Econ 497 Spring 2007
Assignment 1: Complete Markets

The due date for this assignment is Tuesday, January 30.

1. In the PSU-Michigan football game, the possible outcomes are as follows. In the first half, PSU can go ahead by 10 points or Michigan can go ahead by 10 points. In the second half, the change in the score can be 10 points in PSU's favor or 10 points in Michigan's favor. PSU has the better team, so in the first half there is a 70% probability that PSU will take the lead. However PSU's team gets too conservative when ahead, so if it ends the first half in the lead, there is a 50% probability that the score will move in Michigan's favor in the second half. If Michigan's team ends the first half in the lead, the probability remains 70% that the score will move in PSU's favor in the second half.

   (a) Draw the event tree that describes the outcomes in this game. At each node of the tree, enter the PSU lead (the number of points that PSU is ahead, a negative number if Michigan is ahead). What are the probabilities of each final PSU lead (i.e., of each final score, PSU's lead can be positive or negative)?

   (b) Suppose there are bookmakers who allow you to bet at the start of the game on the halftime score. If the bookmakers have no costs and make no profits on average, how much do you have to pay at the start of the game to receive \$1 if PSU is ahead at half-time? How much do you have to pay to receive \$1 if Michigan is ahead?

   (c) Suppose the bookmakers also operate at half-time, and allow you to bet on the final score. If PSU is ahead at half-time, how much do you have to pay at half-time to receive \$1 if PSU wins the game, and how much do you have to pay at half-time to receive \$1 if the game is a tie? What if Michigan is ahead at half-time?

   (d) Show that even if the bookmakers do not offer bets at the start of the game on the final outcome of the game, you can still use a sequence of bets, starting at the beginning of the game, to obtain \$1 if PSU wins the game, or \$1 if Michigan wins, or \$1 if the game is a tie. How much does it cost to obtain each of these payments?

   (e) How do these costs relate to the probabilities in part a)? Under what conditions would you expect this relationship to hold in other more general examples?