Introduction to the Mathematical and Statistical Foundations of Econometrics

Errata to the first edition

April 5, 2007

Page 10, line 2 from bottom:	Replace “smallest” with “largest”
Page 10, line 1 from bottom:	Replace $\geq$ with $\leq$
Page 20, line 5 from bottom:	Replace $P(\{T\}\})$ with $P(\{T\})$
Page 37, line 2 of Section 2.1:	Replace “dice”[1] with “a die”
Page 37, line 4 of Section 2.1:	Replace “dice” with “die”
Page 38, line 16 from top:	Replace “dice” with “die”
Page 49, line 10 from top:	Replace $\mu X(B_j)$ with $\mu_X(B_j)$
Page 50, line 1 from top:	Replace “Chapter 7“ with “Chapter 6“
Page 53, line 10 from bottom:	Replace $X = U_0 . U_2$ with $Y = U_0 . U_2$
Page 84, last equation:	Replace the second **and** last integrals with $\int_{A \cap B_m} \lim_{n\to\infty} Z_n^{(m)}(\omega) dP(\omega)$
Page 89, line 2 from bottom:	Replace “measure” with “measurable”
Page 90, lines 6 and 7 from top:	Replace the function $g(x)$ with $g(x) = \sin^2(\pi x/2) = (\sin(\pi x/2))^2$
Page 113, line 2 in Theorem 5.2:	Replace “*characteristic of Y*” with “*characteristic function of Y*”
Page 131, line 3 of Section 5.7.3:	Replace “have” with “has”
Page 144, line 1 of Theorem 6.7:	Replace “*Let* X_n *a sequence*” with “*Let* X_n *be a sequence*”
Page 146, first line after eq. (6.8):	Replace “Theorem 6.3” with “Theorem 6.10”
Page 150, line 7 from top:	Replace $X_n \to_p .$ with $X_n \to_p X.$
Page 179, line 1 from top:	Replace “Chapter 6 I focused” with “Chapter 6 focused”
Page 185, line 11 from top:	Replace “shock” with “shocks”
Page 185, line 3 of Theorem 7.5:	Replace $\mathscr{F}_{-\infty} = \bigcap_{t=1}^{\infty} \mathscr{F}_{-\infty}^{t}$ with $\mathscr{F}_{-\infty} = \bigcap_{t} \mathscr{F}_{-\infty}^{t}$
Page 188, equation (7.22):	Replace U_t with X_t
Page 188, line 7 from below:	Replace “$t < n$,” with “$t < 0$ as well,”

[1] Note that "dice" is the plural of "die".

Page 190, last equation:	Replace this equation with

$$\partial f_t(\theta_0)/\partial\theta_0^T = \begin{pmatrix} \Sigma_{j=1}^{\infty}\big(\beta_0 + (\beta_0 - \gamma_0)(j-1)\big)\beta_0^{j-2}V_{t-j} \\ -\Sigma_{j=1}^{\infty}\beta_0^{j-1}V_{t-j} \end{pmatrix}$$

Page 191, equation (7.35):	Replace $\beta_0^{2(j-2)}$ in the expressions of the upper-right and lower-left elements of the matrix involved with β_0^{2j-3}
Page 193, Lemma 7.2:	Replace "of random " with "of zero-mean random "
Page 196, line 1 after (7.58):	Replace "σ-mixing" with "α-mixing"
Page 200, Definition 7.A.2:	Replace X_n with x_n
Page 201, line 4 from top:	Replace $n,m \to \infty$ with $k,m \to \infty$
Page 201, line 12 from top:	Replace the sentence "Moreover, it is easy to verify that $E[X] = 0$ and $E[X^2] < \infty$." with the following:

To prove that $E[X^2] < \infty$, observe that for any $K > 0$ and fixed m,

$$E[X^2 I(|X| < K)] = E[(X-X_m)^2 I(|X| < K)] + 2E[(X-X_m)X_m I(|X| < K)] + E[X_m^2 I(|X| < K)]$$

$$\leq E[(X-X_m)^2] + 2\sqrt{E[(X-X_m)^2]E[X_m^2]} + E[X_m^2] = \|X-X_m\|^2 + 2\|X-X_m\|\sqrt{E[X_m^2]} + E[X_m^2] < \infty.$$

Letting $K \to \infty$ it follows from this inequality and the monotone convergence theorem that $E[X^2] < \infty$. To prove $E[X] = 0$, observe that

$$|E[X]| = |E[X] - E[X_m]| \leq E[|X-X_m|] \leq \sqrt{E[(X-X_m)^2]} = \|X-X_m\| \to 0 \text{ as } m \to \infty,$$

where the first equality follows from the fact that $E[X_m] = 0$, and the second inequality follows from Liapounov's inequality.

Page 202, last line:	Replace $1/n^2$ with $2/n^2$, and $1/m^2$ with $2/m^2$
Page 203, line 10 from bottom:	Replace $\|Y_t\|^2 = E[Y_t^2] < \infty$ with $\|\hat{X}_t\|^2 = E[\hat{X}_t^2] < \infty$
Page 204, line 1 from top:	Replace $Z_{t,m} = \Sigma_{j=1}^m \alpha_j U_{t-j}$ with $Z_{t,m} = \Sigma_{j=0}^m \alpha_j U_{t-j}$
Page 204, line 1 from top:	Replace $\alpha_j = \langle X_t, U_{t-j}\rangle = E[X_t U_{t-j}]$ with $\alpha_j = \langle X_t, U_{t-j}\rangle/\sigma_u^2 = E[X_t U_{t-j}]/\sigma_u^2$

Page 204, lines 2-4 from top: Replace the equation with

$$\begin{aligned}\|X_t - Z_{t,m}\|^2 &= \|X_t - \textstyle\sum_{j=0}^{m}\alpha_j U_{t-j}\|^2 \\ &= E[X_t^2] - 2\textstyle\sum_{j=0}^{m}\alpha_j E[X_t U_{t-j}] + \sum_{i=0}^{m}\sum_{j=0}^{m}\alpha_i\alpha_j E[U_i U_j] \\ &= E[X_t^2] - \sigma_u^2\textstyle\sum_{j=0}^{m}\alpha_j^2 \geq 0\end{aligned}$$

Page 204, line 5 from top: Replace $\sum_{j=1}^{\infty}\alpha_j^2 < \infty$ with $\sum_{j=0}^{\infty}\alpha_j^2 < \infty$

Page 204, line 6 from top: Replace $S_{-\infty}^{t-1}$ with $S_{-\infty}^{t}$

Page 204, line 7 from top: Replace $Z_t = \sum_{j=1}^{\infty}\alpha_j U_{t-j} \in S_{-\infty}^{t-1}$ with $Z_t = \sum_{j=0}^{\infty}\alpha_j U_{t-j} \in S_{-\infty}^{t}$

Page 204, line 8 from top: Replace $W_t = X_t - \sum_{j=1}^{\infty}\alpha_j U_{t-j}$ with $W_t = X_t - \sum_{j=0}^{\infty}\alpha_j U_{t-j}$

Page 207, line 3 from bottom: Replace $E[L_1(\theta)/L_1(\theta_0)|\mathscr{F}_0]$ with $E[\hat{L}_1(\theta)/\hat{L}_1(\theta_0)|\mathscr{F}_0]$

Page 218, equation (8.36): Replace = with = −

Page 218, equation (8.38): Replace = − with =

Page 223, last equation: Replace = − with =

Page 224, equation (8.58): Replace = − with =

Page 224, equation (8.59): Replace = − with =

Page 244, lines 2 and 3: The statement "Moreover, if E is an elementary matrix and $P_{i,j}$ is a elementary permutation matrix then $P_{i,j}E = EP_{i,j}$." is false.[2] Therefore, this sentence has to be deleted.

Page 244, Theorem I.7: Consequently, the first part of Theorem I.7 is false. This theorem has to be replaced with the following:

Theorem I.7: *Let $P_{i,j}$, $i \neq j$, be an elementary permutation matrix and let $E_{k,m}(c)$ be a conformable elementary matrix. Then*

(*a*) $i \notin \{k,m\}$ *and* $j \notin \{k,m\}$ *imply* $P_{i,j}E_{k,m}(c) = E_{k,m}(c)P_{i,j}$,

(*b*) $i = k$ *and* $j \neq m$ *imply* $P_{i,j}E_{k,m}(c) = E_{j,m}(c)P_{i,j}$,

(*c*) $i \neq k$ *and* $j = m$ *imply* $P_{i,j}E_{k,m}(c) = E_{k,i}(c)P_{i,j}$,

(*d*) $i = k$ *and* $j = m$ *imply* $P_{i,j}E_{k,m}(c) = E_{m,k}(c)P_{i,j}$.

Consequently, for every elementary matrix E and conformable permutation matrix P there exists an

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2 Thanks to Christian Kascha for pointing this problem out to me.

elementary matrix E_* *such that* $PE = E_*P^{\mathrm{T}}$. *Moreover,* $P^{-1} = P^{\mathrm{T}}$.

Proof: Consider the matrix $P_{i,j}E_{k,m}(c)P_{i,j}$, which is the results of swapping rows i and j of $E_{k,m}(c)$ first and then swapping columns i and j. In case (a) the position of c remains the same, hence $E_{k,m}(c) = P_{i,j}E_{k,m}(c)P_{i,j}$, which implies that $P_{i,j}E_{k,m}(c) = E_{k,m}(c)P_{i,j}$ because $P_{i,j}P_{i,j} = I$. In case (b), swapping rows k and j of $E_{k,m}(c)$ moves c to position (j,m), and subsequent swapping of columns k and j leaves c in the new position (j,m), because $m \neq j$ and $m \neq k$, but restores the elementary matrix structure. Therefore, in this case $E_{j,m}(c) = P_{i,j}E_{k,m}(c)P_{i,j}$. In case (c), swapping rows i and m of $E_{k,m}(c)$ leaves c in position (k,m), and subsequent swapping of columns i and m moves c to the new position (k,i) but the restores the elementary matrix structure. In case (d), swapping rows k and m of $E_{k,m}(c)$ moves c to position (m,m), and subsequent swapping of columns k and m moves c to position (m,k). Q.E.D.

Page 245, lines 8 & 7 from bottom: Replace the two lines between equations (I.25) and (I.26) with: "for instance. It follows now from Theorem I.7(b), equation (I.25) and the trivial equality $P_{i,j} = P_{j,i}$ that"

Page 245, equation (I.26): Replace this equation with:

$$\begin{aligned} P_{2,3}E_{3,1}(½)E_{2,1}(-½)A &= P_{3,2}E_{3,1}(½)E_{2,1}(-½)A \\ &= E_{2,1}(½)P_{2,3}E_{2,1}(-½)A \\ &= E_{2,1}(½)E_{3,1}(-½)P_{2,3}A \\ &= DU. \end{aligned} \tag{I.26}$$

Page 245, last equation: Replace this equation with

$$E_{2,1}(½)E_{3,1}(-½) = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix}$$

Page 246, equation (I.27): Replace this equation with:

$$\left(E_{2,1}(½)E_{3,1}(-½)\right)^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} = L \qquad \text{(I.27)}$$

Page 287, line 2 from bottom: Replace the inequality with

$$\varphi(\lambda a + (1 - \lambda)b) \leq \lambda\varphi(a) + (1 - \lambda)\varphi(b)$$

Page 288, line 2 from top: Replace the inequality with

$$\varphi(\lambda a + (1 - \lambda)b) \geq \lambda\varphi(a) + (1 - \lambda)\varphi(b)$$

Page 295, first **and** last equations: Replace x_n with x_0