make a binding commitment not to sell the objects below the announced reserve price (see, however, McAfee and Vincent (1997) and Skreta (2001), which are discussed in the next section). For some reason, such perfect commitments are often not characteristic of the real world, as the example of the Picasso lithographs above illustrates. (Note that the consignor there had specified that all the items should be sold-there was no reserve price set.) Also some auctioneers in eBay have been observed to reduce the reserve price at the end of the auction

if they fail to sell with the original reserve price ³. They can resell their commodity in eBay for the second time for free if no bids meet the reserve price in the first auction ⁴. Besides the lack of commitment for not selling the unsold stock, reserve prices, announced or secret, appear not to be popular in some sites like eBay. Many bidders generally won't bid on an item with a reserve price ⁵. In the absence of a commonly known perfect commitment not to ever sell below the reserve price, it is by no means evident that a reserve price policy will continue to be optimal. We explore this issue as well in this paper.

As in the lithographs example above, the auctioneer in our model does not control the total number of items he or she has to sell. This distinguishes this problem from that of a monopolistic seller who can choose how much to produce. The auction format used in this paper is the simple uniform price (“second price”) auction, with independent private valued bidders. Since each bidder is assumed to want only one item, this allocates items to the highest value bidders in the symmetric equilibrium.

Our major results are as follows: First, when the “intensity of competition”, which is characterized by the ratio of the number of bidders to the number of items for sale, is below a threshold value, a sequential auction performs better in expected revenue terms than the one-stage auction⁶. Furthermore, when the number of items is large and if either the auctioneer cannot throw away the unsold items, or he cannot commit to do so, the effect of a reserve price to screen out low-valued bidders is greatly reduced, and the sequential auction may generate more expected revenue for the auctioneer by stimulating competition in the initial period. Second, in sequential auctions, by not revealing the number of auctions (or number of total items for sale), an auctioneer might take advantage of bidders' uncertainty about the auctioneer's inventory. In equilibrium, depending on the intensity of competition, as well as bidders' prior beliefs about the auctioneer's type, a “high inventory” auctioneer can choose to

³Auction Tips, <http://www.printweb.org/auctiontips.shtml>

⁴eBay Fee Chart, <http://auctionknowhow.com/resource/fees.shtml>

⁵eBay Sites, http://www.on-line-auction-sites.com/ebay_sites.html;

Valerie's eBay Tips, http://www.unixmama.com/arboreparents/ebay_tips.html

⁶We often deal with the quantity K/n , which is inversely related to the intensity of competition.

mimic a "low inventory" type in the first period. The presence of a second period will reveal a "high inventory" type in this model. Last, we extend this disclosure problem into a T -period setting with the seller having private information about his production capacity. We find that in equilibrium, a "high" type auctioneer specifically creates uncertainty about his/her type by pretending to be a "low" type auctioneer up to a certain period, then randomizing between revelation and non-revelation with time-dependent probabilities. The key difference between this model and other similar, incomplete information analysis is that there are auctions every period of unsold items produced upto that point and the buyers with discount factor 1 all arrive at the beginning of the game.

The remainder of the paper is organized as follows: in the next section, we discuss some relevant literature, then in section 3, we introduce the environment and notation. In section 4, we consider the case where the auctioneer's inventory is common knowledge, and he cannot commit not to sell his inventory. We compare the performance of a sequential auction with that of a simultaneous auction in section 4.1, and with that of an "optimal auction" without commitment to the the reserve price in section 4.3. In section 5 we consider the case where the number of items is known only to the seller. Section 6 discusses how the auctioneer can exploit this uncertainty in a T period setting.

In this paper, we will occasionally refer to "two-period auctions" and sometimes to "two auctions"; these refer to the same object and are both used only for variety.

2 Literature Review

The study of sequential vs. simultaneous auctions was initiated by Weber (1983) [13]. He studied an independent private values environment with each bidder having unit demand and with the seller having multiple items for sale, just as we do. However, Weber's paper has no time discounting and ours does. This turns out, not surprisingly, to make a crucial difference to the results. He finds that the auctioneer's expected payoff is the same from selling all the items in one period, or one in each period. The expected winning price is constant across periods.

Weber’s results appear as a limiting case of the first model in our paper as the discount factor goes to 1.

Menezes (1993)[9] analyzes a 2-period sequential model with a fixed cost of delay but does not consider a comparison of sequential and simultaneous auctions.

Zeithammer (2004) [15] studies bidders’ forward-looking behavior in sequential auctions and finds that buyers “underbid when they expect the seller to offer another auction in the near future”, and the auctioneer decides to sell or not to sell based on bidders’ behavior. But in their model, bidders are limited in two types (H and L), and there is no uncertainty about the auctioneer’s type. Ganuza (2004)[4] studies how much information the auctioneer should reveal about the object for sale in a single object setting. He finds that the auctioneer has incentive to release less information than is efficient in order to promote competition.(The point of contact with our paper is with the uncertainty models in the last two sections here.)

Beam et al.(1996)[1], Pinker et al. (2000) [3] and Karuga etal (2005) [5] use the actual transaction data on certain internet auction sites to determine how many items should be sold in each round of a sequential auction. However, these papers assume naive (non-strategic) bidders who do not consider the future effects of their current actions. Bidders in our model, in contrast, are fully rational.

On the question of lack of commitment to a reserve price in “optimal” auctions, McAfee and Vincent (1997) [8] study the optimal sequential auction when the auctioneer cannot commit never to sell below the initial reserve price, and Skreta (2002) [12] studies the optimal mechanism to sell a single item in a two-period model. However both consider only the sale of a single object, and do not address the comparison of an explicitly sequential auction and one in which a reserve price is used to separate first-period and second-period buyers depending on their (privately known) values⁷. McAdams (2005) [7] also studies the lack of commitment in uniform price auctions but his model is very different from ours. Bidders submit demand functions and the seller decides on the supply after observing the demand functions. In our

⁷We note that this is effectively what a reserve price does, in a two-period model without commitment to the reserve price.

model, the seller first announces the number of items and bidders have unit demand.

3 The Model and Notation

We now describe our first model, where the auctioneer receives K items in inventory. There are n symmetric risk neutral bidders. Bidder i has a unit demand if the price is less than or equal to v_i , his private value, which is drawn (independently of other v_j) from a common, absolutely continuous distribution F on $[0, 1]$. All K , n and $F(\cdot)$ are common knowledge. For simplicity (like the Skreta paper cited earlier), we only consider a two-period case in this first model, and assume that if the auctioneer decides to use a sequential auction, he or she will sell half the inventory $k = K/2$ in each period⁸. (This is also common knowledge). The auctioneer's decision is to choose whether to sell his inventory in one or two periods.

The auctioneer's inventory information K is public information in our first model (thus the bidders know for sure how many auctions there are), or private information in our later models, if the auctioneer decides not to inform the bidders whether there will be future auctions. When K is the auctioneer's private information, the number of items put up for sale in the first auction (k) could convey information about K to the bidders. In the beginning of the auction, the bidders observe the number of items for sale, and decide whether to bid in the current period, then how much to bid. If a bidder wins an item he leaves, and if not, he stays for the second period and decides how much to bid in the second period.

As mentioned in the introduction, the auction format is a sealed bid, uniform price auction, where the market clearing price will be determined by the highest rejected bid in each period. The auctioneer is also assumed to be risk neutral and to have a commonly known reserve value of 0. We first consider the case where no reserve price is set.

The auctioneer and bidders have a common discount factor δ , $0 \leq \delta \leq 1$. We assume that the number of bidders is always greater than the total number of items in the auction, that is, $n > K$. The case where $n \leq K$ is trivial. We use the following notation in the analysis:

⁸This assumption can be relaxed and we address this later in the paper.

1. b_i^t : The bid of bidder i with valuation v_i in period t .
2. Y_i : the i^{th} highest valuation among $n - 1$ bidders.
3. $H_i(v)$: the probability of $Y_i \leq v$, where $H_i(v) = \sum_{l=0}^{i-1} \frac{(n-1)!}{(n-1-l)!l!} F(v)^{n-1-l} (1 - F(v))^l$,
 $i = 1, 2, \dots, n - 1$
4. $h_i(v)$: the probability density function of the i^{th} highest bid among $n - 1$ bidders, where
 $h_i(v) = \frac{(n-1)!}{(n-1-i)!(i-1)!} F(v)^{n-1-i} (1 - F(v))^{i-1} f(v)$, $i = 1, 2, \dots, n - 1$.
5. X_i : the i^{th} highest valuation among n bidders.
6. $G_i(v)$: the probability of $X_i \leq v$, with density function $g_i(v)$ where

$$G_i(v) = \sum_{l=0}^{i-1} \frac{n!}{(n-i)!l!} F(v)^{n-l} (1 - F(v))^l, \quad i = 1, 2, \dots, n$$

7. $g_i(v)$: the probability density function of the i^{th} highest bid among n bidders, where
 $g_i(v) = \frac{(n)!}{(n-i)!(i-1)!} F(v)^{n-i} (1 - F(v))^{i-1} f(v)$, $i = 1, 2, \dots, n$.

Note that, unfortunately, the superscript represents both powers and the index of the time period, and the subscript represents an individual bidder and the index of the order statistic of bidders' values. It is clear from the context which interpretation is intended.

4 Number of Items Commonly Known

In this section we analyze the first model described above. First we compare the performance of a sequential auction with a simultaneous one. In both formats there is no reserve price. Next we compare the sequential auction to an auction with reserve price, under the assumption that the auctioneer cannot commit not to sell below the reserve price.

4.1 Sequential Auctions vs. Simultaneous Auctions

First consider the sequential auction where the auctioneer sells half of his inventory in each of the two periods. We start with an analysis of the second auction, assuming that the k highest

bidders win and leave in the first period, and there are only $n - k$ bidders remaining. (This is actually on the equilibrium path, when $k = \frac{K}{2}$. The only out-of-equilibrium node occurs when fewer than $K/2$ objects are sold in the first period. This does not affect bids in the second period, where bidders bid their valuations.)

Using backward induction, we first consider the second-period auction. Since it is public information then that this is the final period of the auction, by standard auction theory, truth telling is the weakly dominant strategy for the bidder. Thus, bidders bid their true values.

Now consider the first period. All n bidders are present at the beginning of the first period. Since there will be a second period, bidders have the option to win an item at a possibly lower price in the second period. Thus a first-period bid should make the buyer indifferent between winning in the first period or winning in the second. More specifically, a bidder i 's expected payoff is:

$$\begin{cases} v_i - \beta(Y_k) & \text{if } b_i^1 \geq \beta(Y_k) \\ \delta E \max[(v_i - Y_{2k}, 0) \mid \beta^{-1}(b_i^1) \leq Y_k] & \text{otherwise} \end{cases}$$

Proposition 1 *Under the assumption that it's common knowledge that there are two auctions held sequentially, in equilibrium, a bidder i 's optimal first-period strategy is as follows:*

$$\begin{aligned} b_i^1(v_i) &= v_i - \delta \pi_i^2(v_i \mid Y_{2k} < v_i \leq Y_k) \\ &= (1 - \delta)v_i + \delta E[Y_{2k} \mid Y_{2k} < v_i \leq Y_k] \end{aligned}$$

where $\pi_i^2(\cdot)$ denotes the second-period expected profit of i .

A bidder's optimal second-period strategy if he does not win an object in the first-period is to bid $b_i^2(v_i) = v_i$.

Proof. In the first period, a bidder never bids an amount x more than $v_i - \pi^2(v_i \mid Y_{2k} < v_i \leq Y_k)$, because if he does, there is a positive probability that some bid is between $v_i - \pi^2(v_i \mid Y_{2k} < v_i \leq Y_k)$ and x . Then if bidder i wins in the first period, he has to pay more than $v_i - \pi^2(v_i \mid Y_{2k} < v_i \leq Y_k)$, which makes his expected payoff less than $\pi^2(v_i \mid Y_{2k} < v_i \leq Y_k)$. He is therefore better off not bidding in the first period. On the other hand, he will not bid less

than $v_i - \pi^2(v_i|Y_{2k} < v_i \leq Y_k)$, because an increase of ϵ in the bid can increase his probability of winning without affecting the payment, given the bidding strategy above is followed by all the other bidders. \diamond

Since the equilibrium bidding functions in both periods, $b_i^1(v_i)$ and $b_i^2(v_i)$ are increasing in v_i , it is clear that the auction allocates the goods to bidders with the highest K valuations. However, the auction may be *inefficient* in terms of a cost of delay, if the auctioneer chooses a two-period auction when $\delta < 1$.

Under the highest-rejected payment rule, the expected transaction prices in the sequential auction, are the $k + 1$ 'th bidder's bid in the first period, and the $K + 1$ ' bidder's bid in the second period. Thus the revenue that the auctioneer expects in the first period is:

$$\begin{aligned} E[R^1] &= k \cdot E \left[X_{k+1} - \pi_{k+1}^2(X_{k+1}|X_{2k} < X_{k+1} \leq X_k) \right] \\ &= k \cdot E \left[X_{k+1} - (\delta X_{k+1} - \delta E[X_{2k+1}|X_{2k} < X_{k+1} \leq X_k]) \right] \\ &= k \cdot ((1 - \delta) E[X_{k+1}] + \delta E[X_{2k+1}]) \end{aligned}$$

The auctioneer's discounted payoff in the second period is $E[R^2] = \delta k \cdot E[X_{2k+1}]$.

The auctioneer's total revenue $E[R2]$ in the sequential auction is then:

$$\begin{aligned} E[R2] &= E[R^1] + E[R^2] \\ &= k \cdot \{(1 - \delta) E[X_{k+1}] + 2\delta E[X_{2k+1}]\} \end{aligned} \tag{1}$$

From Eq. 1, we can see that the auctioneer's expected revenue not only depends on the relative magnitudes of n and k , but also depends on the value of δ . The discount factor δ has two effects in opposition directions on $E[R2]$, the auctioneer's total expected revenue; the more the bidders discount (the smaller the discount factor), the more aggressively bidders bid in the first period. This can increase the first-period revenue. But at the same time the utility from the second-period auction decreases because of the discounting, this time by the auctioneer. The overall effect of δ depends on the trade-off between these two effects.

Now consider the case when the auctioneer decides to sell all of his items in a single period. It is straightforward to show that the bidders' equilibrium bidding strategies are to bid their

true values. Thus, the auctioneer’s expected revenue can be expressed as:

$$E[R1] = 2k \cdot E[X_{2k+1}] \tag{2}$$

The difference in expected revenue, between using a sequential auction and a simultaneous auction, is the difference between Eq. (1) and Eq. (2):

$$E[R2] - E[R1] = k(1 - \delta)(E[X_{k+1}] - 2E[X_{2k+1}]) \tag{3}$$

When $\delta = 1$, that is, when there is no time discounting, $E[R2] = E[R1]$, the sequential auction produces the same expected revenue as the simultaneous auction, which is the standard result from Weber (83). If $\delta < 1$, then the sign of this difference is determined by $(E[X_{k+1}] - 2E[X_{2k+1}])$. This depends upon the relationship of n and k , given the distribution function F . If $E[X_{k+1}] > 2E[X_{2k+1}]$, then the auctioneer can be better off holding a two-period auction, even though his expected payoff in the second period is very small! This happens when the market competition for the items is so weak that bidders can get the items at a very low price. Then it is better to sell only k items at a higher price by creating some competition in the first period.

So even though both the bidders and auctioneer discount future periods, it may not always be better to sell all the items in one period. Which format performs better is largely determined by the relationship between K and n , which indicates the intensity of competition, as well as the discount factor. Discounting encourages people to shade less, thus to bid more aggressively in the first period. Thus, it stimulates competition in the first period. This is beneficial to the auctioneer. For some combination of k and n , this positive effect can overcome the negative effect and make the auctioneer better off in a sequential auction than in a simultaneous one.

More specifically, define the “intensity of competition” by $\frac{n}{K}$. Assume that bidders’ valuations of the item are i.i.d random variables uniformly distributed between $[0, 1]$, then:

Proposition 2 *When $\delta < 1$, whether or not a sequential auction is better than a simultaneous auction depends on the “intensity of competition”. More specifically, if $\frac{n}{K} > \frac{3}{2}$, it is always better to have a one-period auction. If $\frac{n}{K} < \frac{3}{2}$, it is always better to hold a two-period auction.*

When $k = \frac{1}{3}n$, the expected revenue from the simultaneous auction and sequential auction is the same for the auctioneer.

Please refer to the Appendix A.1 for the proof.

4.2 Optimal Inventory Allocation in Sequential Auctions

Until now we have only considered the case where the auctioneer sells items in identical lots, if a two-period auction is used. If we relax this constraint, the question becomes how to optimally allocate items across the two periods. Let k_1 be the number of items in the auction in the first period, then in the second period there are $K - k_1$ items remaining. Given the bidding strategy specified in Proposition 1, the auctioneer maximizes:

$$\text{Max} \quad k_1 \{E[X_{k_1+1}] - \delta(E[X_{k_1+1}] - E[X_{K+1}])\} + \delta(K - k_1)E[X_{K+1}]$$

Assume that the bidders' values follow a uniform distribution on $[0, 1]$; solving for the the first-order condition and rounding to the smallest integer, we get $k_1^* = \min\{\lfloor \frac{1}{2}n \rfloor, K\}$. This is to say, the optimal amount of inventory to sell in the first period again depends purely on the ‘‘intensity of competition’’: when $\lfloor \frac{1}{2}n \rfloor > K$, the competition is relatively hot and it is better to sell all the inventory in the first period; however, when $\lfloor \frac{1}{2}n \rfloor < K$, the auctioneer needs to use a sequential auction to stimulate competition in the first period, even though future revenue is discounted. The auctioneer sells more items in the first period than in the second period (if there is a second period).

4.3 Reserve Price Without Commitment

It is well-known from auction theory that an optimal mechanism for selling multiple homogeneous items to bidders with independent private values and unit demand, is an auction where all the items are sold at once with an optimal reserve price, which might be different from the seller's valuation. However, in reality, the assumption of full commitment to the reserve price might not always hold. Even if the auctioneer wishes to commit to such a reserve price, there

may be no credible mechanism allowing him to do so. (See the introduction for examples.) Thus, bidders need to take into consideration this possible second auction without reserve price (or reduced reserve price), when they decide whether to bid in the first period ⁹.

Let's consider the case that the auctioneer offers for sale all his (K) items with announced reserve price r in the first period. But bidders know that the auctioneer cannot commit to this reserve price and will resell the unsold items in the next period with reduced reserve price. For simplicity here we only consider a two-period model, so the reserve price in the second period is reduced to 0. Suppose bidders don't know how many other bidders' valuations are above the reserve price. Anticipating there will be a second auction with positive probability, bidders will adjust their bidding strategy and decide whether or not to participate in the first period auction, and if so, how much to bid¹⁰.

Proposition 3 *If $0 \leq \delta \leq \tilde{\delta}$, where $\tilde{\delta} = \frac{(v-r)\text{prob}(n' < K) + \sum_{i=K}^{n-1} (v - E[Y_{K,n-1}|n'=i])\text{prob}(n'=i)}{\sum_{i=0}^{n-1} (v - E[Y_{K,n-1}|n'=i])\text{prob}(n'=i)}$, in equilibrium, there exists a threshold value $\tilde{r} \in [0, 1]$. A bidder with value no less than \tilde{r} will bid his true value in the first period, and bidders whose valuations are lower than \tilde{r} will wait for the second period. That is,*

$$\left\{ \begin{array}{ll} (v - r)\text{prob}(n' < K) + \sum_{i=K}^{n-1} (v - E[Y_{K,n-1}|n'=i])\text{prob}(n'=i) \\ \geq \delta \sum_{i=0}^{n-1} (v - E[Y_{K,n-1}|n'=i])\text{prob}(n'=i) & \text{if } v \geq \tilde{r} \\ < & \text{if } v < \tilde{r} \end{array} \right. \quad (4)$$

and the optimal \tilde{r} is determined by:

$$(\tilde{r} - r) \cdot \text{prob}(n' < K) = \delta \sum_{i=0}^{n-1} (\tilde{r} - E[Y_{K,n-1}|n'=i]) \cdot \text{prob}(n'=i) \quad (5)$$

where $E[Y_{K,n-1}|n'=i]$ represents the K 'th highest bidders valuation among $n - 1$ bidders, given $n' = i$ other bidders' valuations above \tilde{r} .

⁹A similar practice is also observed in retail markets. For example, experienced shoppers know that the current sale is not final even though it is marked "final sale", though waiting to buy later will incur a waiting cost, plus the risk that the good may be sold out. The shopper's decision problem includes the optimal timing to participate seriously.

¹⁰To reiterate, the reserve price here plays the role of a stochastic choice of number of items to be sold in the first period and number to be sold in the second.

The proof is in the Appendix A.2.

Given this strategy, the auctioneer's expected revenue when he or she uses a reserve price r without commitment is:

$$\begin{aligned}
E[R(r)] &= \sum_{i=K+1}^n \tilde{r}^{n-i} (1 - \tilde{r})^i \binom{n}{n-i} K \cdot (E[X_{K+1} | X_{K+1} > \tilde{r} \& n' = i]) \\
&\quad + \sum_{i=0}^K \tilde{r}^{n-i} (1 - \tilde{r})^i \binom{n}{n-i} (i \cdot \tilde{r} + \delta(K - i) \cdot E[X_{K+1} | X_{K+1} < \tilde{r} \& n' = i])
\end{aligned}$$

where $\tilde{r}(r)$ is determined by Eq. (5).

Then the auctioneer's decision problem is to choose the optimal reserve price r to announce, that is, $r^* = \arg \max \{R(r)\}$.

Compare this revenue with the expected revenue from the sequential auction without reserve price. Intuitively, the auction with reserve price works better when δ is small. When $\delta \rightarrow 0$ it approaches the theoretical "optimal auction" with full commitment. However when δ is large, bidders are more willing to wait till the next period for a lower price, which makes the reserve price in the first period less effective in screening out the low-valued bidders, to an extent that the auction with reserve price may perform worse than the sequential auction. Figure 1 shows a numerical example in which for a certain region of δ (when $\delta > 0.1$) the sequential auction is better than the reserve price auction. In this example, $n = 7$, $K = 4$, and bidders' valuations follow a uniform distribution in $[0, 1]$.

From the figure we can see, when $\delta = 0$, which is equivalent to the case where the auctioneer can make a full commitment to the reserve price, an auction with a reserve price performs better than an auction without reserve price. With an increase in δ , however, bidders become more and more patient, thus they are more and more willing to wait to bid in the second period. The effective reserve price is reduced, and the performance of the reserve price auction is reduced. There exists a region of δ where the reserve price auction does not perform as well as the sequential auction without reserve price. When $\delta = 1$, the reserve price auction produces the same equilibrium results as the sequential auction without any reserve prices, as is to be

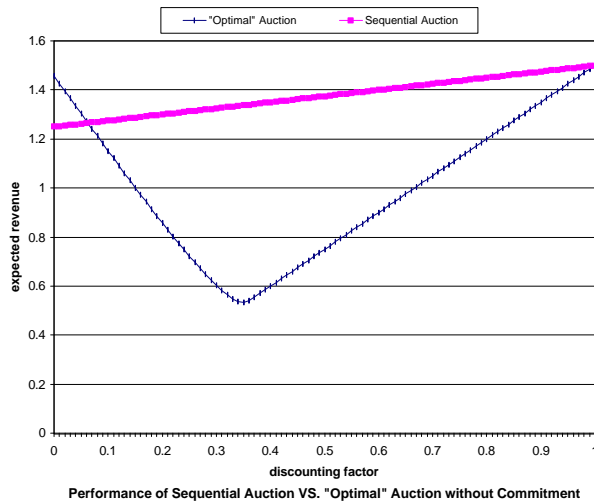


Figure 1: Performance with respect to the change of δ of auction with reserve price without commitment vs. no reserve price.

expected.

5 Bidders Uncertain about Number of Periods

We now consider the case when the auctioneer does not announce how many periods there will be. Under the setup of our model, this is to say that the auctioneer does not tell bidders his inventory information. So when bidders come to the auction site, they know that there can be at most 2 periods, but they don't know whether the current auction has one or two periods. This is the case in many of the online auction sites, where items are sold in identical lots but the total amount of inventory on hand is not known to bidders. Milgrom and Weber (1982) [10] indicated that in a single unit case with affiliated values, the auctioneer is always better off by providing as much value-related information as she can to the bidders. Endogenizing competition might lead to different results (see the discussion earlier of Ganuza (2004)). The decision to communicate how many periods there will be is a somewhat different kind of choice,

but it is interesting to check whether the auctioneer will in any case be better off revealing this information.

We consider the following game: in the beginning, Nature determines whether the auctioneer is a High type (H) who has K_2 items on hand, or a Low type (L) who has only $K_1 < K_2$ items on hand. With probability ρ he is a High type, and with $1 - \rho$ he is a Low type. All of K_1 , K_2 and the belief ρ are common knowledge. The high type auctioneer decides whether or not to pretend to be a low type one, by selling the same number of items in the first period as the low type does. There is no credible way for the Low type auctioneer to signal his type. Only the High type can differentiate himself from the Low type by selling more than the Low type in the first period. Bidders bid based on their beliefs about the total number of items in the auction, or the auctioneer's type. The game is shown in figure ??:

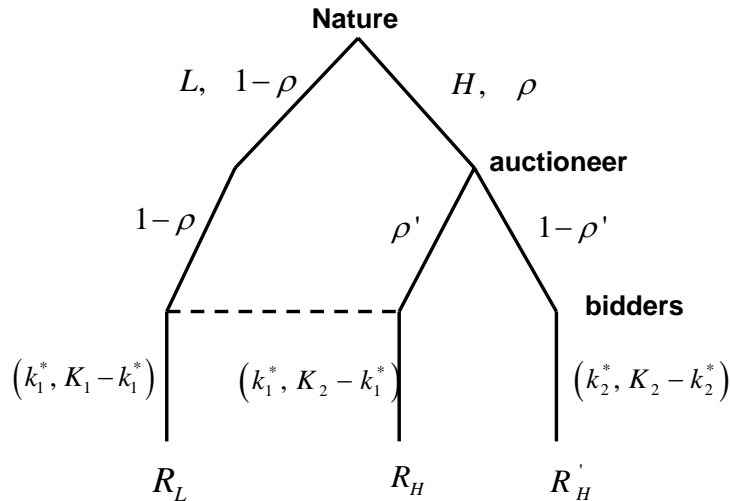


Figure 2: The game when bidders don't know the auctioneer's type

Again we use backward induction to derive the Perfect Bayesian Equilibrium. Let α be the probability that the high type will pretend to be a low type by selling the same number of

items as the low type. Thus when bidders cannot differentiate between these two types, they expect that with probability ρ' there will be a second period, where $\rho' = \frac{\alpha\rho}{\alpha\rho+(1-\rho)}$. Since the low type cannot credibly signal his type, and it is never optimal for him to pretend to be a high type, he will always sell the optimal number of items according to section 4.2.

According to the discussion in section 4.2, there exists an optimal number of items for sale k_1^* , k_2^* for the Low and High type auctioneer, respectively, when there is complete information about the auctioneer's type. Obviously $K_1 \geq k_1^*$ and $K_2 \geq k_2^*$.

If $k_2^* = k_1^*$, then both types of auctioneer sell the same optimal quantity in the first period, and the bidders cannot tell the type of the auctioneer.

If $K_2 > K_1 \geq k_2^* > k_1^*$ or $K_2 \geq k_2^* > K_1 \geq k_1^*$, then the high type auctioneer needs to decide whether or not to pretend to be a low type by selling up to k_1^* items in the first period, or reveal his type by selling k_2^* .

Assuming that bidders valuations follow a uniform distribution in $[0,1]$, we can specify the equilibrium strategy in the following proposition:

Proposition 4 *The auctioneer's equilibrium strategy is:*

1. *If Low type, always sell k_1^* items in the first period.*

2. *If High Type,*

(a) *When $K_2 > K_1 > \frac{n}{2}$, always sell $\frac{n}{2}$ in the first period. In this case, $\alpha = 1$ and*

$$\rho' = \rho.$$

(b) *When $K_2 > \frac{n}{2} > K_1$*

i. *If $\rho \leq \tilde{\rho}$, where $\tilde{\rho} = \frac{0.25n^2(1-\delta)+\delta K_2(n-K_2)-K_1(n-K_1)}{\delta[K_1(n-K_1)+K_2(n-K_2)]}$, pretend to be a Low type*

by selling K_1 items in the first period, where $\alpha = 1$ and $\rho' = \rho$.

ii. *If $\rho \geq \tilde{\rho}$, play a mixed strategy that with probability α sells K_1 items in the first period and with probability $1 - \alpha$ sells $\frac{n}{2}$ items in the first period, where α*

satisfies the equation that $\frac{\alpha\rho}{\alpha\rho+(1-\rho)} = \tilde{\rho}$. That is, α generates a belief ρ' that

makes the bidders play such that the auctioneer is indifferent between revealing her type and pooling with the low type player.

(c) When $\frac{n}{2} > K_2 > K_1$

- i. If $0 \leq \rho \leq \bar{\rho}$, where $\bar{\rho} = \frac{(K_2 - K_1)(n - K_2 - K_1)}{\delta[K_1(n - K_1) + K_2(n - K_2)]}$, pretend to be a Low type by selling K_1 items in the first period, where $\alpha = 1$ and $\rho' = \rho$.
- ii. If $\rho \geq \bar{\rho}$, play a mixed strategy that with probability α sells K_1 items in the first period and with probability $1 - \alpha$ sells $\frac{n}{2}$ items in the first period, where α satisfies the equation that $\frac{\alpha\rho}{\alpha\rho + (1-\rho)} = \bar{\rho}$.

The equilibrium strategy for the bidders is as follows: In the second period (if there is one), $b_i^2 = v_i$ (the assumptions on the number of bidders guarantee that there are more bidders than objects remaining). In the first period, $b_i^1 = v_i - E[V_i^2(\tilde{k}_2)]$, where \tilde{k}_2 is the inventory of items to be auctioned off in the second period and $V_i(\cdot)$ is the expected second-period payoff. The expectation $E[\cdot]$ is taken over the beliefs about \tilde{k}_2 ¹¹.

Intuitively, if ρ is low, both types of auctioneer pool on the “tough” choice, namely choosing the quantity k_1^* . If ρ is higher than $\bar{\rho}$ when $K_2 > \frac{n}{2} > K_1$, or $\bar{\rho}$ when $\frac{n}{2} > K_2 > K_1$, the Low type chooses k_1^* and the High type randomizes between k_1^* and k_2^* . An out-of-equilibrium belief that sustains this equilibrium is that the auctioneer is believed to be of Low type if the number offered for sale is less than or equal to k_1^* , and to be of High type otherwise. The High type seller’s choice of randomization affects the bidders’ strategies in such a way that the seller is indifferent between revealing and not revealing his type. If the market is highly competitive, the auctioneer is better off by revealing himself.

The formal proof is in appendix B.

The structure of the equilibrium above is similar to one encountered in many games of incomplete information like this one. However, the result has interesting implications. If the auctioneer is a high type, it might be beneficial for him or her to hide this information by

¹¹The uniform distribution simplifies matters for explicit calculation because the order statistic is linear in the number of items, so the second term depends on the expectation of \tilde{k}_2 .

pretending to be an auctioneer of a low type. This uncertainty drives up the competition among the bidders, and makes a bidder bid more aggressively for the current auction. Note that this information or the lack thereof does not affect bidders' valuations; it does, however, affect bidders' strategies. Thus it has an effect similar to an overall rise in bidder valuations.

6 T-period Extension

In this section, we consider an extension of the model of the last section to a general finite horizon. The main difference between the two models is that in the last section the seller's inventory was unknown, while in this section the seller's capacity to produce is unknown. This latter problem appears to be similar to the one discussed in Wilson (1985) [14] as the "Picasso problem" and attributed by him to Sridhar Moorthy¹². In this framework, Picasso as a monopolist producing paintings, maximizes his life time revenue by deciding how many paintings to sell each year. The price for his paintings will be low even in his earlier years if collectors anticipate many paintings in the market later on. Thus Picasso has an incentive to pretend to be a low productivity type in his early years¹³.

We simplify the problem by assuming a monopolist producer with a lifetime of T periods, who can be a high type with the capacity to produce 2 items each period; or a low type with a capacity of 1. We assume no inventory is allowed, so he has to sell whatever is produced¹⁴. With probability ρ he is a high type, thus he needs to decide whether or not to produce 1 item to pretend to be a low type, or 2 items and reveal his type. Once he reveals his type, we assume there is no way to change bidders' belief. There are n potential bidders who are present at period 1. We only consider the interesting case where $n > T$. We also assume the presence

¹²We have been unable to locate the original paper.

¹³A problem similar to this is faced by a durable goods monopolist in a market where used goods can be resold (see Bulow (1982)[2]). However, our Picasso model does not have to do with resale but with incomplete information, there is no deterioration in the quality of goods in later periods and we do not need discounting by the bidders.

¹⁴Using inventory instead of capacity will lead to an equilibrium with similar structure.

of dedicated followers and fans makes it impossible to hide paintings once produced. In each period there is a uniform-price auction of the paintings already produced and the winning bids are announced but not the losing ones.

In each period t , the high type Picasso decides whether to sell 2 items, so his type is revealed and bidders realize that he will have a total of $2(T - t + 1) + (t - 1)$ products in his life time, or to sell only 1 item so bidders still believe that he is a low type with probability $1 - \rho$, with expected life time production of $t + (2\rho + 1 - \rho)(T - t)$.

If bidders are non-strategic, this is like an optimal stopping problem in dynamic programming, where the time that the auctioneer decides to reveal his type is the time that the game “stops”. Let $S(t)$ represent the stopping payoff at period t , then $S(t) = 2(T - t + 1)E[X_{2T-t+2}]$. The current period payoff is $g(t) = E[X_{(1+\rho)T-\rho t+1}]$. Thus the optimal stopping time satisfies $S(t) \geq g(t) + S(t+1)$. Assuming that bidders’ valuations follow a uniform distribution on $[0, 1]$, we have $E[X_{2T-t+2}] = \frac{n-2T+t-1}{n+1}$ and $E[X_{(1+\rho)T-\rho t+1}] = \frac{n-(1+\rho)T+\rho t}{n+1}$. The optimal stopping time is $t^* = \max\{0, \frac{(5-\rho)T-n+2}{4-\rho}\}$, since $\frac{(5-\rho)T-n+2}{4-\rho}$ is always greater than T given $T < n$.

The above analysis, however, only considers the bidders as passive players in the game. If the buyers are also strategic players and fully rational, they can calculate what is the optimal t^* and expect that if the seller is a high type, he won’t reveal his type before t^* . Thus the belief about his capacity remains the same as the prior until period t^* , instead of updating every period. Only after period t^* , when the auctioneer reveals his type with positive probability, will the bidders’ belief about the capacity be updated. Knowing about this, the seller needs to modify his optimal producing/selling strategy. Basically, he can utilize the structure that bidders use to update their beliefs, and tries to pretend to be a low type for a longer period of time, till a period t_2 (different from t^*), after which it’s no longer optimal for the seller to pretend to be a low capacity type. That means, if the seller continues selling 1 product until period t_2 , the buyers will believe that he is a low type for sure, since there is no more benefit for him to continue to pretend any more. Thus $g(t_2) = E[X_{T+1}]$. Moreover, t_2 satisfy the condition that $S(t_2) \geq g(t_2) + S(t_2+1)$, or more specifically, $\frac{2(T-t_2+1)(n-2T+t_2-1)}{n+1} \geq \frac{n-T+2(T-t_2)(n-2T+t_2)}{n+1}$.

The optimal t_2^* is $\min[\frac{5T-n+2}{4}, T]$, which is larger than the original stopping time t^* . Thus t_2^* is the last period that the producer can profit from pretending to be a low type.

For the formal analysis, assume the high type auctioneer uses a mixed strategy before t_2^* , pretending to be a low type with probability α_t at period t and revealing with complementary probability. The equilibrium strategy for the high type auctioneer is:

Proposition 5 *There exist two cutoff times t_1^* , t_2^* , where $t_1^* = \max\{\lfloor \frac{(5-\rho)T-n+2}{4-\rho} \rfloor, 0\}$ and $t_2^* = \min\{\lceil \frac{5T-n+2}{4} \rceil, T\}$. Before t_1^* , the high type auctioneer will always produce 1 and sell 1; and after t_2^* , he will reveal his type by producing 2 and selling 2. Between t_1^* and t_2^* , he will produce 1 with probability α_t , and produce 2 with probability $1-\alpha_t$, where $\alpha_t = \frac{1-\rho}{\rho} \frac{5T-n-4t+2}{-4T+n+3t-2}$.*

Accordingly, the bidders' equilibrium strategy is:

1. when observing 2 items for sale at period t , realize that the auctioneer is of High type, and bid $E[Y_{t-1+2(T-t+1)} | Y_{t-1+2(T-t+1)} \leq v_i]$
2. When observing 1 items in period t ,
 - (a) If $t < t_1^*$, bid according to the belief that the auctioneer is of High type with probability ρ , thus $b_i(v_i) = E[Y_{\tilde{t}} | Y_{\tilde{t}} \leq v_i]$, which is constant in t , where $\tilde{t} = 2\rho(T - t_2^* + 1) + (1 - \rho)T + \sum_{t_1^*}^{t_2^*} (\alpha_t \rho + 1)$.
 - (b) If $t_1^* \leq t \leq t_2^*$, bid according to the belief that the auctioneer is of High type with probability $\alpha_t \rho$, thus $b_i(v_i) = E[Y_{\hat{t}} | Y_{\hat{t}} \leq v_i]$, where $\hat{t} = 2\rho(T - t_2^* + 1) + (1 - \rho)T + (t - t_1^* + 1) + \sum_t^{t_2^*} (\alpha_t \rho + 1)$;
 - (c) If $t > t_1^*$, realize that the auctioneer is of Low type with probability 1, thus $b_i(v_i) = E[Y_T | Y_T \leq v_i]$
 - (d) In the last period, bid one's value.

Proof. Since the high type auctioneer uses a mixed strategy before t_2^* , pretending to be a low type with probability α_t at period t and revealing with complementary probability,

when observing 1 item for sale at period $t < t_2^*$, the bidders always anticipates that with probability $\rho'_{t+1} = \frac{\alpha_t \rho_t}{\alpha_t \rho_t + 1 - \rho_t}$ the current seller is a high type. Given this belief, the seller is indifferent between revealing at period t , or revealing at period $t + 1$, which is also known to the bidders. Given this belief, a bidder i 's bidding function takes into account this potential gain ($E[Y_T | Y_T < v_i] - \rho'_{t+1}(v_i - E[Y_{2T-t} | Y_{2T-t} < v_i])$) if the auctioneer is high type. Thus the auctioneer's current period payoff is $g(t) = (1 - \rho'_{t+1})E[X_{T+1}] + \rho'_{t+1}E[X_{2T-t+1}]$. Since the auctioneer is indifferent between hiding and revealing between period t_1^* and t_2^* , $S(t) = S(t + 1) + g(t)$, that is: $\frac{2(T-t_2+1)(n-2T+t_2-1)}{n+1} = \frac{2(T-t_2)(n-2T+t_2)+(1-\rho')(n-T)+\rho'(n-2T+t)}{n+1}$. From this we get $\alpha_t = \frac{1-\rho_t}{\rho_t} \frac{5T-n-4t+2}{-4T+n+3t-2}$. We see $\alpha_t = 1$ when $t = t_1^* = \frac{(5-\rho)T-n+2}{(4-\rho)}$, thus t_1^* is the first period that the high type auctioneer begins to use a non-degenerate mixed strategy; and $\alpha_t = 0$ at $t_2^* = \frac{5T-n+2}{4}$, which confirms our results before. \diamond

This proposition shows that the high type seller reveals his type earlier when the market is more competitive (the larger the n , the smaller the t_1^*). Comparing to the case of passive bidders, the auctioneer's action of hiding his type won't change bidders beliefs before t_1^* . Moreover, the auctioneer can enjoy the uncertainty about his capacity information longer after t^* , until period t_2^* , because only after t^* will bidders begin to seriously update their belief about his type.

More specifically, after t_1^* , if the auctioneer continues hiding, bidders believe he is a high type auctioneer with an increasing probability over time, but still smaller than ρ , until period t_2^* , when his type is revealed. As a result, the price is constant before t_1^* , and increasing between t_1^* and t_2^* . This can be seen from the increasing current period payoff $g(t)$ from t_1^* to t_2^* . Thus even though the belief the auctioneer is a high type is increasing between t_1^* and t_2^* , the price is increasing during this period of time, because the fact that the auctioneer sells only 1 item in each period reduces the total inventory. Any period in which the auctioneer offers 2 items reveals him as the High type and the game continues as a full information game.

7 Conclusions & Extensions

This paper has considered the simplest version of an important problem in order to gain intuition that can, we hope, be transferred to more complex settings. An auctioneer has K items for sale and bidders know that he has to sell all of the items. The auctioneer has to decide whether to sell them in two lots or one. Both the auctioneer and bidders are impatient. We find that when bidders have complete information about how many periods there will be, whether or not to sell the items in one period or two largely depends on the “intensity of competition” in the market. When the market is less competitive, the auctioneer can earn more from selling in two periods. To sell all the items at once with a reserve price does not necessarily perform better than the sequential auction without reserve price, if the auctioneer cannot fully commit not to sell the remaining items in the future without a reserve price. We then study the case when the auctioneer does not reveal how many auctions he is going to hold, and when the bidders believe with probability ρ that he is an auctioneer with high level of inventory. In this case we find that when ρ is small enough, the auctioneer can realize his monopoly power by pretending to be a Low type, selling items in two periods. This shows that it is not always better for the auctioneer to offer this piece of information to the bidders. The reason is that even in an independent private value setting, the uncertainty about a subsequent auction affects the bidders’ strategies, and the auctioneer can take advantage of this common effect to increase market competition among bidders.

We then extend the 2 period case with uncertainty to T periods, though with a capacity constraint for the auctioneer/producer rather than the feature that potential buyers are unable to observe inventories. We find an equilibrium structure where a high type auctioneer pretends to be a low type one initially, reveals with some probability thereafter, and then reveals for sure. As a result, the market price for these products increases over the time of possible revelation.

Much future work, of course, remains to be done. For example, it might be interesting to add uncertainty in the market in different periods, for example, a stochastic number of bidders, so that both the auctioneer and the bidders are not sure about the future when they

make decisions in the first period. However we feel that the more general case should not be qualitatively different from the one we consider.

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A Number of items commonly known

A.1 Auctioneer's choice of either one period or two-period auction

proposition 2. The difference of revenue between holding a two-period auction and a one-period auction is:

$$E[R2] - E[R1] = k \cdot ((1 - \delta) E[X_{k+1}] - 2(1 - \delta)E[X_{2k+1}]) \quad (6)$$

When $\delta = 1$,

$$E[R2] - E[R1] = k \cdot ((1 - 1) E[X_{k+1}] - 2(1 - 1)E[X_{2k+1}]) = 0 \quad (7)$$

so the auctioneer gets the same results no matter whether he sells the items in one period or two periods, which is the standard result of Weber(83). More specifically if we assume that bidders' values follow a uniform distribution, then

$$\begin{aligned} E[R2] &= k \cdot \{(1 - \delta) E[X_{k+1}] + 2\delta E[X_{2k+1}]\} \\ &= k \cdot \left\{ (1 - \delta) \frac{n-k}{n+1} + 2\delta \frac{n-2k}{n+1} \right\} \\ &= \frac{k}{n+1} \{(1 + \delta)n - (1 + 3\delta)k\} \end{aligned}$$

When $n > 3k$, $E[R2]$ is increasing in δ ; when $n < 3k$, $E[R2]$ is decreasing in δ . So given the distribution of the bidders values, the effect of the discounting factor on the sequential auction depends on the market competition. \diamond

proposition 2. The difference of revenue between holding a two-period auction and a one-period auction is:

$$\begin{aligned} E[R2] - E[R1] &= E[R2] - E[R1] \\ &= k \cdot ((1 - \delta) E[X_{k+1}] - 2(1 - \delta)E[X_{2k+1}]) \\ &= k \cdot (1 - \delta) (E[X_{k+1}] - 2E[X_{2k+1}]) \\ &= \frac{k \cdot (1 - \delta)}{n+1} (3k - n) \end{aligned}$$

so when $k < \frac{1}{3} \cdot n$, $E[R2] > E[R1]$, the auctioneer is better off by offering two-period auction. \diamond

A.2 Reserve Price Without Commitment

Proposition 3. First show that there exists such a \tilde{r} . We can reinterpret Eq. 4 as,

$$(v - r) \cdot \sum_{i=0}^{K-1} \text{prob}(n' = i) + \sum_{i=K}^n (v - E[Y_{K,n-1}|n' = i]) \text{prob}(n' = i)$$

Vs. $\delta \sum_{i=0}^n (v - E[Y_{K,n-1}|n' = i]) \cdot \text{prob}(n' = i)$

If $0 \leq \delta < \tilde{\delta}$, where $\tilde{\delta} = \frac{(v-r)\text{prob}(n' < K) + \sum_{i=K}^n (v - E[Y_{K,n-1}|n' = i]) \text{prob}(n' = i)}{\sum_{i=0}^n (v - E[Y_{K,n-1}|n' = i]) \text{prob}(n' = i)}$, when $v = r$, $LHS = 0$, while $RHS > 0$, thus $LHS(r) < RHS(r)$. When $v = 1$, $LHS > RHS$. Since both LHS and RHS are increasing in v (this can be seen from

$$0 \leq \frac{\partial LHS}{\partial v} = \delta \cdot \left(1 - \left(1 - \frac{H^2(v) - h(v) \int_0^v H(y) dy}{H^2(v)} \right) \right) < 1$$

Furthermore, it can be shown that the LHS is increasing faster than RHS (because the RHS is multiplied with an extra $\delta < 1$), thus, there exists a unique non-negative v^* which makes $LHS = RHS$. And thus threshold v is determined by Eq. 5.

Next we will show it is public perfect Nash Equilibrium to follow the strategy specified in proposition 3. Suppose that all the bidders except i follows this strategy. Now for bidder i , if his valuation is less than \tilde{r} , then he has no incentive to deviate by bidding in the first period because his only chance of winning is when $n' < K$; then he pays at least r for the winning item. But then according to Eq. 4, his expected payoff is lower than winning in the second period and paying a lower price. When $n' \geq K$, he cannot win bidding in either period. So bidding in the first period won't help him.

If a bidder i 's valuation is higher than \tilde{r} , then if he bids in the first period, he has no incentive to deviate from bidding his true value because by bidding lower there is positive probability that he will not be among the winners, and by bidding higher there is positive probability that he is going to pay more than his true value if he wins. According to Eq. 5, given that he will bid his true value, he has no incentive to deviate to wait till the second period. This concludes our proof. \diamond

B Bidders uncertain about number of periods

proposition 4. Assuming bidders' value follows a uniform distribution in $[0, 1]$,

1. When $K_2 > K_1 > \frac{n}{2}$, it is optimal to sell $\frac{n}{2}$ items in the first period, for both the High or Low type bidders. Thus the high type bidder will always pretend, with $\alpha = 1$ and $\rho' = \rho$.
2. When $K_2 > \frac{n}{2} > K_1$. In this case, the Low type auctioneer's best strategy is to sell K_1 items in the first period. For the high type bidder, he can choose to pretend to be a low type by selling K_1 items, or to reveal by selling $\frac{n}{2}$ items (there is no benefit to sell more than $\frac{n}{2}$ items in the first period). If he pretends, his expected revenue will be:

$$K_1\{(1 - \delta\rho'E[X_{K_1+1}] + \delta\rho'E[X_{K_2+1}]\} + (K_2 - K_1)\delta\rho'E[X_{K_2+1}] \quad (8)$$

If he chooses to reveal by selling $\frac{n}{2}$ items in the first period, his expected revenue is:

$$\frac{n}{2}(1 - \delta)E[X_{n/2+1}] + K_2\delta E[X_{K_2+1}] \quad (9)$$

For the high type auctioneer to feel indifferent between these two options, ρ' should satisfy Eq.(8)=Eq.(9), which gives the threshold value $\tilde{\rho} = \frac{0.25n^2(1-\delta)+\delta K_2(n-K_2)-K_1(n-K_1)}{\delta[K_1(n-K_1)+K_2(n-K_2)]}$. Since $\rho \geq \rho'$, if $\rho \leq \tilde{\rho}$, the high type auctioneer will pretend to be a low type with probability of 1. If $\rho \geq \tilde{\rho}$, there exists an α satisfying $\frac{\alpha\rho}{\alpha\rho+(1-\rho)} = \tilde{\rho}$, which makes Eq.(8)=Eq.(9). Thus the high type auctioneer will play mixed strategy where he pretends to be a low type bidder with probability α .

3. $\frac{n}{2} > K_2 > K_1$: this case works the same way as the previous one. The only difference is that, if deciding to reveal, the high type auctioneer will sell K_2 items instead of K_1 items in the first period. Thus, if revealing, his expected revenue will be:

$$K_2E[X_{K_2+1}] \quad (10)$$

Thus the threshold value $\bar{\rho}$ is determined by Eq.(8)=Eq.(10).

◇

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