

Information revelation in the Diamond-Dybvig banking model

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Abstract

Three recent papers, Green and Lin, Peck and Shell, and Andolfatto et. al., study optima in almost identical versions of the Diamond-Dybvig model. They differ about what agents know when they make early withdrawals. We view all three as special cases of a framework in which the planner chooses how much to reveal. It is shown that (i) the Peck-Shell conclusion, the best weakly implementable outcome can be subject to a bank run, is robust to a planner choice about what to reveal; (ii) the solution to the strong implementability problem can be something other than reveal nothing and reveal everything.

1 Introduction

The Diamond-Dybvig model [2] treats the structure of a demand deposit contract as a solution to a mechanism-design problem—as an optimum chosen by a benevolent planner. The model is a two-date, real (nonmonetary) economy in which ex ante identical consumers are the only agents. There are two frictions in the model. First, consumers face uncertainty about their realized types—impatient or patient—and their realized types are private information. Second, there is sequential service—early withdrawal requests have to be dealt with as they arrive.

Prior to learning their types and the order in which they will arrive to make early withdrawals, consumers exchange their endowment of the resource for a deposit contract. The model's banking sector can be viewed as extending callable loans in the form of the resource to the model's business sector, which invests the resource in the commonly available and productive constant-returns-to-scale intertemporal technology. The early withdrawals of deposits are necessarily matched by calls on loans and real investment liquidation. Two nice features of the model are that deposit contracts have the *demand* feature—

meaning that the consumers decide when to withdraw—and display a low rate of return relative to that of the technology.

The new question studied here is how much of the history of early withdrawal requests, a history which the planner necessarily knows, should be revealed as that history is unfolding. In the context of a fractional reserve banking system, the analogous question is how much of the time path of system-wide reserve losses should be revealed (in real time) as that path evolves.

We were led to that question by the seemingly disparate results in three recent papers: Green and Lin [4] (GL), Peck and Shell [7] (PS), and Andolfatto et al [1] (ANW). All three explore optima in almost identical versions of the model. The common ingredients are a finite number of agents, independent-across-agents determination of preference types (and, hence, aggregate risk), and an exogenous and random order that determines the sequence in which early withdrawal requests are made. The versions differ about what agents know when they turn up in order to make early withdrawals. In GL, each knows his place in the ordering; in ANW, each knows that and the announcements of those who preceded him; in PS, each knows nothing. Here, we view these versions as special cases of a more general framework.

In the more general framework and as in PS, each agent starts out knowing nothing. However, the planner—who, as noted above, deals with the agents in order and who, therefore, necessarily knows each agent’s place in the ordering and the previous announcements—can choose how much to reveal. From that perspective, GL study optima under the restriction that the planner reveals place in the ordering, PS study optima under the restriction that the planner reveals nothing, and ANW study optima under the restriction that the planner reveals everything.

Two kinds of implementability constraints are common to all versions of the model. One is physical feasibility implied by the intertemporal technology and sequential service. The other is incentive constraints that arise from the private information. The incentive constraints depend on what the planner reveals and on whether the planner is trying to solve a *weak implementability* problem or a *strong implementability* problem. (Recall that an allocation is weakly implementable if it is the outcome of *some* equilibrium; it is strongly implementable if it is the outcome of *every* equilibrium.)

In terms of our framework, the results in the above papers can be summarized as follows. For environments in which the incentive constraints implied by planner revelation of everything are nonbinding, GL show that if the planner reveals either place in the ordering or everything, then the first best (best

subject only to physical feasibility) is strongly implementable. ANW show that if the planner reveals everything, then weak implementability implies strong implementability—whether or not incentive constraints bind. These strong implementability results imply that there are no bank runs. PS show that there exist settings such that if the planner reveals nothing, then the best weakly implementable allocation is not strongly implementable. In particular, there is another equilibrium that is a bank run and gives a worse outcome.

Thus, from our perspective, ANW fail to consider the possibility that the planner could achieve a better outcome by withholding information, while PS fail to consider the possibility that the planner could achieve an equally good and unique outcome by revealing some information. These are the issues addressed here.¹

First, we set out the model and a nesting result: the less the planner reveals, the larger, in the weak sense, is the set of weakly implementable allocations. Then we study two examples, one of which is the pertinent PS example. The examples demonstrate two things. First, the PS conclusion survives in our more general framework. That is, the PS result is robust to permitting the planner a choice about what to reveal. Second, the solution to the strong implementability problem can be something other than reveal nothing and reveal everything.

2 Environment

There are N ex ante identical agents, two dates, 1 and 2, and there is one good per date. The economy is endowed with an amount $Y > 0$ of date-1 good and has a constant returns-to-scale technology with gross rate-of-return $R > 1$. That is, if C_i denotes total date- i consumption, then $C_2 \leq R(Y - C_1)$.

The sequence of events and actions is as follows. Let $\mathbf{N} = \{1, 2, \dots, N\}$ be the set of possible places in the ordering and let $T = \{impatient, patient\} \equiv \{i, p\}$ be the set of possible preference types. First nature selects a queue, denoted $t^N = (t_1, t_2, \dots, t_N) \in T^N$, where $t_k \in T$ is the type of the k -th agent in the ordering. Nature's draw is from a probability distribution $g : T^N$. (In the examples, all 2^N queues are equally probable.) If an agent observed the realized queue, which is not the case, then the agent would know his own place in the ordering and preference type, his own $(k, t) \in \mathbf{N} \times T$, and would know the preference types of others by place in the ordering. The realized queue is

¹The issue of how much to reveal does not arise in GL. Their result is general in one sense: because the first best is achieved, it follows that revealing the information is undominated. However, their result does not apply to the large class of environments with binding incentive constraints. See Lin [6] for the sense in which the general case has binding incentive constraints.

observed by no one, not even the planner, but each agent privately observes his type in the set T .² Each agent maximizes expected utility. An agent of type $t \in T$ has utility function $u(\cdot, \cdot, t)$, where the first argument is date-1 consumption and the second is date-2 consumption. For a given t , u is increasing and concave. Then agents meet the planner in the order determined by the queue.³ In a meeting, the planner knows the vector of announced types of the agents with earlier places in the ordering (and the agent's place in the ordering). The planner announces part of what he knows to the agent. Then the agent announces an element of T . The outcome of the meeting is the agent's date-1 consumption. After all the date-1 meetings occur, the planner simultaneously assigns date-2 consumption to each agent.⁴

3 Weak and strong implementability

Formally, the planner's information revelation policy is a vector of partitions $P = (P_0, P_1, \dots, P_{N-1})$, where the announcement to the k -th agent in line is the element (set) in the partition P_{k-1} that contains t^{k-1} , the realized history of prior announcements.⁵ Here, P_{k-1} is a partition of $H = T^0 \cup T^1 \cup \dots \cup T^{N-1}$, the set of all possible *histories* of announced preference types, where $T^0 = \{\text{null history}\}$ (because there is no history of announcements that precedes the first agent in line) and $T^{k-1} = \{i, p\}^{k-1}$, $k \geq 2$. In order to describe the particular partitions used in GL, PS, and ANW, we need to distinguish between a set A and the set that enumerates the elements of A , which we denote $[A]$; i.e., if $A = \{a_1, a_2, a_3\}$, then $[A] = \{a_1\}, \{a_2\}, \{a_3\}$. The least coarse partition is $P_{k-1} = \{[T^{k-1}], H - [T^{k-1}], \phi\}$. This is the ANW specification and implies that the planner reveals everything to the k -th agent in line. The most coarse partition is $P_{k-1} \equiv \{H, \phi\}$. This is the PS partition and implies that the k -th agent is told nothing. The partition that corresponds to the GL specification is $P_{k-1} = \{T^{k-1}, H - T^{k-1}, \phi\}$, which implies that the planner announces k to

²Thus, *queue* here is used not in the sense of a line of people, each of whom is in touch with those nearby. Instead, our queue is like the order in which people arrive at a drive-up window.

³We are studying a model in which all types meet the planner at date 1. This is the version that ANW study and that PS study in one of their appendices. We prefer this version because in a version with many types, as described in Lin [6], almost everyone would have to meet the planner.

⁴Notice that we give the planner control of all the resources and that the only decision that an agent makes is an announcement of type in the set T . This suffices for our purposes. There are generalizations of the model in which each agent starts out owning Y/N and can, before he learns his type in T , defect to autarky. That option is not relevant for what we do.

⁵Thus, we do not permit the planner to randomize announcements.

the k -th agent in line.⁶

A mechanism is (P, c) , where $c = (c^1, c^2, \dots, c^N)$, $c^k = (c_1^k, c_2^k)$, and $c_1^k : T^k \rightarrow \mathbb{R}_+$ is date-1 consumption of the k -th agent in the ordering and $c_2^k : T^N \rightarrow \mathbb{R}_+$ is date-2 consumption of that agent. The domain of c is agent announcements. We say that c is feasible if for all $t^N \in T^N$, $R(Y - \sum_{k=1}^N c_1^k) \geq \sum_{k=1}^N c_2^k$. Let C denote the set of all feasible c .

For a given (P, c) , a strategy for the k -th agent in the ordering is $\sigma_k : P_{k-1} \times T \rightarrow T$, where the first argument is what the planner announces and the second is the true type of the agent. We let $\sigma^k = (\sigma_1, \sigma_2, \dots, \sigma_k)$.

Given (P, c) and σ^N , we let $g_k(t)$ be the conditional distribution over T^N of the k -th agent in the ordering who is type t . Here an element of T^N is a queue as defined above and $g_k(t)$ is derived from g (the ex ante distribution over queues), (P, c) , and σ^N via Bayes' rule. Now we can define equilibrium.

Definition 1 *Given (P, c) with $c \in C$, the strategy σ^N and the associated belief given by $g_k(t)$ is a perfect Bayesian equilibrium if for each $(k, h^{k-1}, t) \in (\mathbf{N} \times H^{k-1} \times T)$ and each $\tilde{\sigma}_n \in T$,*

$$\begin{aligned} & E_{g_k(t)} u[c_1^n(\sigma^{n-1}, \sigma_n), c_2^n(\sigma^{n-1}, \sigma_n, \sigma_{n+1}^N), t] \\ & \geq E_{g_k(t)} u[c_1^n(\sigma^{n-1}, \tilde{\sigma}_n), c_2^n(\sigma^{n-1}, \tilde{\sigma}_n, \sigma_{n+1}^N), t], \end{aligned} \quad (1)$$

where $E_{g_k(t)}$ denotes expectation with respect to the distribution $g_k(t)$.

The next definition relies on the revelation principle.

Definition 2 *Given P , $c \in C$ is weakly implementable (by truth-telling) if for each $(k, h^{k-1}, t) \in (\mathbf{N} \times H^{k-1} \times T)$ and each $\tilde{t} \in T$,*

$$\begin{aligned} & E_{g_k(t)} u[c_1^n(t^{n-1}, t), c_2^n(t^{n-1}, t, t_{n+1}^N), t] \\ & \geq E_{g_k(t)} u[c_1^n(t^{n-1}, \tilde{t}), c_2^n(t^{n-1}, \tilde{t}, t_{n+1}^N), t]. \end{aligned} \quad (2)$$

In (2), t_{n+1}^N is a random variable because it is the part of the queue that has not been revealed when the planner encounters the k -th person. In addition, part of (t^{n-1}, n) may be random. Its distribution depends on how much the planner reveals.

For our purposes, the following restrictive notion of strong implementability suffices.

⁶In appendix 1, these partitions are set out explicitly for the case $N = 3$.

Definition 3 Given P , $c \in C$ is strongly implementable if truth-telling is the unique equilibrium for (P, c) .

Motivated in part by the comparisons between PS, GL, and ANW, we compare different P 's according to *coarseness*. The following nesting result is an immediate consequence of the law of iterated expectations.

Claim 1 Let P' be coarser than P'' . If $c \in C$ satisfies definition 2 for $P = P''$, then c satisfies definition 2 for $P = P'$.

The planner's objective in this model is ex ante expected utility; namely,

$$W(\sigma^N; P, c) = E_g u[c_1^n(\sigma^{n-1}, \sigma_n), c_2^n(\sigma^{n-1}, \sigma_n, \sigma_{n+1}^N), t]. \quad (3)$$

The planner's *weak implementability problem* is to choose (P, c) to maximize W subject to $c \in C$ and satisfaction of definition 2. The planner's *strong implementability problem* adds satisfaction of definition 3 as an additional constraint.⁷

4 Two examples

For each of two examples, we compute the best weakly implementable allocation and the best strongly implementable allocation. One example is the PS appendix B example and the other is a slight variant of it. The PS example has $N = 2$, $Y = 6$, $R = 1.05$, equally likely queues, and $u(c_1, c_2, \textit{impatient}) =$

⁷ANW mistakenly claim that weak implementability implies strong implementability if the planner reveals only place in the ordering. Their mistake can be explained using the above formulation.

Suppose that $N = 3$ and that π is the (independently and identically distributed) probability that a person is patient. Let c satisfy definition 2 when agents learn only their place in the ordering. In a truth-telling equilibrium, the second agent knows that with probability π the first agent announced patient and with probability $1 - \pi$ announced impatient. Our nesting implies that (2) is a weighted average of two "underlying" incentive conditions; one associated with the first agent announcing patient and the other associated with him announcing impatient, with weights π and $1 - \pi$, respectively.

Consider now a candidate (run) equilibrium where the first two agents announce impatient independent of type and the last agent announces truthfully—the only possibility for a run equilibrium with $N = 3$ when the ordering is revealed by the planner. Will the second agent defect? If patient, then that agent will announce patient only if the underlying incentive condition associated with the first agent announcing impatient is slack, which, of course, is not implied by (2). Finally, inequality (2) says nothing about what the first agent does if he believes that the second will always announce impatient.

More generally, if there are N agents and each learns only his place in the ordering, then the incentive constraint (for truthful revelation) for the n -th agent is an average of 2^{n-1} underlying incentive conditions, one condition for each possible history of the previous $n - 1$ agent announcements. Again, satisfaction of the average does not imply satisfaction of the 2^{n-1} separate constraints. In particular, the incentive condition for agent n associated with the previous $n - 1$ agents announcing impatient may not be satisfied. If not, then a bank run is possible.

$Av(c_1)$ and $u(c_1, c_2, \textit{patient}) = v(c_1 + c_2)$, with $v(x) = -x^{-1}$ and $A = 10$. The alternative is identical except that $v(x) = \ln x$. We denote by w^k date-1 consumption for the first person to announce *impatient* whose position in the ordering is $k \in \{1, 2\}$. Given (w^1, w^2) , the other components of the allocation are determined residually from the resource constraint.

Table 1 reports (w^1, w^2) and ex ante welfare of the best weakly and strongly implementable allocations under the different information revelation possibilities. (The full-information benchmark, in which the planner can observe the agent's type, is reported in the first row and other expected utilities are expressed relative to the normalized expected utility for it.) With $N = 2$, there are only three such schemes, which correspond to those studied by PS, GL, and ANW. Moreover, with $N = 2$, weak implementability implies strong implementability under reveal-place-in-the-ordering and reveal-everything.⁸ Hence, there is only one allocation described for each of those schemes. Also, under reveal-nothing, there is one condition for strong implementability; namely, that a *patient* type reports truthfully even if he thinks that the other agent if *patient* will announce *impatient*.⁹

Table 1. Optimal allocations: (w^1, w^2) and ex ante welfare		
Information Assumption	The PS example $v(x) = -x^{-1}$	The alternative $v(x) = \ln x$
No private information	(3.4483, 4.5850) 1.0000	(3.8710, 5.4545) 1.0000
reveal nothing (weak solution)	(3.0940, 3.2011) 0.9447	(3.0952, 3.1981) 0.9121
reveal nothing (strong solution)	(3.1500, 3.1500) 0.9444	(3.3075, 3.0000) 0.9106
reveal place in the ordering	(2.9758, 3.3144) 0.9435	(2.9842, 3.3075) 0.9117
reveal everything	(2.9925, 3.1500) 0.9346	(2.9962, 3.1500) 0.9044
Autarky	(3.0000, 3.0000) 0.9247	(3.0000, 3.0000) 0.8966

Under reveal-nothing, the weak and strong solutions differ for the PS example, as they report. They also differ for the alternative example. However, from

⁸To understand uniqueness when $N = 2$ and only place in the ordering is revealed, suppose allocation (w^1, w^2) is weakly implementable. If the candidate equilibrium has the first agent announcing "impatient" independent of type, then a patient second agent will announce "patient" because $R(Y - w^1) > Y - w^1$. Then, weak implementability implies that a patient first agent will defect from proposed equilibrium play and will announce truthfully. As explained in the last footnote, this result does not hold for $N > 2$.

⁹The planner's maximization problem for each of the information assumptions reported in Table 1 is explicitly described in appendix 2.

the perspective of our model, a comparison between rows 2 and 3 is not enough. What would suffice is to confirm that the best weakly implementable allocation under the other information schemes gives lower ex ante welfare. And given nesting, it is enough to show that the solution to the weak implementability problem under reveal-place-in-the-ordering is worse than under reveal-nothing. As shown in the fourth row of the table, both examples accomplish what PS set out to show—even from the broader perspective taken here. That is, even when we search over alternative information-revelation schemes by the planner, it remains true that there are settings in which the best weakly implementable allocation is not strongly implementable.¹⁰

The alternative example shows that the solution to the strong implementability problem can be something other than reveal-nothing or reveal-everything. Hence, it shows that solving the strong implementability problem can involve a nontrivial choice about what the planner should reveal.

5 Concluding remarks

The model above contains two extreme assumptions about the queue, assumption that are best discussed against the background of the following related specification. Suppose that each agent gets a random draw, (t, τ) , where, as above, t is the preference type and where $\tau \in [0, 1]$ is the *time* (of day) at which the agent will encounter the planner. One extreme assumption is that each agent does not observe his τ . (As PS say, the agent does not have a clock.) If agents privately observe τ , then they have different priors about ordering in the queue even if the planner reveals nothing. A second extreme assumption is that agents cannot take costly actions to influence the time at which they meet the planner. If they could take such actions, then we suspect that optima would display less dependence on ordering in the queue—dependence that we tend not to see.

6 Appendix 1: The partitions when $N = 3$.

As in the text, let $H = T^0 \cup T^1 \cup T^2$, where $T^0 = \{\text{null history}\}$ and $T^{k-1} = \{i, p\}^{k-1}$, $k \geq 2$. For $N = 3$, $H = \{T^0, i, p, (i, i), (i, p), (p, i), (p, p)\}$. Here are the partitions for the three information schemes in ANW, GL, and PS.

¹⁰In both examples, at least some incentive constraints bind. The role of bindingness is highlighted in Ennis and Keister [3] in which incentives do not bind. In their model, the first best allocation is weakly implementable under both reveal-nothing and reveal-place-in-the-ordering, but is strongly implementable under the latter and not under the former.

For ANW (reveal everything), they are

$$\begin{aligned} P_1 &= \{T^0, \{i, p, (i, i), (i, p), (p, i), (p, p)\}, \phi\} \\ P_2 &= \{\{i\}, \{p\}, \{T^0, (i, i), (i, p), (p, i), (p, p)\}, \phi\} \\ P_3 &= \{\{(i, i)\}, \{(i, p)\}, \{(p, i)\}, \{(p, p)\}, \{T^0, i, p\}, \phi\}. \end{aligned}$$

For GL (reveal place in line), they are

$$\begin{aligned} P_1 &= \{T^0, \{i, p, (i, i), (i, p), (p, i), (p, p)\}, \phi\} \\ P_2 &= \{\{i, p\}, \{T^0, (i, i), (i, p), (p, i), (p, p)\}, \phi\} \\ P_3 &= \{\{(i, i), (i, p), (p, i), (p, p)\}, \{T^0, i, p\}, \phi\}. \end{aligned}$$

For PS (reveal nothing), they are

$$P_1 = P_2 = P_3 = \{H, \phi\}.$$

7 Appendix 2: Maximization problems for the examples

For the examples, the planner's objective function is

$$\begin{aligned} U(w^1, w^2) &= (1 - \pi)^2 (Au(w^1) + Au(Y - w^1)) + \\ &\quad \pi(1 - \pi) (Au(w^1) + v((Y - w^1)R)) + \\ &\quad \pi(1 - \pi) (Au(w^2) + v((Y - w^2)R)) + \pi^2 2v(YR/2), \end{aligned}$$

where π is the (independently and identically distributed) probability that a person is patient, which is 0.5 in the examples.

For the kind of preferences in the examples, which are the same as those originally studied, truth-telling is potentially binding only for patient people. Therefore, the planner's reveal nothing, weak implementability (definition 2) problem is maximize U subject to

$$\begin{aligned} &(1 - \pi) (.5) \{v[(Y - w^1)R] + v[(Y - w^2)R]\} + \pi v\left(\frac{Y}{2}R\right) \\ &\geq .5\{v(w^1) + [(1 - \pi)v(Y - w^1) + \pi v(w^2)]\}. \end{aligned} \tag{4}$$

When nothing is revealed, the patient person believes that the other person is impatient with probability $1 - \pi$ and patient with probability π ; and that with probability 0.5 he is either first or second in the ordering.

The planner's reveal-nothing, strong implementability problem is maximize U subject to (4) and

$$v[(Y - w^1)R] + v[(Y - w^2)R] \geq v(w^1) + v(Y - w^1). \quad (5)$$

Constraint (5) says that a patient person prefers to announce truthfully when everyone else reports that they are impatient, given that it is equally likely that the person is first or second in the ordering.

The planner's reveal-place-in-the-ordering problem is maximize U subject to

$$(1 - \pi)v[(Y - w^2)R] + \pi v\left(\frac{Y}{2}R\right) \geq v(w^1) \quad (6)$$

and

$$(1 - \pi)v[(Y - w^1)R] + \pi v\left(\frac{Y}{2}R\right) \geq (1 - \pi)v(Y - w^1) + \pi v(w^2). \quad (7)$$

Constraint (6) applies to a patient agent who is first in the ordering, while constraint (7) applies to one who is second in the ordering.

Finally, the planner's reveal-everything maximization problem is maximize U subject to (6) and

$$\frac{Y}{2}R \geq w^2. \quad (8)$$

Constraint (6) applies to a patient agent who is first in the ordering, and constraint (8) applies to a patient agent who is second in the ordering and follows a person who announced *patient*. (The constraint for a patient agent who is second and who follows a person who announced impatient is $(Y - w^1)R \geq Y - w^1$, which is always satisfied.)

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