Optimal inflation in a model of inside money

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Article history:
Received 26 June 2012
Revised 25 June 2013
Available online xxxx

JEL classification:
E52
E58

Keywords:
Inside-money
Inflation
Monitoring
Optima

1. Introduction

There are, by now, several models of outside money in which some inflation produced by lump-sum transfers is optimal (see Levine, 1991; Kehoe et al., 1992; Molico, 2006; Green and Zhou, 2005; Deviatov, 2006). In those models, the transfers have a beneficial effect on extensive margins by altering the money holdings of those who trade in a way that more than offsets their harmful effect on intensive margins implied by the decrease in the return on money. We study optima in a model of inside money, essentially the model in Cavalcanti and Wallace (1999). By way of examples, we show that inflation can also be optimal in that model.

CW was designed to compare inside and outside money as alternative monetary systems and to do so in a way consistent with the prominent role given to monitoring in recent work in monetary theory. The main result of that work is that imperfect monitoring is necessary to give money a role (see Kocherlakota, 1998 and Wallace, 2011). Because there are no general results that give both necessary and sufficient conditions for money to have a role, many models assume no monitoring at all: they assume that each person’s previous actions are private information to that person. (All the outside-money models cited above make that assumption, explicitly or implicitly.) However, such models are too extreme to allow for the study of inside money—which is always a credit instrument whose amount has to be controlled in some way or other. CW bridge the gap between models with no monitoring and models with perfect monitoring, models which have no role for money, by assuming that some exogenous fraction of the population, labeled $m$-people, are perfectly monitored and that the rest, labeled $n$-people, are not monitored at all. The $m$-people are the potential issuers of inside money. CW study steady

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states in which they impose the requirement that there is neither inflation nor deflation. Here, we also study steady states, but without imposing that inflation is zero.

Inside money in the model can be interpreted as trade-credit instruments (or privately issued banknotes or so-called company scrip) that are (i) potentially issued by m-people when they buy from n-people, (ii) used by n-people in trade among themselves, and (iii) potentially redeemed or accepted by all m-people when they sell to n-people—as in a network of banks, each of which can be induced to accept the banknotes issued by other banks. (One such network was the Suffolk banking system [see Rolnick et al., 2002]; another was the National Banking System [see Friedman and Schwartz, 1963, Chapter 2].)

As regards inflation, in models with divisible money, a standard normalization holds the stock of money fixed and represents inflation by an proportional tax on money holdings. Our approach is the same, except that the discreteness of money in our model—each person’s money holding is constrained to be in the set \( \{0, \ldots, \} \)—forces us to use a probabilistic version of such a tax: a person who ends up after trade with a unit of money loses it with some probability. This way of modeling inflation, which was devised by Victor Li (see 1994 and 1995) and has been used by others, has the same effects on incentives to acquire money as inflation in a model with divisible money. A literal interpretation is that money is made of stuff such that each unit disintegrates at each date with a probability that the planner chooses.

Inflation in the model comes about in connection with the trades that occur in meetings between people who can issue inside money, m-people, and those who cannot, n-people. Inflation occurs if the m-people spend more in such meetings when they are buyers than when they are sellers. Moreover, the role of inflation bears some resemblance to its role in the above models of outside money. Although individual money holdings are in the set \( \{0, 1\} \), inflation allows the post-trade distribution of money holdings to differ from the pre-trade distribution in a way that turns out to be valuable in terms of ex ante welfare—representative-agent welfare before both monitored status and initial money holdings are assigned.

Interpreted as a model of private banknote issue, the finding that inflation can be optimal casts doubt on some of the rules that were imposed to limit note issue in actual nineteenth-century private banknote systems. Some of those systems had multiple safeguards designed to prevent continuing net money creation. For example, under the National Banking System, banknote issue had to be fully backed by holdings of government debt, was limited by bank capital, and was payable on demand into specie. In addition, our finding casts doubt in a new way on the widespread view that inflation is harmful in the absence of nominal rigidities—new because the inflation comes about through a continuing process of net positive private money creation.

2. The model

The background setting is borrowed from Trejos and Wright (1995) and Shi (1995). Time is discrete and there is a nonatomic measure of people each of whom maximizes expected discounted utility with discount factor \( \beta \in (0, 1) \). Production and consumption occur in pairwise meetings that occur at random in the following way. Just prior to each meeting, each person looks forward to being a consumer (a buyer), who meets a random producer (seller) with probability \( 1/K \), looks forward to being a producer who meets a random consumer with probability \( 1/K \), and looks forward to no pairwise meeting with probability \( 1 - 2/K \), where \( K \geq 2 \). The period utility of someone who becomes a consumer and consumes \( y \in \mathbb{R}_+ \) is \( u(y) \), where \( u \) is strictly increasing, strictly concave, differentiable, and satisfies \( u(0) = 0 \). The period utility of someone who becomes a producer and produces \( y \in \mathbb{R}_+ \) is \( -c(y) \), where \( c \) is strictly increasing, convex, and differentiable and \( c(0) = 0 \). In addition, \( y^* = \arg \max_{y \geq 0} [u(y) - c(y)] \) is positive. Production is perishable; it is either consumed or lost. In addition, either \( u \) is bounded above or \( c \) is such that \( y \) is bounded above, an assumption that allows us to invoke the principle of one-shot deviations.

People in the model are ex ante identical but, as in CW, a fraction \( \alpha \) become permanently monitored (m-people), while the rest are permanently nonmonitored (n-people). For m-people, histories and money holdings are common knowledge; for n-people, they are private. However, the monitored status and consumer–producer status of people in a pairwise meeting are common knowledge. And, no one except the planner can commit to future actions.

Each person and the planner have printing presses capable of turning out indivisible and durable objects. Those turned out by the printing press of any one person are distinguishable from those turned out by other peoples’ printing presses. That allows us to prevent an m-person who defects from issuing additional money. Finally, each person’s holding of money issued by others is restricted to be in \( \{0, 1\} \).

3. Implementable allocations and the optimum problem

We limit the search for an optimum to allocations that are steady states and are symmetric in two senses. First, all people in the same situation take the same action, an action that can be a lottery. Second, all monies issued by other

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2 If \( k \) is an integer that exceeds two, then, as is well known, it can be interpreted as the number of goods and specialization types in Trejos and Wright (1995) and Shi (1995).

3 One interpretation is that a fraction \( \alpha \) realize a zero cost of attaining \( m \) status and that the rest realize a prohibitively high cost of attaining \( m \) status. In other work, we have explored examples with a less extreme assumption about the distribution of the cost of attaining \( m \) status.
and the people who have not defected and any money issued by the planner are treated as perfect substitutes.\footnote{Given a bound on discrete money holdings, trade can be enhanced by distinguishing among monies—say, by color (see Aiyagari et al., 1996). Here we ignore that possibility because the only role of restricting money holdings to $\{0,1\}$ is to limit the number of unknowns in our optimum problem. If we permit distinctions among monies, then that role is compromised.} We assume that all monies issued by $n$-people are worthless.

The planner’s objective is ex ante expected utility, where $\alpha$ is the probability of becoming an $m$-person and where the probabilities of starting with money are given by the fractions of $m$- and $n$-people with a unit of money—fractions which the planner chooses.\footnote{Also, although the planner could, in principle, randomize and treat some $m$-people as $n$-people, that is never optimal (see Wallace, 2011).} The planner also chooses trades in meetings (as a function of the states of the producer and the consumer in the meeting); and chooses the inflation rate (the probability that money held after meetings disintegrates, which occurs independently across different units of money). We also permit the planner to make transfers of money. The planner is constrained by the steady-state restriction and by self-selection constraints that follow from our specification of private information and of punishments.

We assume that the only feasible punishment is permanent banishment of an individual $m$-person to the set of $n$-people, which includes loss of the ability to issue money. (A general result [see Wallace, 2011] is that in an optimum $m$-people never hold money issued by others before meetings.) Underlying this assumption about punishment is free exit at any time from the set of $m$-people into the set of $n$-people and the ruling out of global punishments—like the shutting down of all trade in response to individual defections. We also allow both individual defection ($IR$) and cooperative defection by the pair in any meeting.

Let $S = \{m,n\} \times \{0,1\}$ be the set of individual states, where $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$ denote generic elements of $S$. (Notice that $s_2 = 1$ means having money issued by someone else.) The state of a meeting is denoted $(s, s')$, where $s$ is the state of producer and $s'$ is the state of the consumer. A constant allocation consists of a specification of the variables listed in Table 1.

The planner chooses the variables in the table subject to the constraints set out below. The first three variables, the $y$’s and the $\lambda$’s, describe what happens in meetings; the next two, disintegration and transfers, describe what happens after meetings. Finally, the $\theta$’s describe the state of the economy at a start of a date.\footnote{One could also study Pareto allocations by varying the weight attached to different people in the welfare criterion.\footnote{Readers who are comfortable with the above description of the planner’s problem can turn directly to the next section which contains the results.}}

### 3.1. Feasibility and steady state conditions

The $\lambda$’s describe money-holding transitions in meetings, while the $\phi$’s describe transfers at the end of a date. Both must be lotteries on $\{0,1\}$ and must satisfy the obvious feasibility restrictions; in particular, money is not created in a meeting between two $n$-people. Disintegration of money occurs after meetings, but before transfers.

The $\lambda$’s, $\xi$, and the $\phi$’s imply the following transition probabilities of a person’s money holding from the start of one date to the start of the next date. The probability that a person in state $(s_1, i) \in S$ transits to state $(s_1, j) \in S$ is

$$t^{s_1}(i,j) = \frac{1}{K} \sum_{s' \in S} \theta^{s'} \left[ (\lambda^s_p)^{s'}(0) + \lambda^s_c^{s'}(1) \right] \Psi \Phi^{s_1}(j),$$

(1)

where $\lambda^s_p^{s'} = (\lambda^s_p^{s'}(0), \lambda^s_p^{s'}(1))$, $\lambda^s_c^{s'} = (\lambda^s_c^{s'}(0), \lambda^s_c^{s'}(1))$, $\delta_i$ is the two-element unit vector in direction $i + 1 \mod 2$,

$$\Psi = \begin{bmatrix} 1 & 0 \\ \xi & 1 - \xi \end{bmatrix},$$

(2)

and $\Phi^{s_1}(j) = (\phi^{s_1}(0)(j), \phi^{s_1}(1)(j))$. If $T^{s_1}$ denotes the $2 \times 2$ matrix whose $(i,j)$-th component is $[t^{s_1}(i,j)]$, then the steady-state requirement is

$$(\theta^{s_1}(0), \theta^{s_1}(1)) T^{s_1} = (\theta^{s_1}(0), \theta^{s_1}(1)),$$

(3)

for $s_1 \in \{m,n\}$, where $\theta^{(m,0)} + \theta^{(m,1)} = \alpha$ and $\theta^{(n,0)} + \theta^{(n,1)} = 1 - \alpha$.

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**Table 1**

Notation.

| $y^{s_1}$ | production by $s$ and consumption by $s'$ in a meeting |
| $\lambda_p^{s_1}$($i$) | prob. that end-of-meeting money is $i$ for producer |
| $\lambda_c^{s_1}$($i$) | prob. that end-of-meeting money is $i$ for consumer |
| $\phi^{s_1}$($i$) | prob. next-date money is $i$ if post-inflation state is $s$ |
| $\theta^{s_1}$ | fraction in state $s$ before meetings |
| $\nu^{s_1}$ | expected discounted utility for $s$ before meetings |
3.2. Incentive constraints

It is convenient to first define discounted expected utility before meetings. For \( s \in S \), we have the following Bellman equation:

\[
v^s = \frac{1}{K} \sum_{s' \in S} \theta^{s'} \left[ \pi^p(s, s') + \pi^c(s', s) + (K - 2) \pi^0(s) \right],
\]

(4)

where

\[
\pi^p(s, s') = -c(y^{s',s}) + \beta \lambda_p^{s'} \psi \phi^{s1} v^{s1},
\]

(5)

\[
\pi^c(s, s') = u(y^{s',s}) + \beta \lambda_c^{s'} \psi \phi^{s1} v^{s1},
\]

(6)

and

\[
\pi^0(s) = \beta \delta_2 \psi \phi^{s1} v^{s1}.
\]

(7)

Here, \( \delta_2 \) is the \( 1 \times 2 \) unit vector in direction \( s_2 + 1 \mod 2 \),

\[
\phi^{s1} = \begin{bmatrix} \phi^{(s1,0)}(0) & \phi^{(s1,0)}(1) \\ \phi^{(s1,1)}(0) & \phi^{(s1,1)}(1) \end{bmatrix},
\]

(8)

and \( v^{s1} = (v^{(s1,0)}, v^{(s1,1)})' \). The \( \pi \)'s are payoffs including continuation values; \( \pi^p(s, s') \) is the payoff from being a state-\( s \) producer in a meeting with a state-\( s' \) consumer; \( \pi^c(s', s) \) is the payoff from being a state-\( s \) consumer in a meeting with a state-\( s' \) producer; and \( \pi^0(s) \) is the payoff from not being in a meeting. Given the variables in the first six rows of Table 1, Blackwell's sufficient conditions for contraction imply that \( v^* \) exists and is unique. We express the incentive constraints in terms of the \( v^* \)'s. This is legitimate because the principle of one-shot deviations applies to this model.

There are truth-telling constraints only for \( n \)-people with money when they are consumers. They are

\[
\pi^c(s, (n, 1)) \geq u(y^{s,(n,0)}) + \beta(\xi, 1 - \xi) \Phi^n v^n.
\]

(9)

This potentially binds only when the producer is an \( m \)-person.

The individual rationality constraints for meetings are

\[
\pi^p((s1, 0), s') \geq \beta \phi^{(n,0)} v^n \quad \text{and} \quad \pi^p((s1, 1), s') \geq \beta(\xi, 1 - \xi) \Phi^n v^n,
\]

(10)

\[
\pi^c(s, (s1', 0)) \geq \beta \phi^{(n,0)} v^n \quad \text{and} \quad \pi^c(s, (s1', 1)) \geq \beta(\xi, 1 - \xi) \Phi^n v^n,
\]

(11)

and

\[
\pi^0(m, 0) \geq \beta \phi^{(n,0)} v^n \quad \text{and} \quad \pi^0(m, 1) \geq \beta(\xi, 1 - \xi) \Phi^n v^n.
\]

(12)

Because punishment of an \( m \)-person is loss of \( m \)-status, the defection payoffs, the right-hand sides of these inequalities, are always expected discounted utilities for \( n \)-people.

We also have a constraint which says that \( m \)-people prefer the transfers intended for them to defecting to \( n \)-status just prior to those transfers; namely,

\[
\phi^{(m,s2)} v^m \geq \phi^{(n,s2)} v^n.
\]

(13)

As noted above, the optimum is such that \( m \)-people do not start a date with money issued by others. In terms of our notation, this implies \( \phi^{(m,1)} = (1, 0) \).

We also allow cooperative defection for people in meetings. Those constraints, which turned out not to be relevant for our examples, are that the trades be in the pairwise core for those in a meeting when they take as given the continuation values.

3.3. The planner's problem

The planner chooses the variables in Table 1 to maximize

\[
\sum_{s \in S} \theta^s v^s = \sum_{s \in S} \sum_{s' \in S} \theta^s \theta^{s'} [u(y^{s's}) - c(y^{s's})] / (K(1 - \beta))
\]

subject to all the relevant constraints. The equality in (14) is a standard result and is obtained from (4). It follows that \([u(y^*) - c(y^*)] / (K(1 - \beta)) = v^*\), ex ante welfare if \( y^* \) is produced and consumed in every meeting, is an upper bound on the planner's objective.
Table 2
Welfare and aggregates.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 1/4$</th>
<th>$\alpha = 1/2$</th>
<th>$\alpha = 3/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i\in S} \theta^i v_i/v^*$</td>
<td>0.428</td>
<td>0.598</td>
<td>0.791</td>
</tr>
<tr>
<td>$v_{000}^\alpha/v^*$</td>
<td>0.697</td>
<td>0.792</td>
<td>0.895</td>
</tr>
<tr>
<td>$v_{010}^\alpha/v^*$</td>
<td>0.184</td>
<td>0.207</td>
<td>0.244</td>
</tr>
<tr>
<td>$v_{100}^\alpha/v^*$</td>
<td>0.656</td>
<td>0.736</td>
<td>0.839</td>
</tr>
<tr>
<td>$v_{110}^\alpha/v^*$</td>
<td>0.325</td>
<td>0.371</td>
<td>0.398</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.082</td>
<td>0.104</td>
<td>0.114</td>
</tr>
</tbody>
</table>

4. Examples

In order to learn a bit about the properties of optima, we compute optima for some examples. Our examples are arbitrary except in two respects. First, if everyone were monitored ($\alpha = 1$), then the first-best would be implementable. (That is, only the presence of $n$-people prevents the first-best outcome from being attained.) Second, if no one were monitored ($\alpha = 0$), then it would be desirable to pay interest on money if doing so were feasible. These conditions can be expressed as assumptions about the discount factor, $\beta$, for given $u$, $c$, and $K$.

If $\alpha = 1$, then $y^*$ in every meeting is implementable if and only if

$$\frac{u(y^*)}{c(y^*)} \geq 1 + K(1 - \beta)/\beta. \tag{15}$$

(This comes from the constraint $c(y) \leq \beta(u(y) - c(y))/(K(1 - \beta))$. Notice that the defection payoff is permanent autarky when $\alpha = 1$.) If $\beta^*$ denotes the $\beta$ for which (15) holds at equality, then we assume $\beta \geq \beta^*$. If, instead, $\alpha = 0$, then trade occurs only when the producer has no money and the consumer has money, a trade meeting. The output level $y^*$ in every trade meeting is implementable if and only if

$$\frac{u(y^*)}{c(y^*)} \geq 1 + \frac{K(1 - \beta)/\beta}{1 - \theta}, \tag{16}$$

where $\theta$ is the fraction with a unit of money. Let $\beta^{**}$ be the value of $\beta$ for which (16) holds at equality when $\theta = 1/2$, the magnitude of $\theta$ that maximizes the frequency of trade meetings. Obviously, $\beta^{**} > \beta^*$. As is well known, if $\beta > \beta^{**}$, then the optimum in the $\alpha = 0$ economy is $y = y^*$ and $\theta = 1/2$. If, however, $\beta < \beta^{**}$, which is what we assume, then the optimum has $\theta < 1/2$ and $y < y^*$.

8 In that case, it would be desirable to pay interest on money if doing so were feasible because it would give an additional payoff to acquiring money.

In accord with both restrictions, we set $\beta = (\beta^{**} + \beta^*)/2$. Throughout, we fix $u$, $c$, and $K$ as follows: $u(y) = 1 - e^{-10y}$, $c(y) = y$, and $K = 3.9$ This specification implies $y^* = (\ln 10)/10 \approx 0.23$, $\beta^* \approx 0.51$, and $\beta^{**} \approx 0.67$. Therefore, we set $\beta = 0.59$.

We report results for $\alpha \in \{1/4, 1/2, 3/4\}$. In addition to having all $m$-people enter meetings without money (issued by others), one common feature of the examples is that there are no end-of-period transfers of money to $n$-people. Welfare and some aggregates at the optimum are in Table 2.

The first four rows report welfare as a fraction of first-best welfare. As expected, ex ante welfare, given in the first row, is increasing in $\alpha$. Also, the fraction of $n$-people with money is substantially less than one-half. Finally, the inflation rate is positive.

To better understand the positive inflation rate, it helps to see the trades that occur. Table 3 shows output (relative to the first best level) in each of the five meetings in which positive output is possible. (As noted above, there are no $(m, 1)$ people in meetings, $(n, 1)$ people cannot be induced to produce, and there is no production in the $(n, 0)$ $(n, 0)$ meeting.)

The table does not show the transfer of money from the consumer to the producer because that turned out to be extremely simple. Although we allow for lottery, they are not optimal for these examples. In the first-, second-, and third-

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7 Inequality (16) is obtained from the constraint $c(y) \leq \beta[u(y) - c(y)]/(K(1 - \beta))$, where $v^{01}(y) = \frac{1}{K}u(y) + \beta v^{00} + (1 - \frac{1}{K})\beta v^{01}(y)$ and $v^{00}(y) = \frac{1}{\beta}[-c(y) + \beta v^{01}(y)] + (1 - \frac{1}{\beta})\beta v^{00}(y)$.

8 With only $n$-people, having $\theta < 1/2$ is the only way to loosen the producer’s participation constraint. It does so by reducing the expected number of periods during which money is held before a trading opportunity occurs and comes at the expense of a reduction in the frequency of trade meetings.

9 We chose this functional form for $u$ mainly because the optimization program works better with a finite marginal utility at zero. Otherwise, it is completely arbitrary, as is the parameter that determines $u(0)$, except that we want it to be sufficiently large relative to $c'(0)$.

10 The optimum problem is solved using the General Algebraic Modeling System (GAMS), which is designed for the solution of large linear, nonlinear, and mixed integer optimization problems. It consists of a language compiler and a large menu of stable integrated high-performance solvers. The solvers are divided into two groups: local solvers (which are fast, but do not guarantee that the global solution is located) and global solvers (which are slow, but are very likely to find the global optimum). The global solver used is a Branch-And-Reduce Optimization Navigator (BARON) solver. BARON uses a deterministic algorithm of the branch-and-bound type, which is guaranteed to find the global optimum under very general conditions. These conditions include bounds on variables and the functions of them that appear in the nonlinear programming problem to be solved.

The results have been independently confirmed by Ricardo Cavalcanti.

Please cite this article in press as: Deviatov, A., Wallace, N. Optimal inflation in a model of inside money. Review of Economic Dynamics (2013), http://dx.doi.org/10.1016/j.red.2013.06.003
fourth-row meetings, a unit of money is transferred from the consumer to the producer. (In the other meetings, no money is transferred.) In all but the third-row meeting, the producer’s IR constraint is binding. In other words, in all those meetings, the optimum has the consumer make a take-it-or-leave-it offer. (When the producer is an m-person, rejecting the offer means becoming an n-person without money.)

When the producer is an n-person (the first two rows), output (the level) is \(\beta(1 - \xi)(v^{n1} - v^{m0})\), where \(\xi\) is the inflation rate and \(v^{n1}\) is pre-meeting welfare of an n-person with \(i\) units of money. As in a version with \(\alpha = 1\), when the producer is an m-person (the last three rows), production is always a gift that is sustained by the threat of banishment to the set of n-people without money. In the meetings in the last two rows, output (the level) is \(\beta(v^{m0} - v^{m0})\). Although no constraint is binding in the third-row meeting, a larger output there would reduce \(v^{m0}\) and increase \(v^{m0}\), and, would, therefore, lead to a violation of the IR constraints for all the other meetings. As this suggests, all the binding constraints depend on all the trades.

Inflation is implied by the expenditures of money in the second- and fourth-row meetings. The inflow into money holdings of n-people occurs via the second-row meeting and is proportional to the fraction of n-people without money. The outflow from money holdings of n-people occurs via the fourth-row meeting and is proportional to the fraction of n-people with money. Because the fraction of n-people with money is less than one-half, the inflow exceeds the outflow, which produces inflation.

The inflow into money holdings of n-people—and, therefore, inflation—could be reduced by having a lottery in the second-row meeting—by having money go to the producer only with some probability. However, that would reduce output in that meeting. And, although inflation gives rise to a lottery over whether the producer ends up with money, the two lotteries are not equivalent because the inflation lottery hits every n-person with money at the end of trade, including those who acquired money earlier.

Also, notice that the price of goods in the first- and second-row meetings is higher than that in the fourth-row meeting. In other words, an n-person faces a lower average price as a buyer than as a seller. Other things equal, this makes the acquisition of money by n-people more desirable.

Finally, we should emphasize that the optimum cannot be achieved by having our planner be the sole issuer of a uniform money. A planner with a uniform money can insure that each m-person starts each period with a unit of money. However, if money is uniform, then when the m-person is a producer in a meeting, the defection payoff would be that of an n-person with money, rather than the lower value of an n-person without money. Because the producer’s IR constraint in the last two rows of Table 2 is binding at the lower defection payoff, this optimum cannot be achieved with uniform money—unless the planner could impose a global punishment like shutting down the entire economy or could punish a defecting m-person with permanent autarky. In our version, because the money held by the m-person is special to that person, it can be rendered worthless in the case of a defection without any sort of global punishment. That is the only advantage of inside money over uniform outside money in this model.

5. Concluding remarks

We present essentially one example, and one that is certainly not intended to be a realistic calibration of any economy. Therefore, it is reasonable to ask what can we learn from it. At a minimum, the example provides a robust counter-example to the view that inside-money economies should be regulated so as to avoid inflation. The example is robust in three senses. First, the model was not originally constructed to study inflation. Second, we did not search over parameters to find an example with positive optimal inflation. And, third, it is obvious that the positive-inflation result holds for parameters that are close to those we selected.

We admit that limiting money holdings to \([0, 1]\) is special in that it does not give scope to policies that resemble the Friedman rule. If money holdings were richer—say, in the set \([0, 1, 2, \ldots, B]\) for a large B—then deflation could be approximated by having m-people earn more than they spend in meetings with n-people and by having the excess returned to n-people roughly in proportion to their holdings. (In the B = 1 case, the excess can only go to those with 0, which does not approximate deflation.) However, we doubt that anything like a Friedman-rule deflation would emerge as an optimum in such a model. First, spending by m-people serves a purpose in the model. Therefore, taxing them by having them earn more than they spend is not costless in terms of welfare. Second, as we saw in the examples above, money holding by n-people can be rewarded through different prices in the different kinds of meetings.

Aside from computational simplicity, there is little justification for our focus on steady states. The same welfare problem could be formulated without limiting the set of allocations to constant allocations. We suspect that our finding about

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<td>((n, 0)/(n, 1))</td>
<td>.606</td>
<td>.663</td>
<td>.739</td>
</tr>
<tr>
<td>((n, 0)/(m, 0))</td>
<td>.606</td>
<td>.663</td>
<td>.739</td>
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<td>.107</td>
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<td>((n, 0)/(n, 1))</td>
<td>.717</td>
<td>.818</td>
<td>.911</td>
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Please cite this article in press as: Deviatov, A., Wallace, N. Optimal inflation in a model of inside money. Review of Economic Dynamics (2013). http://dx.doi.org/10.1016/j.red.2013.06.003
inflation for steady states will carry over to the more general problem because there is no reason why a zero net inflow to holdings of \( n \)-people should be a feature of an optimal path.\(^{11}\)

Finally, a counter-example is only as interesting as the underlying model. Therefore, our results are interesting only to the extent that CW is an attractive model of inside money. Although it is easy to point to extreme features of that model, we do not see attractive alternatives to it. Also, we see no reason to think that generalizations of it would give radically different answers.

References


\(^{11}\) In work which has only recently been brought to our attention, Bertolai et al. (2012) have studied nonstationary optima in the \( \alpha = .5 \) example, but without lotteries. Their optimum has a limiting positive inflation rate, but one that is only about 1/3 of what we find.