Strategic Choice of Sequencing of Negotiations: A Model with One-Sided Incomplete Information[1]

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Abstract

This paper considers the effect of linkage between bargains that a single buyer has with two suppliers of different items. The buyer's valuation is private information but it is commonly known that one of the items is worth twice as much to the buyer as the other. Each bargain uses the extensive form of Section 5 of Fudenberg and Tirole (1983). We find that it is optimal for the buyer either to bargain simultaneously or to bargain first with the less important seller. The gains from sequencing come from being tough with the less important seller. The D1 refinement is modified and used to select equilibria in the bargaining game.

1 Introduction

The problem addressed in this paper arises in many different contexts, in which a bargainer has to engage in a sequence of negotiations, with the negotiations being of possibly differing importance and conducted with different partners. An example that has already been studied in the literature (Marshall and Merlo([9]) and Banerji ([2]))is that of negotiations between the United Auto Workers union and the major car companies in the United States. The UAW faces a sequence of negotiations with opponents of clearly unequal sizes and has the choice of whether to negotiate simultaneously or sequentially, and, if sequentially, to choose the order in which the negotiations take place. Banerji's work concentrates on two different models, one in which the union negotiates the total wage bill with each firm, and the other in which only the wage is negotiated and employment is chosen by the firms after the negotiations. The second model is driven by the fact that the car companies compete with each other, and that a high wage agreed on by one of them and publicly known, increases the level of profit for each of the other firms for each cost structure of these other firms. Therefore, later bargainers have more to give away, and this results in an upward pressure on negotiated wages in the later bargains.His first model is similar to a strategic bargaining version of the Marshall-Merlo model, which uses Nash bargaining. In the Marshall-Merlo "pattern bargaining" paper, the negotiated outcome in the first encounter affects the status quo point for future negotations with other firms.

A somewhat related issue in a different context is discussed by Winter([13]), in which the same parties negotiate on two issues, one of which is much more important than the other. He concludes that it is better for the parties concerned to tackle the more important issue first, so as to reduce the incentive for early posturing to get future advantage.

Other contexts could include a management negotiating with unions in two geographically different locations, or trade negotiations, say between the United States and Brazil, India and Japan. Since Japan is the most important of the three, it might be thought that the optimal sequence from the US viewpoint might be to deal with Japan last, since threats are less costly to carry out with the first two and might convince the third of the "toughness" of the party involved.[1]

[1]Of course, Japan could equally well disregard any sanctions against the first two

A somewhat different question of sequencing (and one less relevant to this paper) is informally discussed by Sebenius ([11]). Here the issue is the formation of the appropriate coalition and whom one needs to speak with first in order to bring such a coalition about. In the multilateral trade context, again from the US point of view, securing the agreement of the G-7 countries first and dealing with everyone else once the rich countries have formed a coalition with forward commitment, seems to be a commonly used strategy. In this paper, we do not consider such forward commitments or coalition formation, though this is certainly not to downplay the importance of considering such a problem.

Our paper focuses on a context in which a single buyer has to buy one object from each of two separate sellers. The value of one of the objects to the buyer is significantly higher than that of the other; the values are linearly related (deterministically) but are known only to the buyer. The sellers have probabilistic information about the values. (We shall expand on this description in the next section.) An example of the situation we have in mind would be the arrival of a new procurement Vice-President to a company. The new person carries his reputation with him, but he is operating in a different corporate environment, so there is uncertainty about how hard a bargainer he (or the procurement division in general) will be. In deciding how to conduct the negotiations with various suppliers, the new Vice-President might decide whether to negotiate first with the seat belt supplier or with the supplier of the anti-lock brakes used in the vehicles sold by the new VP's new company (or to deal with them simultaneously, since we assume the VP is able to conduct two negotiations simultaneously). The company's profitability and the procurement division's new goals will obviously be a common feature in both negotiations, and will affect the buyer's willingness to pay for each item.

The remainder of the paper is organised as follows: In the next section, we describe the model in detail and our main results in brief .[2]Section 3 contains the heart of the analysis and the calculation of the equilibria under various scenarios. Section 4 discusses the choice of sequence and Section 5 concludes.

countries, as not being relevant to a relationship of much greater scope and size, thus making posturing in earlier negotiations counter-productive.

[2]We put off a summary of the results until the next section, because the necessary notation has not been developed yet.

2 The Model and a Preview of the Results

We consider a model with a single buyer, B, and two sellers, S_1 and S_2. The buyer's value v_1 for $S_1's$ item is b, and for $S_2's$ item, $v_2 = 2b$. This means that B has a demand of one unit of $S_1's$ good, if the price is less than or equal to b; likewise, he has a unit demand for $S_2's$ good for a price at most $2b$. We assume that the sellers' reservation prices are identical and set equal to zero . The buyer's value b is known to the buyer, but not to the sellers, and takes on one of two values $\overline{b}$ (type h) or $\underline{b}$ (type ℓ) with initial probabilities π_h and π_ℓ respectively, with $\pi_h + \pi_\ell = 1$. The values are related as follows: $\overline{b} > \underline{b} > 0$. All players discount time at the common discount factor δ. The preceding is assumed to be commonly known.

The game being played is as follows: The buyer decides initially on a sequence of play; this could be (i)simultaneous,(ii) S_1S_2 and (iii)S_2S_1. Given the sequence chosen, the buyer makes an offer to S_i, who either accepts or rejects. If S_i rejects, she makes a counter-offer in the next period to the buyer, who accepts or rejects. If B rejects, he then makes an offer to S_{-i} (the other seller) in the next period and the same sequence of acceptance/ rejection and counter-offer follows. If the buyer has chosen to play both sellers simultaneously, both bargains take place in the first two periods.

We assume that any agreement in the first negotiation is observable to the second seller in the sequence, as is a failure to agree in the first negotiation.

The payoffs are as follows, if p_i is the agreed-upon price in the negotiation with seller i in period t_i :

$\sum_i \delta^{t_i-1}(v_i - p_i)$ to the buyer,[3]

$\delta^{t_i-1}p_i$ to S_i, if agreement; 0 otherwise.

The foregoing reflects some modelling choices that we have made in order to keep the model as simple as possible. The model has two types, since this suffices to demonstrate the effect we have in mind. In general, models with two types might not be representative of the realistic situations where there is a continuum or a general finite number of types; however, we see no reason to believe that the basic intuition of our results does not extend to the more complicated setting. (We would be extremely surprised if it did not.)

The bargaining between a buyer and a seller consists of at most one offer by each side with the buyer moving first in each bargain. This is clearly

[3] This, of course, assumes two agreements. The payoffs for the cases with zero agreements or one agreement can be written analogously.

not the most general depiction of bargaining with incomplete information between two players; a large literature exists on this problem.[4] However, it is well-known that the general problem of alternating offers bargaining is a complex one, and it is not our intention to repeat complex analyses that have already been done. We have once again chosen a simple model of bilateral bargaining, since we embed this in a larger problem, but one that allows at least some opportunity for information to be revealed in the course of the bargaining, and for this information to be used by the uninformed party. If we had chosen to model the bargaining in more detail, we would no doubt have had to contend with even more equilibria than we already have; in addition, instead of no agreement in our finite horizon model, we would get delayed agreement in an infinite horizon model. One might argue that effectively there is a finite horizon for each bargain in the context we have in mind; if within a fixed time no agreement has been reached, the buyer breaks off and invokes his outside option, which may be to produce the item himself at a greater cost, rather than hold up production for the negotiation to conclude. However, the costs and benefits implicit in such an argument are not present in our model, so we have nothing more precise to say about it.

The discount factor plays a role here, even in a four-period model, in that it penalises the buyer for sequencing the sellers rather than negotiating with them simultaneously. Simultaneous negotiation would, of course, lead to both negotiations being concluded in two periods rather than four, and the benefit to the buyer from this should be accounted for in the analysis.

We are now in a position to state our results informally. We use the notion of perfect Bayes equilibrium (or sequential equilibrium) and we will employ a "divinity" refinement in the analysis. Our analysis is done for the more interesting case of δ sufficiently high, but less than 1.

If π_h is "low", i.e. below a threshold value, the optimal sequence for the

[4] For different models, see Sobel and Takahashi ([12]), Fudenberg, Levine and Tirole ([7]), (Chatterjee and L. Samuelson ([4],[5]), Ausubel et al ([1]), and Cho ([6]). Ausubel et al.([1]) is a survey in which they discuss a variety of papers including their own important contributions to the literature. This is just a sample; other papers on this topic have appeared and continue to appear. The model we use for each single bargain is mentioned in Section 5 of Fudenberg and Tirole [8]. However, the notes mentioned there are not available any more with the authors. The paper [8] was published before the refinements literature took off and the original notes are unlikely to have contained any discussion of D1, even for the one-stage model it considered.

buyer is $S_2 S_1$, that is to negotiate first with the more important buyer. However, under these circumstances, negotiating simultaneously is the preferred alternative.

If π_h is "high", i.e. above the threshold, the optimal sequence is $S_1 S_2$. This is true, moreover, for an interesting reason. It is not just the case that the buyer gains in the second negotiation by signalling toughness in the first negotiation. The gains from sequencing in the manner indicated actually come from the first negotiation, in that S_1 is aware of the buyer's extra incentive to play tough, and modifies her strategy to be less aggressive.

There is an additional inefficiency as a result of the sequencing. If π_h is "high" and δ is "low", the type ℓ buyer might obtain only one of the two objects with positive probability, even though it is commonly known that gains from trade exist.

3 The Analysis

3.1 The Buyer and One Seller

We first analyze the two-period bargaining game between the buyer and a single seller; this analysis will lead us to that of the complete model. For convenience, we do this assuming the seller is S_1. Throughout the analysis, "equilibrium" will refer to "sequential equilibrium" (or " perfect Bayes equilibrium", since we will not formally check the consistency conditions).

The following proposition describes all the equilibria of this game.

Proposition 1 *(i) If $\pi_h \leq \frac{\underline{b}}{\overline{b}}$, a pooling equilibrium exists. Both types h and ℓ offer $p_\ell = \delta \underline{b}$, and S_1 accepts. If S_1 rejects following any buyer offer, she proposes a price of $\underline{b}$ and buyer ℓ accepts any price $\leq \underline{b}$, buyer h accepts any price $\leq \overline{b}$ and rejects otherwise. If a price $< p_\ell$ is offered by the buyer, the seller rejects and offers $\underline{b}$. If a price above p_ℓ is offered by the buyer, the seller accepts if the price is less than or equal to $\underline{b}$ or above $\delta \overline{b}$, and rejects otherwise. Seller beliefs that make such behaviour optimal are, for example, π_h does not change for any price offer $\leq \underline{b}$ and changes to 1 for any greater offer.[5] The equilibrium payoffs are $(v - \delta \underline{b})$ to the buyer and $\delta \underline{b}$ to the seller.*

[5] The seller might easily also have accepted all offers $\geq p_\ell$. Since buyers want low offers, p_ℓ would still have been offered in equilibrium. However, beliefs for values of $p > \underline{b}$ cannot be consistent with a positive probability of acceptance, buyer rationality and $\pi_h < 1$.

(ii) If π_h lies between $\frac{\underline{b}}{\bar{b}}$ and $\frac{\underline{b}}{\delta\bar{b}}$, a pooling equilibrium exists. Both buyer types offer $\pi_h\delta\bar{b}$, and the seller accepts. If the seller rejects any offer, she proposes $\bar{b}$. The type h buyer accepts any offer $\leq \bar{b}$, and the type ℓ buyer accepts any price at or below $\underline{b}$. If the buyer offer exceeds $\pi_h\delta\bar{b}$, the seller accepts, except for price offers above $\underline{b}$ and below $\delta\bar{b}$, which are rejected, with beliefs analogous to the case above. If the buyer offer is less than $\pi_h\delta\bar{b}$, the seller rejects, believing π_h does not change. The equilibrium payoffs can be calculated as above, replacing p_ℓ by $\pi_h\delta\bar{b}$.

(iii) For any x and $\pi(x)$ such that $x\epsilon[\delta\underline{b}, \underline{b}]$, $x = \delta\pi(x)\bar{b}$ and for $\pi_h > \pi(x)$, there exists a semi-separating equilibrium with the buyer type ℓ offering $p = x$, the buyer type h randomising between $\delta\bar{b}$ and p with probabilities just sufficient to make $\pi_h(p) = \pi(x)$[6], and the seller randomising between acceptance (probability a) of $p = x$ and rejection so as to make

$$(\bar{b} - x)a = \bar{b} - \delta\bar{b} \tag{1}$$

, and accepting $\delta\bar{b}$ or higher with probability 1. If the seller rejects, she offers $\bar{b}$, unless $x = \delta\underline{b}$, in which case she randomises between $\underline{b}$ and $\bar{b}$,with probability ρ for the higher offer. A price $\widetilde{x}$ in $(x, \underline{b}]$ is accepted by the seller with probability $\widetilde{a}$ such that $(\bar{b} - \widetilde{x})\widetilde{a} = \bar{b} - \delta\bar{b}$, with accompanying belief $\pi_h = \pi(\widetilde{x})$.[7] Any price greater than $\underline{b}$ leads to the belief $\pi_h = 1$, and the seller rejects any offer less than $\delta\bar{b}$. Any price less than x does not change the seller's belief that $\pi_h = \pi(x)$, and she rejects and offers $\bar{b}$.

Remark 1 *Note that there are multiple equilibria and that there is no purely separating equilibrium.*

Proof. (Sketch)By inspection, the strategies specified are best against each other and rational, given out-of-equilibrium beliefs as specified. To check in (iii) above that there is no incentive for the buyer to deviate from x, note that type h is indifferent between x and any higher price. Type ℓ compares $(\underline{b} - x)a$ to $(\underline{b} - \widetilde{x})\widetilde{a}$ or $(\underline{b} - x)\frac{\bar{b}-\delta\bar{b}}{(\bar{b}-x)}$ to $(\underline{b} - \widetilde{x})\frac{\bar{b}-\delta\bar{b}}{(\bar{b}-\widetilde{x})}$. Substituting for a and $\widetilde{a}$, from 1 we need to show that

[6] This is slightly infelicitous notation, but just means that the posterior probability given an observed price of p is equal to the preassigned probability $\pi(x)$.

[7] Alternatively, the seller could react to a price of $\widetilde{x}$ by rejecting, based on a belief that $\pi_h = 1$.

$$\{(\underline{b}-x)\frac{\overline{b}-\delta\overline{b}}{(\overline{b}-x)}/(\underline{b}-\widetilde{x})\frac{\overline{b}-\delta\overline{b}}{(\overline{b}-\widetilde{x})}\}>1, \tag{2}$$

so that type ℓ is better off not deviating. Simplifying 2we get

$$(\underline{b}-x)(\overline{b}-\widetilde{x})/\{(\overline{b}-x)(\underline{b}-\widetilde{x})\},$$

$$>1, \text{ as } \widetilde{x}>x.$$

We need also to show there are no other equilibria. There can be no pooling or semi-separating equilibria with buyer offers above $\underline{b}$. The preceding argument shows that there cannot be mixed-strategy equilibria with more than two equilibrium offers by the buyer, as type ℓ will choose the lowest such offer and this will lead to the seller rejecting all intermediate offers. Finally, there cannot be separating equilibria with pure strategies being employed by the buyer. For if there were, the pure offers would have to be $\delta\underline{b}$ and $\delta\overline{b}$ for types ℓ and h respectively, and this would be subject to deviation by type h. ■

In order to proceed with the remainder of the paper, which involves ultimately comparing equilibrium payoffs, it is necessary to argue for a particular equilibrium in this simple two-period bargaining game. We shall invoke a slightly modified D1 requirement ([3][**?**]), followed by an ad hoc argument of plausibility for those readers who do not like the refinements literature.

The D1 refinement restricts beliefs off the equilibrium path in the following way: Consider an out-of-equilibrium action m, types ℓ and h, and a response by the receiver of $a(m)$. Let the equilibrium payoff of type t be $E(t)$, and the payoff if $a(m)$ is used after message m as $\Xi(a(m),m,t)$. Consider the sets of actions $A(\ell)=\{\ a(m)\mid\Xi(a(m),m,\ell)>E(\ell)\}$ and $\widehat{A}(h)=\{a(m)\mid\Xi(a(m),m,h)\geq E(h)\}$. If $\widehat{A}(h)\subset A(\ell)$, then the beliefs should accord probability 0 to type h.

For the ensuing analysis, we need an additional condition, namely that the probability ρ, with which the seller offers $\overline{b}$ in the event she rejects the buyer offer x with $\pi_h(x)=\frac{\underline{b}}{\overline{b}}$, is equal to 1. The arguments for this condition are twofold, one relating to the equilibrium path and the other to out-of-equilibrium moves. In one of the equilibria described above, $a\epsilon(0,1)$ and ρ is potentially positive and less than 1. This occurs when the equilibrium price is $p_\ell=\delta\underline{b}$. For prices above p_ℓ, there exist other semi-separating equilibria with $\rho=1$. A continuity argument would ensure that as $x\rightarrow p_\ell$, $\rho(x)\rightarrow$

$\rho(p_\ell)$, whence $\rho(p_\ell) = 1$. Off the equilibrium path, the argument is somewhat different. If $\pi_h(x) > \frac{\underline{b}}{\overline{b}}, \rho = 1$ in any optimal seller strategy. Therefore, $\rho\epsilon(0,1)$ potentially when $\pi_h(x) \leq \frac{\underline{b}}{\overline{b}}$. When this inequality holds, any optimal response by the seller must accept an offer $x > p_\ell$, i.e. $a = 1$. Thus, off the equilibrium path, in any optimal seller strategy given her beliefs, either $\rho = 1$ or $a = 1$, in which case the value of ρ makes no difference to any payoff. Henceforth, we shall assume that $\rho = 1$ whenever $\pi_h(x) \geq \frac{\underline{b}}{\overline{b}}$ and $\pi_h > \frac{\underline{b}}{\overline{b}}$.

Proposition 2 *Under D1, with the additional condition that $\rho = 1$,the following constitutes the unique equilibrium of the game.*

(i) If $\pi_h \leq \frac{\underline{b}}{\overline{b}}$, both types h and ℓ offer $p_\ell = \delta\underline{b}$, and S_1 accepts. If S_1 rejects following any buyer offer, she proposes a price of $\underline{b}$ and buyer ℓ accepts any price $\leq \underline{b}$, buyer h accepts any price $\leq \overline{b}$ and rejects otherwise. If a price $<p_\ell$ is offered by the buyer, the seller rejects and offers $\underline{b}$. If a price above p_ℓ is offered by the buyer, the seller accepts if the price is less than or equal to $\underline{b}$ or above $\delta\overline{b}$, and rejects otherwise.

(ii) For $\pi_h > \frac{\underline{b}}{\overline{b}}$, the buyer type ℓ offers $p_\ell = \delta\underline{b}$, the buyer type h randomises between $\delta\overline{b}$ and p_ℓ with probabilities just sufficient to make $\pi_h(p_\ell) = \frac{\underline{b}}{\overline{b}}$, and the seller randomises between acceptance (probability a) of p_ℓ and rejection so as to make

$$(\overline{b} - x)a = \overline{b} - \delta\overline{b} \tag{3}$$

, and accepts $\delta\overline{b}$ or higher with probability 1. If the seller rejects, she randomises between $\underline{b}$ and $\overline{b}$, with probability ρ for the higher offer. Any price offer greater than the equilibrium price leads to the belief $\pi_h = 1$, and the seller then rejects any offer less than $\delta\overline{b}$.

The beliefs of the seller that support this equilibrium are as follows: If an offer $\widetilde{x} <$ the equilibrium offer is observed, the seller believes $\pi_h(\widetilde{x}) = 0$. If an offer above the equilibrium price is observed, the seller believes that $\pi_h(\widetilde{x}) = 1$.[8]

Proof. We need to show that D1 excludes beliefs that sustain the other equilibria. Consider the equilibria with buyer offer $x\epsilon(\delta\underline{b}, \underline{b}]$ with positive probability of acceptance and seller equilibrium beliefs $\pi_h(x) > \frac{\underline{b}}{\overline{b}}$, with a

[8]The promised ad hoc argument for the plausibility of these beliefs is monotonicity- a higher offer increases the probability of the high type and a lower offer increases the probability of a low type buyer.

positive probability a^* of acceptance[9]. Suppose an offer $\widetilde{x} < x$ is observed. Using the notation above, $A(\ell) = \{a \mid (\underline{b} - \widetilde{x})a \geq (\underline{b} - x)a^*\}$ and $\widehat{A}(h) = \{a \mid (\bar{b} - \widetilde{x})a > (\bar{b} - x)a^*\}$. Let $a_\ell = \frac{(\underline{b}-x)a^*}{(\underline{b}-\widetilde{x})}$, and $a_h = \frac{(\bar{b}-x)a^*}{(\bar{b}-\widetilde{x})}$. Since $\bar{b} > \underline{b}$ and $x > \widetilde{x}$, $a_\ell = \frac{(\underline{b}-x)a^*}{(\underline{b}-\widetilde{x})} < \frac{(\bar{b}-x)a^*}{(\bar{b}-\widetilde{x})} = a_h$. Therefore $\widehat{A}(h) \subset A(\ell)$. Therefore, by D1, the players should assign 0 probability to type h, and hence probability 1 to type ℓ. But if this is the seller's belief every offer above $\delta\underline{b}$ will be accepted with probability 1, and hence the deviation to $\widetilde{x} < x$ is profitable for both types. Hence equilibria of the sort being considered do not satisfy D1; we henceforth discard these.

It remains to show that the equilibrium identified in the proposition does satisfy D1. For $\pi_h \leq \frac{\underline{b}}{\bar{b}}$, consider an offer $\widetilde{x} > \delta\underline{b}$. Now $\widetilde{x} > x$, and the situation is reversed, so that $\pi_\ell = 0$. Given this belief, it is easy to see that the prescribed seller behaviour is optimal. If $\pi_\ell = 1$, a price less than $\delta\underline{b}$ is rejected by the seller who then offers $\underline{b}$, thus giving both types of buyer a zero payoff, hence making the potential downward deviation unprofitable. ■

3.2 Sequential Bargaining with Two Sellers.

We now consider the complete game where the buyer negotiates sequentially with the two sellers, and the game can potentially continue for four periods. However, if there is an agreement with the first seller in the first period, the buyer can start the second negotiation in the next period. We initially consider the sequence S_1S_2. We assume that the modified D1 criterion discussed in the previous subsection has been applied to rule out all but one equilibrium in the negotiation with the second seller. Once again, let π_h be the initial probability of a high value buyer. The equilibria here are calculated on the assumption of sufficiently high values of δ in order to avoid possibly uninteresting special cases. The cutoff value of δ depends on the values of $\bar{b}, \underline{b}$; the nature of this dependence is made clear in the analysis.

Proposition 3 *The following constitutes a (modified D1) equilibrium for the game in which the buyer plays S_1S_2 in sequence.*

(i) If $\pi_h \leq \frac{\underline{b}}{\bar{b}}$, both types of buyer offer $\delta\underline{b}$ to S_1, who accepts it. The buyer then offers $2\delta\underline{b}$ to S_2 who accepts it.

[9] The probability t^* of the seller offering $\bar{b}$ after rejecting is 1, since π_h is larger than the value that makes the seller indifferent between offering $\bar{b}$ and offering $\underline{b}$.

If the buyer deviates and offers a lower price (to either seller), the sellers believe the buyer is type ℓ, reject and offer $\underline{b}$ (or twice that, for the second seller) in the next period, which both types of buyer accept. If a buyer rejects an offer of $\underline{b}$, the second seller's actions do not change.

If the buyer offers more than the equilibrium pooling price, the sellers change their beliefs to $\pi_h = 1$ and offer $\bar{b}$ (or $2\bar{b}$ for the second seller) after rejecting any buyer offer less than $\delta\bar{b}(2\delta\bar{b})$.[10]

If the seller offers any price greater than the equilibrium price, the buyer rejects and continues with the prescribed equilibrium behaviour.

(ii) If $1 > \pi_h > \frac{\underline{b}}{\bar{b}}$, and δ is sufficiently high, the buyer makes a pooling offer to the seller of $\delta^2\underline{b}$, which the seller accepts. If the seller rejects this offer, she offers $\delta\underline{b}$, which both buyer types accept. The equilibrium in the game with the second seller proceeds as in part (ii) of the previous proposition. (If δ is "low", the buyer initially makes a pooling offer to the seller, who rejects and makes a separating offer; the game with the second seller has $\pi_h = 1$ or 0.)

The out-of-equilibrium behaviour in the game with the first seller is as follows: If the buyer offer is different from the equilibrium offer, the seller believes, depending on the offer, either that $\pi_h = 0$ or $\pi_h = 1$ or π_h is unchanged, and makes the optimal decision given this belief. The second seller plays the equilibrium strategy of the previous subsection, given the new beliefs.

Proof. In case (i) above, S_1 and S_2 have the same initial beliefs. If the type h buyer deviates and offers $\widetilde{x} \neq \delta\underline{b}$ or rejects a seller offer of $\underline{b}$, the sellers either continue to believe $\pi_h \leq \frac{\underline{b}}{\bar{b}}$, and their behaviour remains unchanged, or they believe $\pi_h > \frac{\underline{b}}{\bar{b}}$. (The belief of course depends on the offer that gave rise to it. Monotonicity of beliefs would imply that an offer lower than the equilibrium would not increase π_h and a higher offer would not reduce it.) In this event, we are in case (ii), and the expected payoff for the high type buyer is $\delta(\bar{b}-\delta\underline{b})+\delta^2.2.\bar{b}(1-\delta)$, while the equilibrium payoff is $(\bar{b}-\delta\underline{b})+\delta.2.(\bar{b}-\delta\underline{b})$. The equilibrium payoff is clearly greater, and therefore there is no reason for the buyer to make a rejected initial offer to S_1.[11]

(ii) We now turn to the more interesting part of this proposition, namely that for high δ, the equilibrium is pooling even for $\pi_h > \frac{\underline{b}}{\bar{b}}$. We assume

[10] This is consistent with a modified version of D1 as before. This is worked out in the appendix.

[11] We check in the appendix that a modified D1 is satisfied by this equilibrium.

the equilibrium of the previous subsection holds with the second seller, and consider the first seller's offer after she has rejected a buyer offer with a belief that $\pi_h > \frac{\underline{b}}{\overline{b}}$. We first investigate whether an offer exists that would induce type h to accept and type ℓ to reject. Such an offer x would have to satisfy the following conditions:

$$\overline{b} - x + \delta.2.\overline{b}(1-\delta) \geq 0 + \delta.2.(\overline{b} - \delta\underline{b}), \tag{4}$$

and

$$\underline{b} - x \leq 0 + \delta.2.\underline{b}(1-\delta). \tag{5}$$

In (5), the low type gets 0 future payoff from being identified as a high type. These two equations give us:

$$\overline{b}(1-2\delta^2) \geq x \geq \underline{b}(1-2\delta). \tag{6}$$

For there to be such an offer x, we must have

$$2\overline{b}\delta^2 - 2\underline{b}\delta - (\overline{b} - \underline{b}) \leq 0. \tag{7}$$

The left-hand-side of (7) is increasing in δ (since $4\overline{b} - 2\underline{b} > 0$), and is negative at $\delta = 0$ and positive ($2\overline{b} - 2\underline{b} - \overline{b} + \underline{b} = \overline{b} - \underline{b} > 0$) at $\delta = 1$. Therefore, there exists a $\delta^{\#}$, such that for all $\delta > \delta^{\#}$, (7) does not hold, and therefore such an offer x does not exist. For low values of δ, such an offer might exist.

Therefore, for sufficiently high δ, there must be a pooling equilibrium, a price offer by the seller that induces identical behaviour by the two types of buyer. We consider, first, an offer that both types of buyer would accept. Let this offer be denoted by x^*. We undertake an analysis using the modified D1 criterion to fix beliefs for out-of-equilibrium actions, in this case a rejection. Such a rejection might affect the beliefs of S_2 who then modifies her actions. We assume that the offer $2\delta\underline{b}$ occurs in the supports of the (mixed) strategies played by types h and ℓ. Let a^* be the equilibrium probability of acceptance of $2\delta\underline{b}$ by the seller S_2 and a the probability of acceptance if $S_1's$ offer of x^* is rejected by the buyer. Then a rejection is profitable for buyer with value b, if

$$b - x^* + \delta.2.(b - \delta\underline{b})a^* < 0 + \delta.2.(b - \delta\underline{b})a. \tag{8}$$

This gives us

$$a > \frac{b - x^*}{\delta.2.(b - \delta\underline{b})} + a^*. \tag{9}$$

Let a_h, a_ℓ denote the cutoff value of $a - a^*$ in (9) above, with b replaced by $\overline{b}, \underline{b}$ respectively. Then, as before, if $a_h > a_\ell$, the set of actions under which type h would be better off is a subset of the set of actions for which type ℓ would be better off and a rejection would lead the second seller to believe the buyer's type is ℓ.

The quantity $\frac{a_h}{a_\ell} = \frac{\frac{\overline{b}-x^*}{\delta.2.(\overline{b}-\delta\underline{b})}}{\frac{\underline{b}-x^*}{\delta.2.(\underline{b}-\delta\underline{b})}} = \frac{(\overline{b}-x^*)/(\underline{b}-x^*)}{(\overline{b}-\delta\underline{b})/(\underline{b}-\delta\underline{b})}$. The numerator and denominator are both of the form $\frac{\overline{b}-y}{\underline{b}-y}$. Such a quantity increases with y. Therefore, if $x^* > \delta\underline{b}$, $\frac{a_h}{a_\ell} > 1$, and if $x^* < \delta\underline{b}$, $\frac{a_h}{a_\ell} < 1$. In the first case, a rejection will therefore lead to the belief that $\pi_h = 0$, and in the second case that $\pi_h = 1$. Using these out-of-equilibrium beliefs, we check whether each case is possible in equilibrium.

Type h then accepts in equilibrium, given $x^* > \delta\underline{b}$, provided

$$\overline{b} - x^* + 2\delta\overline{b}(1 - \delta) \geq 0 + 2\delta(\overline{b} - \delta\underline{b}), \tag{10}$$

and type ℓ accepts if

$$\underline{b} - x^* + 2\delta\underline{b}(1 - \delta)a^* \geq 0 + 2\delta\underline{b}(1 - \delta). \tag{11}$$

The previous two equations together imply that

$$x^* \leq \min\{\underline{b} - 2\delta\underline{b}(1 - \delta)\frac{\delta(\overline{b} - \underline{b})}{\overline{b} - \delta\underline{b}}, \overline{b} - 2\delta^2(\overline{b} - \underline{b})\}. \tag{12}$$

Therefore, we must have

$$x^* - \delta\underline{b} \leq \overline{b} - 2\delta^2(\overline{b} - \underline{b}) - \delta\underline{b}. \tag{13}$$

The RHS of (13) is decreasing in δ, takes the value $\overline{b}$ at $\delta = 0$, and $-(\overline{b} - \underline{b})$ at $\delta = 1$. Therefore, there exists some value $\widetilde{\delta}$, such that for $\delta > \widetilde{\delta}$, the RHS is negative, and is therefore inconsistent with the hypothesis that $x^* > \delta\underline{b}$. Therefore, at least one of the equilibrium conditions will not hold, and this case is therefore ruled out for high δ.

For $x^* < \delta\underline{b}$, it is easy to see that the belief that a rejection implies type h is sufficient to deter a rejection by either type of buyer. Given all seller offers of this kind will be accepted by both types, the seller should, in equilibrium, make the highest possible such offer. But a “highest such offer” does not exist. At $x^* = \delta \underline{b}$, there is no restriction on out-of-equilibrium beliefs, but we assume the beliefs are continuous from below in x^* in order to obtain an equilibrium satisfying the modified D1.

The final step in the proof is to demonstrate that the buyer will make a pooling offer of $\delta^2 \underline{b}$ in the first period and that the seller will accept. If the seller rejects, we are in the case considered above. We again consider the likely beliefs if an offer other than $\delta^2 \underline{b}$ is made by the buyer in the first period. If the seller's beliefs either do not change or change to $\pi_h = 1$, the buyer (both types) would be worse off. Consider therefore, the situation where the sellers' subsequent actions are based on the belief that $\pi_h = 0$, and apply the modified D1 criterion to the probability of acceptance a in the first period.

The type h buyer's gain in payoff *relative* to equilibrium for a given a , $R(a, h)$, is given by

$a\{\bar{b} - x + 2\delta(\bar{b} - \delta\underline{b})\} + (1-a)\{\delta(\bar{b} - \underline{b}) + 2\delta^2(\bar{b} - \delta\underline{b})\}$
$-\{\bar{b} - \delta^2\underline{b} + 2\delta(\bar{b} - \delta\underline{b})a^*\}$
$= a\{\bar{b} - \underline{b} + \underline{b} - x + 2\delta(\bar{b} - \underline{b} + \underline{b} - \delta\underline{b})\}+$
$(1-a)\{\delta(\bar{b} - \underline{b}) + 2\delta^2(\bar{b} - \underline{b} + \underline{b} - \delta\underline{b})\}$
$-\{\bar{b} - \underline{b} + \underline{b} - \delta^2\underline{b} + 2\delta(\bar{b} - \underline{b} + \underline{b} - \delta\underline{b})a^*\}$
$= a(\bar{b} - \underline{b})(1+2\delta) + (1-a)\delta(\bar{b} - \underline{b})(1+2\delta) - \{(\bar{b} - \underline{b})(1+2\delta a^*)\}$
$+R(a, \ell)$, where $R(a, \ell)$ is the gain in payoff for type ℓ. Thus,

$$R(a, h) = R(a, \ell) + (\bar{b} - \underline{b})\{a(1+2\delta) + (1-a)\delta(1+2\delta) - (1+2\delta a^*)\}. \quad (14)$$

If we show the expression in the curly brackets is positive for sufficiently high values of δ, we will have demonstrated that for every a, $R(a, h) > R(a, \ell)$, and therefore any such deviation must come from type h. This will ensure that no buyer type will wish to deviate from the equilibrium offer. The expression in the curly brackets is

$$a[2\delta(1 - a^*)] + (1-a)[\delta - 1 + 2\delta(\delta - a^*)].$$

The first term is clearly always positive, and the second is positive for sufficiently high δ. (Note that a^* itself a function of δ.) Thus the belief that $\pi_h = 0$ is not consistent with D1 ■

We now consider the reverse sequence $S_2 S_1$. The nature of the equilibrium changes somewhat, because now the buyer is not as concerned about the high-value future as the current offer. Once again, we retain the same assumptions made in the previous propositions regarding the use of modified D1 as the equilibrium selection criterion. Again the proposition, significantly the third part, holds for δ sufficiently close to 1.

Proposition 4 *(i) If* $\pi_h \leq \frac{\underline{b}}{\overline{b}}$*,both types of buyer offer* $2\delta\underline{b}$ *and* $\delta\underline{b}$ *respectively to sellers* S_2 *and* S_1 *in successive periods; both offers are accepted. If the seller rejects, she offers* $2\underline{b}$ *or* $\underline{b}$ *respectively, and the buyer accepts. If the buyer rejects* S_2'*s offer,* π_h *does not change and* S_1 *makes optimal decisions thereafter. If the buyer offers a price to* S_2 *in* $(2\delta\underline{b}, 2\delta\overline{b})$ *or below* $2\delta\underline{b}$*, the seller rejects. (Similarly* S_1 *rejects offers below* $\delta\underline{b}$ *or in* $(\delta\underline{b}, \delta\overline{b})$*).*

(ii) If $\pi_h \in (\frac{\underline{b}}{\overline{b}}, \frac{x_p}{x_s})$[12]*, both types of buyer will offer* δx_p *to the seller* S_2*, who will accept. If the seller rejects, she will offer* x_p*, which both types of buyer will accept. The equilibrium in the second bargain will be the semi-separating one already outlined in the first proposition. Out-of-equilibrium actions will generate optimal responses, given beliefs consistent with the modified D1 criterion.*[13]

(iii) For $\pi_h > \frac{x_p}{x_s}$*, both types of buyer offer* $\widetilde{x} < \min(\underline{b}, \delta\pi_h x_s)$*; if the offer* $\geq \delta\pi_h x_s$*, the seller believes that* $\pi_h = 1$ *and accepts if the offer is* $\delta\overline{b}$*, otherwise the seller rejects. The seller's beliefs are unchanged by offers below* $\delta\pi_h x_s$*. If the seller rejects, she offers* x_s*, type* h *accepts and type* ℓ *rejects. On the equilibrium path,* π_h*=0, if the game ends without an agreement between the buyer and* S_2*; the second bargain proceeds accordingly.*[14]

Proof. (i) Suppose that $\pi_h \leq \frac{\underline{b}}{\overline{b}}$. If this is also the belief at the beginning of the second bargain, there will be a pooling equilibrium with seller S_1. Consider the offer to be made by S_2 in the event she has to move; any offer of $2\underline{b}$ or less will be accepted by both types, any offer above this will be

[12] The quantities x_p and x_s will be explicitly calculated in the proof.

[13] An alternative equilibrium, which cannot be ruled out by the refinement criterion, is for both types of buyer to make an offer $x_2 < \delta x_p$,which is rejected by the seller, who then offers x_p. Any deviating offer acceptable to the seller under her beliefs π_h will then signal that $\pi_h = 1$, and be rejected unless it exceeds $\delta\overline{b}$. We do not consider this equilibrium in the sequel, because the one in the text dominates it for all types of player.

[14] This equilibrium is consistent with the modified refinement criterion we have used. For some values of $\overline{b} - \underline{b}$, and for δ sufficiently different from 1, there could be another equilibrium in which both types of buyer offer the pooling offer $\delta\pi_h x_s$ in the first period, and the seller accepts, with the second bargain being adjusted to take into account the non-revelation. In general, it is not clear that such a pooling offer would give typeℓ a non-negative payoff, if accepted. As $\delta \to 1$, the region in which such an equilibrium exists disappears.

rejected by type ℓ,and can therefore only be $2\bar{b}$. But given the value of π_h, $2\underline{b}$ is better for the seller. Therefore, the offer of $2\delta\underline{b}$ is optimal for both types of buyer in the first period of the game.

We now consider the other cases, where $\pi_h > \frac{\underline{b}}{\bar{b}}$. Consider the seller in the second period of the first bargain. She can make a separating offer x_s. This must satisfy the following conditions:

$$2\bar{b} - x_s + \delta(\bar{b} - \delta\bar{b}) \geq 0 + \delta(\bar{b} - \delta\underline{b}) \tag{15}$$

and

$$2\underline{b} - x_s + \delta(\underline{b} - \delta\underline{b})a^* \leq 0 + \delta(\underline{b} - \delta\underline{b}). \tag{16}$$

Here a^* has the usual meaning (in this paper) of equilibrium probability of acceptance of an offer of $\delta\underline{b}$. This gives

$$2\underline{b} - \delta^2 \frac{(\underline{b} - \delta\underline{b})(\bar{b} - \underline{b})}{(\bar{b} - \delta\underline{b})} \leq x_s \leq 2\bar{b} - \delta^2(\bar{b} - \underline{b}). \tag{17}$$

Therefore, such an x_s exists for all values of δ, unlike in the case of the S_1S_2 sequence. The value of x_s will obviously coincide with the upper bound to be

$$x_s = 2\bar{b} - \delta^2(\bar{b} - \underline{b}). \tag{18}$$

(ii)We now consider whether a pooling offer x_p could exist. Just as in the previously analysed sequence, the use of modified D1 indicates that a rejection of an offer $x_p > 2\delta\underline{b}$ will be believed to signal a type ℓ buyer, and the rejection of an offer less than $2\delta\underline{b}$ a type h buyer.[15] Therefore a seller will choose, from (17), to set

$$x_p = 2\underline{b} - \delta^2 \frac{(\underline{b} - \delta\underline{b})(\bar{b} - \underline{b})}{(\bar{b} - \delta\underline{b})}. \tag{19}$$

We note that the quantity $\frac{x_p}{x_s} =$

$$\frac{2\underline{b} - \delta^2 \frac{(\underline{b} - \delta\underline{b})(\bar{b} - \underline{b})}{(\bar{b} - \delta\underline{b})}}{2\bar{b} - \delta^2(\bar{b} - \underline{b})}.$$

[15]If rejection is believed to come from a type h, the actions and beliefs are inconsistent.

Subtracting from this $\frac{2\underline{b}}{2\bar{b}}$, we get

$$\begin{aligned}&\delta^2(\bar{b}-\underline{b})[2\underline{b}-2\bar{b}\frac{(\underline{b}-\delta\underline{b})}{(\bar{b}-\delta\underline{b})}]\\&=\frac{\delta^2(\bar{b}-\underline{b})}{(\bar{b}-\delta\underline{b})}[2\delta\underline{b}(\bar{b}-\underline{b})]>0.\end{aligned}$$

Therefore, there is a range of values of π_h in between $\frac{\underline{b}}{\bar{b}}$ and $\frac{x_p}{x_s}$, such that the seller will make the pooling offer x_p in this range, and x_s above this range.

We now consider the buyer's first offer to S_2, given that if S_2 rejects, she offers either $x_p, x_s, \underline{b}$ or $\bar{b}$, depending on her beliefs. We note, as before, that perfectly separating offers cannot be in equilibrium. We check candidate pooling offers x_2 for $\frac{x_p}{x_s} \geq \pi_h > \frac{\underline{b}}{\bar{b}}$. In equilibrium, we must have

$$x_2 \geq \delta x_p, \tag{20}$$

in order for the seller to accept the offer. Suppose a deviation to some offer $\widetilde{x}$ signals that the buyer is type ℓ and suppose the deviating offer is at least $2\delta\underline{b}$ and is therefore acceptable given these beliefs. To deter such a deviation, we must have

$$2b - x_2 + \delta(b-\delta\underline{b})a^* \geq 2b - 2\delta\underline{b} + \delta(b-\delta\underline{b}),$$

for $b = \underline{b}, \bar{b}$. This gives

$$x_2 \leq 2\delta\underline{b} - \delta(b-\delta\underline{b})(1-a^*),$$

for both values of b,and therefore

$$x_2 \leq 2\delta\underline{b} - \delta(\bar{b}-\delta\underline{b})(1-a^*). \tag{21}$$

$>$From the definition of x_p and (20), we get

$$\begin{aligned}x_2 &\geq 2\delta\underline{b} - \delta^2(\underline{b}-\delta\underline{b})(1-a^*)\\&> 2\delta\underline{b} - \delta(\bar{b}-\delta\underline{b})(1-\\&a^*).\end{aligned}$$

Therefore, there cannot exist such a pooling offer with the specified out-of-equilibrium beliefs. Consider now the case where a deviation to $\widetilde{x}$ does not change beliefs . It is clear that $x_2 = \delta x_p$ is sustainable as an equilibrium. If $\pi_h = 1$, then the seller will reject anything less than $2\delta\bar{b}$, and this cannot be a pooling offer unless δ is small.

Let us consider semi-separating equilibria of the kind considered in the one-stage game. Suppose type h is to be rendered indifferent between revealing and pooling by S_2 choosing to accept $2\delta\underline{b}$ with probability a_2^*;then it must be the case that

$$\bar{b}(1-\delta)(2+\delta) = a_2^*[(2+\delta)(\bar{b}-\delta\underline{b})]+ \\ (1-a_2^*)\delta[2(\bar{b}-\underline{b})+\delta(\bar{b}-\delta\underline{b})]. \tag{22}$$

This gives

$$a_2^* = \frac{\bar{b}(1-\delta)(2+\delta)-\delta[2(\bar{b}-\underline{b})+\delta(\bar{b}-\delta\underline{b})]}{[(2+\delta)(\bar{b}-\delta\underline{b})]-\delta[2(\bar{b}-\underline{b})+\delta(\bar{b}-\delta\underline{b})]}.$$

However, the numerator of this expression is decreasing in δ, and for δ sufficiently high, the quantity becomes negative. Therefore such semi-separating equilibria cannot be sustained for high values of δ.

(iii) Consider now the case where the seller makes an optimal separating offer x_s. It is easy to check that, given the beliefs, it is optimal for both types of buyer to make a rejected offer. The remaining part of the equilibrium can be checked from the previous discussion. ■

4 Comparison of Sequences.

We now turn to the first move in the game where the buyer chooses the sequence in which to negotiate. We collect the equilibrium payoffs derived in the last section in a proposition.

Proposition 5 *(i) If $\pi_h \leq \frac{\underline{\underline{b}}}{\bar{b}}$,the sequence S_1S_2 gives the following payoffs to: Type h : $(\bar{b}-\delta\underline{b})(1+2\delta)$ and to Type ℓ : $(\underline{b}-\delta\underline{b})(1+2\delta)$. The sequence S_2S_1 gives for: Type h :$(\bar{b}-\delta\underline{b})(2+\delta)$ and for Type ℓ : $(\underline{b}-\delta\underline{b})(2+\delta)$. Therefore, because of discounting, both types of buyer prefer S_2S_1, and, in fact, prefer simultaneous negotiation to both sequences.*

(ii) If $\pi_h \in (\frac{\underline{\underline{b}}}{\bar{b}}, 1)$, the sequence S_1S_2 gives Type h : $(\bar{b}-\delta^2\underline{b})+2\delta\bar{b}(1-\delta)$, and gives Type ℓ : $(\underline{b}-\delta^2\underline{b})+2\delta\underline{b}(1-\delta)(\bar{b}(1-\delta)/(\bar{b}-\delta\underline{b}))$.

The sequence S_2S_1, for $\pi_h \leq \frac{x_p}{x_s}$ gives the buyer of Type h : $(2\bar{b}-\delta(2\underline{b}-\delta^2\frac{(\underline{b}-\delta\underline{b})(\bar{b}-\underline{b})}{(\bar{b}-\delta\underline{b})}))+\delta\bar{b}(1-\delta)$, and of Type ℓ : $(2\underline{b}-\delta(2\underline{b}-\delta^2\frac{(\underline{b}-\delta\underline{b})(\bar{b}-\underline{b})}{(\bar{b}-\delta\underline{b})}))$

$+\delta\underline{b}(1-\delta)(\bar{b}(1-\delta)/(\bar{b}-\delta\underline{b}))$.

Buyers of both types prefer S_2S_1 to S_1S_2 for values of δ sufficiently close to 1.

(iii) For $\pi_h > \frac{x_p}{x_s}$, sequence S_1S_2 has the same payoffs as in (ii). Sequence S_2S_1 has the following payoffs for type $h : \delta(2\bar{b}-x_s)+\delta^2(\bar{b}(1-\delta)) = \delta^2(\bar{b}-\delta\underline{b})a^*$, (see (17)), and for type $\ell : \delta^2(\underline{b}-\delta\underline{b})$. The difference in payoffs between S_1S_2 and S_2S_1 is:

$(\bar{b}-\delta^2\underline{b})+\delta(\bar{b}-\delta\underline{b})a^*-\delta^2(\bar{b}-\delta\underline{b})a^* > 0$, *for type h, and*

$(\underline{b}-\delta^2\underline{b})+\delta(\underline{b}-\delta\underline{b})a^*-\delta^2(\underline{b}-\delta\underline{b}) > 0$, *for type ℓ.*

Proof. The proof is by direct calculation of the expressions involved. For example, the last line of the proposition is obtained as follows:

$(\underline{b}-\delta^2\underline{b})+\delta(\underline{b}-\delta\underline{b})a^*-\delta^2(\underline{b}-\delta\underline{b})$

$>(\underline{b}-\delta^2\underline{b})-\delta^2(\underline{b}-\delta\underline{b})$

$=\underline{b}\{(1-\delta)(1+\delta)-\delta^2(1-\delta)\}$

$=\underline{b}(1-\delta)\{1+\delta-\delta^2\}$

>0. ■

What we have shown here is only a partial explanation for the phenomena mentioned in the introduction. As long as π_h is below $\frac{\underline{b}}{\bar{b}}$, the equilibrium is pooling for both sequences, and it is optimal to play both partners simultaneously, because of discounting. For somewhat higher values of π_h, the optimal sequence is to play the bigger partner first. Finally above a cutoff in the neighbourhood of $\frac{\bar{b}}{\bar{b}+\underline{b}}$, it becomes optimal for the buyer to switch the sequence and negotiate with the small seller first.

5 Conclusions

Even though much of the paper has dealt with the formalism of equilibrium refinements, the basic intuition is as follows. Normally, the privately informed party in the bargaining can hope to make or receive pooling offers, which do not change the probability distribution of types, when the uninformed party believes it likely that the informed party is "tough". In the case considered in this paper, the informed player has the option of sequencing bargains she is involved in, and this additional option enables her to enforce a pooling equilibrium for a wider range of initial beliefs than would be the case if no sequencing option were available. Thus the *first* uninformed bargainer in the sequence pays to allow the informed party to refuse to reveal weakness to the second, more important, uninformed party in the sequence. This is at least a possible explanation for the Clinton administration's invocations of Super 301during its term.

This paper has assumed that the terms of the first agreement are available to the second uninformed negotiator. We have supposed that the second negotiator could, for example, simply ask the first one and the first uninformed party would have no cause to dissemble. Sometimes, as in the union negotiations, they could be a matter of public record (as the papers of Banerji and Marshall and Merlo, which take very different approaches to this problem also assume). Performing a similar exercise under alternative assumptions would also be instructive.

Finally, we would like to stress that the main point of this paper is not D1; this is only a means to an end. The end is to explain the interesting effect of the ability to sequence negotiations.

6 Appendix

In order to check that the equilibrium is the only one that satisfies a modified version of D1, we perform the following calculation, assuming that the seller offer if she rejects the buyer offer will be $\delta \underline{b}$, that the buyer offers to the next seller will include $\delta \underline{b}$, and that such an offer will be accepted with probability a^*. Alternative scenarios, where the seller offers $\underline{b}$ or $\bar{b}$ clearly do not provide any incentive by the buyer to deviate. Let a be the probability with which S_1 accepts a deviating offer x from the buyer.

Then, a buyer with value b will have an incentive to switch if

$$\begin{aligned} &\{(b-x)+2\delta(b-\delta\underline{b})a^*\}a+(1-a)\{\delta(b-\delta\underline{b})+2\delta^2(b-\delta\underline{b})a^*\} \\ &>(b-\delta\underline{b})(1+2\delta). \end{aligned} \tag{23}$$

Here, $a^* = \frac{\bar{b}(1-\delta)}{\bar{b}-\delta\underline{b}}$. For b, a deviation to x would be beneficial if

$$a > \frac{(b-\delta\underline{b})(1+\delta-2\delta^2 a^*)}{(b-\delta\underline{b})(1-\delta+2\delta a^*-2\delta^2 a^*)+\delta\underline{b}-x}.$$

Let the right hand side be denoted by $\underline{a}$; we wish to determine the sign of $d\underline{a}/db$. The numerator of this quantity is given by

$$\{(b-\delta\underline{b})(1-\delta+2\delta a^*-2\delta^2a^*)+\delta\underline{b}-x\}(1+\delta-2\delta^2a^*)- (b-\delta\underline{b})(1+\delta-2\delta^2a^*)(1-\delta+2\delta a^*-2\delta^2a^*)$$

$$= (b-\delta\underline{b})\{(1-\delta+2\delta a^*-2\delta^2a^*)(1+\delta-2\delta^2a^*)- (1+\delta-2\delta^2a^*)(1-\delta+2\delta a^*-2\delta^2a^*)\}+(\delta\underline{b}-x)(1+\delta-2\delta^2a^*)$$

$$= (\delta\underline{b}-x)(1+\delta-2\delta^2a^*) \tag{24}$$

This is negative if $x>\delta\underline{b}$, and positive if the strict inequality is reversed.

Therefore an offer x less than $\delta\underline{b}$ should lead to a belief that the buyer is type ℓ and an offer greater than $\delta\underline{b}$ analogously that the buyer is of type h, as is required by the equilibrium. Therefore the seller in equilibrium will reject the offer and offer $\underline{b}$ or $\overline{b}$,which then makes such a deviation unprofitable.

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