Introduction to the Mathematical and Statistical Foundations of Econometrics

Remaining corrections and improvements in the 2004 and 2007 editions[1]

January 26, 2012

Page 9, line 2 from top: Replace the first line of the equation with

$$P(\{k\}) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}} = \frac{\dfrac{K!(N-K)!}{\mathbf{k!}(K-k)!(n-k)!(N-K-n+k)!}}{\dfrac{N!}{n!(N-n)!}}$$

(the actual correction is the bold **k!**)

Page 10, line 3 from below: Replace $[0.5-1/q, 0.5+1/q]$ with $(0.5-1/q, 0.5+1/q]$

Page 11, line 2 from top: Replace $[0.5-1/q, 0.5+1/q]$ with $(0.5-1/q, 0.5+1/q]$

Page 11, line 4 from top: Replace $[0.5-1/q, 0.5+1/q]$ with $(0.5-1/q, 0.5+1/q]$

Page 13, line 7 from below: Replace “each” with “at least one”

Page 13, line 6 from below: Replace “they belong” with “it belongs”

Page 18, line 8 from top: Replace “probability” with “unique probability”

Page 18, line 10 from top: Delete this line

Page 21, line 5 of Def. 1.8: Replace $\mathscr{B}^{k,}$ with $\mathscr{B}^{k}$,

Page 25, line 10 from top: Replace $\mu([-\infty, a))$ with $\mu((-\infty, a))$

Page 31, exercise 16: Replace “Definition 1.12" with “Definition 1.11"

Page 36, lines 6 and 7: Replace these two lines with the following:[2]

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[1] Page and line numbers refer to the 2007 edition. Thanks to Dongkoo Kim and Soo Hyun Oh for pointing out many errors.

[2] It was stated that the proof of Lemma 1.B.4. is too difficult and too long. However, there is a much shorter proof in

Jeffrey S. Rosenthal: *A First Look at Rigorous Probability Theory* (2000), World Scientific Publishing Company, p.14.

Thanks to David Jinkins for pointing this out to me.

Proof: Suppose that there exist two extensions of outer measures, P_1 and P_2, to probability measures on $\{\Omega, \mathscr{F}\}$ such that for all sets $B \in \mathscr{F}_0$, $P_1(B) = P_2(B) = P(B)$. Then by the definition of outer measure, for all sets $A \in \mathscr{F}$,

$$\begin{aligned} P_1(A) &= \inf_{A \subset \cup_{i=1}^{\infty} B_i,\ B_i \in \mathscr{F}_0} \sum_{i=1}^{\infty} P(B_i) = \inf_{A \subset \cup_{i=1}^{\infty} B_i,\ B_i \in \mathscr{F}_0} \sum_{i=1}^{\infty} P_2(B_i) \\ &\geq \inf_{A \subset \cup_{i=1}^{\infty} B_i,\ B_i \in \mathscr{F}_0} P_2\left(\cup_{i=1}^{\infty} B_i\right) \geq \inf_{A \subset \cup_{i=1}^{\infty} B_i,\ B_i \in \mathscr{F}_0} P_2(A) \\ &= P_2(A) \end{aligned}$$

and similarly, $P_2(A) \geq P_1(A)$. Thus, $P_2(A) = P_1(A)$ for all sets $A \in \mathscr{F}$. Q.E.D.

Page 41, lines 3-4: Replace $\leq$ with $<$ (4 times), or replace these lines with

(*b*) $\{x \in \mathbb{R}: f_1(x) \leq y\} = \cap_{n=1}^{\infty} \cup_{j=1}^{\infty} \{x \in \mathbb{R}: g_j(x) < y + n^{-1}\} \in \mathscr{B}$.

(*c*) $\{x \in \mathbb{R}: h_1(x) \leq y\} = \cap_{m=1}^{\infty} \cap_{n=1}^{\infty} \cup_{j=n}^{\infty} \{x \in \mathbb{R}: g_j(x) < y + m^{-1}\} \in \mathscr{B}$.

Page 41, line 16 from top: Replace $\left(\inf_{\substack{x \in B(j,m,n) \\ *}} g(x_*)\right)$ with $\left(\inf_{x_* \in B(j,m,n)} g(x_*)\right)$

Page 42, line 2 of Th. 2.7: Replace "limit" with "pointwise limit"

Page 42, line 1 of section 2.3: Replace $\sum_{j=1}^{m}$ with $\sum_{j=0}^{m}$

Page 42, line 3 of section 2.3: Replace $\sum_{j=1}^{m}$ with $\sum_{j=0}^{m}$ twice

Page 43, line 1 of Def. 2.5: Replace $\{\mathbb{R}^k \mathscr{B}^k\}$ with $\{\mathbb{R}^k, \mathscr{B}^k\}$

Page 45, equation (2.11): Replace $\leq$ with $<$

Page 46, line 2 from below: Replace $X(\Omega)$ with $X(\omega)$

Page 46, line 2 of Th. 2.12: Replace " with probability 1" with ", i.e., $Y(\omega) \neq 0$ for all $\omega \in \Omega$,"

Page 47, line 2 of Def. 2.9: Replace "of *P*" with "to *P*"

Page 49: Replace the proof of Theorem 2.18 with the following:

Let $g(x)$ be a nonnegative Borel measurable function on $\mathbb{R}$, and let X be a random variable defined on the probability space $\{\Omega,\mathscr{F},P\}$, with induced probability measure $\mu_X(.)$ defined on the Borel sets in $\mathbb{R}$. Let $S_n(g)$ be the set of all nonnegative simple functions $g_n(x) = \sum_{j=1}^n a_j I(x \in B_j)$ with $a_j \le \inf_{x \in B_j} g(x)$, so that $g_n(x) \le g(x)$ pointwise in x. Without loss of generality we may assume that $0 = a_1 < a_2 < ..... < a_n$ and $\cup_{j=1}^n B_j = \mathbb{R}$. Then by definition,

$$\int g(x)d\mu_X(x) = \sup_{g_* \in \cup_{n=1}^\infty S_n(g)} \int g_*(x)d\mu_X(x).$$

Let $Y = g(X)$, and denote by $S_n(Y)$ be the set of all nonnegative simple random variables $Y_n(\omega) = \sum_{j=1}^n b_j I(\omega \in A_j)$ with $b_j \le \inf_{\omega \in A_j} Y(\omega)$, so that $Y_n(\omega) \le Y(\omega)$ pointwise in $\omega \in \Omega$. Again, without loss of generality we may assume that $0 = b_1 < b_2 < ..... < b_n$ and $\cup_{j=1}^n A_j = \Omega$. Then by definition,

$$\int g(X(\omega))dP(\omega) = \int Y(\omega)dP(\omega) = \sup_{Y_* \in \cup_{n=1}^\infty S_n(Y)} \int Y_*(\omega)dP(\omega),$$

Each simple function $g_n(x) = \sum_{j=1}^n a_j I(x \in B_j) \in S_n(g)$ corresponds to a simple random variable

$$Y_n^*(\omega) = g_n(X(\omega)) = \sum_{j=1}^n a_j I(X(\omega) \in B_j) = \sum_{j=1}^n a_j I(\omega \in C_j) \in S_n(Y)$$

where $C_j = \{\omega \in \Omega\colon X(\omega) \in B_j\} \in \mathscr{F}$, such that

$$\int Y_n^*(\omega)dP(\omega) = \sum_{j=1}^n a_j P(C_j) = \sum_{j=1}^n a_j \mu_X(B_j) = \int g_n(x)d\mu_X(x).$$

Hence,

$$\int g_n(x)d\mu_X(x) \le \sup_{Y_* \in \cup_{n=1}^\infty S_n(Y)} \int Y_*(\omega)dP(\omega) = \int Y(\omega)dP(\omega)$$

and thus,

$$\int g(x)d\mu_X(x) = \sup_{g_* \in \cup_{n=1}^\infty S_n(g)} \int g_*(x)d\mu_X(x) \le \int Y(\omega)dP(\omega).$$

On the other hand, each simple random variable $Y_n(\omega) = \sum_{j=1}^n b_j I(\omega \in A_j) \in S_n(Y)$ corresponds to a simple function $g_n^*(x) = \sum_{j=1}^n b_j I(x \in B_j) \in S_n(g)$ such that

$$g_n^*(X(\omega)) = \sum_{j=1}^n b_j I(X(\omega) \in B_j) \ge Y_n(\omega)$$

To see this, let

$$B_j = \{x \in \mathbb{R}\colon g(x) \in [b_j,b_{j+1})\} \text{ for } j = 1,...,n-1,\ B_n = \{x \in \mathbb{R}\colon g(x) \in [b_n,\infty)\}.$$

Then

$$g_n^*(X(\omega)) = \sum_{j=1}^n b_j I(X(\omega) \in B_j) = \sum_{j=1}^{n-1} b_j I(b_j \le g(X(\omega)) < b_{j+1}) + b_n I(g(X(\omega)) \ge b_n)$$

$$= \sum_{j=1}^{n-1} b_j I(b_j \le Y(\omega) < b_{j+1}) + b_n I(Y(\omega) \ge b_n)$$

$$\geq \sum_{j=1}^{n-1} b_j I(b_j \leq Y_n(\omega) < b_{j+1}) + b_n I(Y_n(\omega) \geq b_n)$$

$$= \sum_{j=1}^{n} b_j I(Y_n(\omega) = b_j) = \sum_{j=1}^{n} b_j I(\omega \in A_j) = Y_n(\omega).$$

Hence,

$$\int Y_n(\omega)dP(\omega) \leq \int g_n^*(X(\omega))dP(\omega) = \int g_n^*(x)d\mu_X(x)$$

$$\leq \sup_{g_* \in \cup_{n=1}^{\infty} S_n(g)} \int g_*(x)d\mu_X(x) = \int g(x)d\mu_X(x)$$

and thus

$$\int Y(\omega)dP(\omega) = \sup_{Y* \in \cup_{n=1}^{\infty} S_n(Y)} \int Y_n(\omega)dP(\omega) \leq \int g(x)d\mu_X(x).$$

Consequently,

$$\int g(X(\omega))dP(\omega) = \int Y(\omega)dP(\omega) = \int g(x)d\mu_X(x) \qquad (2.16)$$

for all nonnegative Borel measurable functions $g(x)$, and therefore (2.16) holds also for all Borel measurable functions $g(x)$. Q.E.D.

Page 50, line 1 of Def. 2.13:	Replace *m's* with *m-th*
Page 50, line 2 of Def. 2.13:	Replace $E[\lvert X - \mu_x\rvert^m]$ with $E[(X - \mu_x)^m]$
Page 51, line 2 from top:	Replace "Function" with "function"
Page 56, proof of Th. 2.21:	The proof of Theorem 2.21 can be shorted considerably as follows:[3]

Let a be a continuity point of $F(x)$ and $G(y)$. Replace b in (2.34) with $a + 1/n$, i.e., replace $\varphi(x)$ with

$$\varphi_n(x|a) = \begin{cases} 0 & \text{if } x \geq a + 1/n, \\ 1 & \text{if } x < a, \\ n(a - x) + 1 & \text{if } a \leq x < a + 1/n. \end{cases} \qquad (2.34)$$

Note that $\lim_{n\to\infty}\varphi_n(x|a) = I(x \leq a)$. It follows therefore from Theorem 2.17 that $\lim_{n\to\infty}E[\varphi_n(X|a)] = E[I(X \leq a)] = F(a)$ and $\lim_{n\to\infty}E[\varphi_n(Y|a)] = E[I(Y \leq a)] = G(a)$.

[3] Thanks to Bruno Sultanum Teixeira for suggesting this.

Because $E[\varphi_n(X|a)] = E[\varphi_n(Y|a)]$, it follows that $F(a) = G(a)$.

Page 57, line 2 from top: Replace $E[\varphi(X)]$ with $E[\varphi(Y)]$

Page 57, line 15 from top: Replace P_j with p_j

Page 59, line 1 of exercise 17: Replace $E(E)$ with $E(X)$

Page 74, Th. 3.5: Replace [4] $P[E(Y|\mathscr{T}) = E(Y)] = 1$ with $E[Y|\mathscr{T}] \equiv E[Y]$

Page 74, Proof of Th. 3.5: Replace the proof with the following:

Denote $Z = E[Y|\mathscr{T}]$. It is left as an exercise to prove that $Z = E[Y]$ a.s., along the same lines as the proofs of Theorems 3.2 and 3.4. Next, recall from Definition 3.1 that Z is measurable $\mathscr{T}$, i.e., for any Borel set B the set $\{\omega \in \Omega: Z(\omega) \in B\}$ is contained in $\mathscr{T}$. Hence, for any Borel set B we have either $\{\omega \in \Omega: Z(\omega) \in B\} = \Omega$ or $\{\omega \in \Omega: Z(\omega) \in B\} = \varnothing$. Now let B be the singleton $\{E[Y]\}$. Then either $\{\omega \in \Omega: Z(\omega) = E[Y]\} = \Omega$ or $\{\omega \in \Omega: Z(\omega) = E[Y]\} = \varnothing$. The latter is excluded by $Z = E[Y]$ a.s. , hence $Z(\omega) \equiv E[Y]$ for all $\omega \in \Omega$. In other words, Z = $E[Y]$ holds exactly.

Page 75, line 8 from top: Replace $= \mathscr{F}_{X,Z}$ with $\subset \mathscr{F}_{X,Z}$

Page 76, line 4 from below: Replace “discrete” with “simple”

Page 77, line 4 of part (b): Insert the following sentence after “monotonic.”:

The sequence $X_n(\omega)$ can be constructed similar the simple function $g_n(x)$ in the proof of Theorem 2.6, with $g(x)$ replaced by $X(\omega)$.

Page 83, Exercise 10: Replace “Borel-measurable” with “continuous”

Page 89, line 9 from below: Replace the expression for $\varphi_{\mathrm{NB}(m,p)}(t)$ with

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[4] Thanks to Renxiang Dai, Tilburg University, for suggesting this.

$$\varphi_{\mathrm{NB}(m,p)}(t) = \left(\frac{p}{1 - (1 - p)e^{i.t}}\right)^m$$

$$= \left(\frac{p}{1 - (1 - p)\cos(t) - i.(1 - p)\sin(t)}\right)^m$$

$$= \left(\frac{p\big(1 - (1 - p)\cos(t) + i.(1 - p)\sin(t)\big)}{p^2 + 2(1 - p)(1 - \cos(t))}\right)^m$$

Page 96, last line: Replace $m(i.t)$ with $m_{N(0,1)}(i.t)$

Page 97, line 2 from top: Replace $m^{/}(i.t)$ with $m^{/}_{N(0,1)}(i.t)$

Page 97, line 2 from top: Replace $m^{//}(i.t)$ with $m^{//}_{N(0,1)}(i.t)$

Page 99, line 12 from top: Replace $N(1, n/y)$ with $N(0, n/y)$

Page 101, line 9 from below: Replace the expression for $\varphi_{U[a,b]}(t)$ with

$$\varphi_{U[a,b]}(t) = \frac{\exp(i.b.t) - \exp(i.a.t)}{i.(b - a)t}$$

$$= \frac{(\sin(b.t) - \sin(a.t)) - i(\cos(b.t) - \cos(a.t))}{(b - a)t}$$

Page 104, line 1: Replace APPENDICES with APPENDIXES [5]

Page 115, line 3 from below: Replace $\begin{pmatrix} 1 & B^{\mathrm{T}} \\ 0 & I_k \end{pmatrix}$ with $\begin{pmatrix} 1 & \beta^{\mathrm{T}} \\ 0 & I_k \end{pmatrix}$

Page 140, equation (6.1): Replace the left-hand side of the inequality with

$$E\left[\left|(1/n)\sum_{j=1}^{n}(Z_j - E(Z_j))\right|\right]$$

Page 141, equation (6.2): Replace $\sum_{k=1}^{j-1}$ with $\sum_{k=1}^{j}$ twice

[5] According to the British-English spelling.

Page 141, line 2 below eq. (6.2): Replace $\sum_{k=1}^{j-1}$ with $\sum_{k=1}^{j}$

Page 141, line 2 below eq. (6.2): The "easy" equality $\sum_{k=1}^{j} k\alpha_k = \sum_{k=1}^{j}\sum_{i=k}^{j}\alpha_i$ appears to be not that obvious. Therefore, here is the proof:

$$\sum_{k=1}^{j} k\alpha_k = \alpha_1 + (\alpha_2 + \alpha_2) + (\alpha_3 + \alpha_3 + \alpha_3) + \dots + (\alpha_j + \alpha_j + \alpha_j + \dots. + \alpha_j)$$

$$= (\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_j) + (\alpha_2 + \alpha_3 + \dots + \alpha_j) + (\alpha_3 + \dots + \alpha_j) + \dots. + \alpha_j$$

$$= \sum_{i=1}^{j}\alpha_i + \sum_{i=2}^{j}\alpha_i + \sum_{i=3}^{j}\alpha_i + \dots + \sum_{i=j}^{j}\alpha_i = \sum_{k=1}^{j}\sum_{i=k}^{j}\alpha_i$$

Page 142, line 3 in Th. 6.3: Replace *in c* with *at x = c*

Page 143, Proof of Th. 6.5: Replace the sentence "Again, without loss of generality we may assume that P[X = 0] = 1 and that X_n is a nonnegative random variable." with the following:

It will be shown first that $X_n \to_p X$ and $\lim_{M\to\infty}\sup_{n\geq 1}E[|X_n|I(|X_n| > M)] = 0$ imply that $|X_n - X|$ is uniformly integrable. To show this, we need to show that $\sup_{n\geq 1}E[|X_n|] < \infty$ and $E[|X|] < \infty$, as follows. Let $M > 0$. Then

$$E[|X_n|] = E[|X_n|.I(|X_n| \leq M)] + E[|X_n|.I(|X_n| > M)] \leq M + \sup_{n\geq 1}E[|X_n|.I(|X_n| > M)$$

Because X_n is uniformly integrable, for an arbitrary $\varepsilon > 0$ we can choose an M_ε such that the second term is less than ε. Consequently, $\sup_{n\geq 1}E[|X_n|] \leq M_\varepsilon + \varepsilon < \infty$.

To show that $E[|X|] < \infty$, choose $K > 0$ and $\varepsilon > 0$ arbitrary. Then

$$E[|X|.I(|X| \leq K)] = E[|X|.I(|X_n - X| > \varepsilon).I(|X| \leq K)] + E[|X|.I(|X_n - X| \leq \varepsilon).I(|X| \leq K)]$$

$$\leq K.P[|X_n - X| > \varepsilon] + E[|X_n - X|.I(|X_n - X| \leq \varepsilon).I(|X| \leq K)]$$

$$+ E[|X_n|.I(|X_n - X| \leq \varepsilon).I(|X| \leq K)]$$

$$\leq K.P[|X_n - X| > \varepsilon] + \varepsilon + \sup_{m\geq 1}E[|X_m|]$$

Letting $n \to \infty$ it follows from $X_n \to_p X$ that $E[|X|.I(|X| \leq K)] \leq \varepsilon + \sup_{m\geq 1}E[|X_m|]$ and next, letting $K \to \infty$, it follows that $E[|X|] \leq \varepsilon + \sup_{n\geq 1}E[|X_n|]$. Because ε was arbitrary, it follows now that $E[|X|] \leq \sup_{n\geq 1}E[|X_n|] < \infty$.

To show that $|X_n - X|$ is uniformly integrable, observe that for arbitrary $M > 0$ and $K > 0$,

$$E[|X_n - X|.I(|X_n - X| > M)] = E[|X_n - X|.I(|X_n - X| > M).I(|X_n| \le K).I(|X| \le K)]$$
$$+ E[|X_n - X|.I(|X_n - X| > M).I(|X_n| > K).I(|X| \le K)]$$
$$+ E[|X_n - X|.I(|X_n - X| > M).I(|X_n| \le K).I(|X| > K)]$$
$$+ E[|X_n - X|.I(|X_n - X| > M).I(|X_n| > K).I(|X| > K)]$$
$$\le 4.K.P[|X_n - X| > M] + 2\sup_{m\ge 1}E[|X_m|.I(|X_m| > K)] + 2.E[|X|.I(|X| > K)]$$

Hence,

$$\limsup_{n\to\infty}E[|X_n - X|.I(|X_n - X| > M)] \le 2.\sup_{m\ge 1}E[|X_m|.I(|X_m| > K)] + 2.E[|X|.I(|X| > K)]$$

and thus, letting $K \to \infty$, it follows that $\lim_{n\to\infty}E[|X_n - X|.I(|X_n - X| > M)] = 0$.

Next, choose an arbitrary $\varepsilon > 0$, and pick an $M_0 > 0$. Then there exists a natural number $n_0(\varepsilon)$ such that $E[|X_n - X|.I(|X_n - X| > M_0)] < \varepsilon$ for all $n > n_0(\varepsilon)$, and therefore also

$$E[|X_n - X|.I(|X_n - X| > M)] < \varepsilon \text{ for all } n > n_0(\varepsilon) \text{ and } M > M_0.$$

Hence,

$$\lim_{M\to\infty}\sup_{n\ge 1}E[|X_n - X|.I(|X_n - X| > M)] \le \varepsilon + \max_{1\le n\le n_0(\varepsilon)}\lim_{M\to\infty}E[|X_n - X|.I(|X_n - X| > M)]$$
$$= \varepsilon$$

where the equality follows from the fact that $E[|X_n - X|] < \infty$. Because ε was arbitrary, it follows now that $|X_n - X|$ is uniformly integrable. Therefore, without loss of generality we may now assume that $P[X = 0] = 1$ and that X_n is a nonnegative random variable.

Page 144, last line of Th. 6.7: Replace this line with

$P(X \in B) = 1$. *Then* $\Psi(X_n) \to \Psi(X)$ *a.s.*

Page 146, first line below (6.8): Replace "Theorem 6.3" with "Theorem 6.10"

Page 147, equation (6.11): Replace equation (6.11) with:

$$E[g(X_1,\theta)] = \int_{-\infty}^{\infty} \frac{\exp(-(x+\theta_0-\theta)^2/2)/\sqrt{2\pi}}{\pi(1+x^2)}dx$$
$$= \int_{-\infty}^{\infty} f(x-\theta+\theta_0)h(x|0)dx = \gamma(\theta-\theta_0), \tag{6.11}$$

Page 149, line 1 from top: Replace the sentence "*where B is a closed and bounded subset of* $\mathbb{R}^k$ *containing c*" with "*where B is an open subset of* $\mathbb{R}^k$ *containing c*" [6]

Page 150, line 7 from top: Replace $X_n \to_p .$ with $X_n \to_p X.$

Page 150, Proof of Th. 6.17: Replace the proof of Theorem 6.17 with the following:

Proof: Let X_n and X be random vectors in $\mathbb{R}^k$. It follows from Theorem 6.B.3 in Appendix 6.B that for each subsequence n_j of n there exists a further subsequence n_{j_m} such that $X_{n_{j_m}} \to X$ a.s. as $m \to \infty$, so that by Theorem 6.7,

$$\varphi\left(X_{n_{j_m}}\right) \to \varphi(X) \text{ a.s. as } m \to \infty,$$

for every bounded and continuous function $\varphi(x)$ on $\mathbb{R}^k$. This implies, by Theorem 6.B.3, that $\varphi(X_n) \to_p \varphi(X)$, which by Theorem 6.4 implies that $\lim_{n\to\infty} E[\varphi(X_n)] = E[\varphi(X)]$. Theorem 6.17 now follows from Theorem 6.18. Q.E.D.

Page 153, line 9 from top: Replace $|E[\Phi(X_n,Y_n)] - E[\Phi(X_n,c)|$ with $|E[\Phi(X_n,Y_n)] - E[\Phi(X_n,c)]|$

Page 154, line 12 from top: Replace $\mathbb{R}^{k\times k}$ with $\mathbb{R}^{k^2}$

Page 157, last line of Th. 6.25: Replace $\varphi(X_n)$ with $\Phi(X_n)$

Page 159, line 7 from below: Replace "Theorem 6.1" with "Theorem 6.28"

Page 161, last line: Replace "Theorem 6.21" with "Theorem 6.12"

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[6] The reason is that the singleton $B = \{c\}$ is closed and bounded, and that for this case Theorem 6.12 may not hold. If B is open then there exists a $\delta > 0$ such that $\{x \in \mathbb{R}^k: \|x-c\| < \delta\} \subset B$, which is an essential element of the proof of Theorem 6.12.

Page 180, line 7 from top: Replace “The exact proof” with “A more exact proof”

Page 180, line 10 from top: Replace “It is possible” with “Suppose it is possible”

Page 180, line 11 from top: Insert the following sentences after “minimal.”:

This is not always possible. See http://econ.la.psu.edu/~hbierens/WOLD.PDF for a counter example.

Page 182, lines 8-9 from top: Replace these lines with the following:

Multiplying these equations by U_{t-m} for some integer $m \geq 1$ and then taking expectations it follows straightforwardly from (7.4), (7.5) and (7.10) that $\delta_m = 0$; hence, $W_t = \sum_{j=1}^{\infty}\beta_j W_{t-j}$ with probability 1. Q.E.D.

Page 185, line 3 of Th. 7.5: Replace ∅ with 0

Page 194, last line: Replace the leading + with −

Page 195, lines 4-6 from top: Replace these lines with the following statements:

Note that by Lemma 7.1,

$$|\exp(-x^2/2 + r(x))| = \exp(-x^2/2)\,|\exp(\mathrm{Re}[r(x)] + i.\mathrm{Im}[r(x)])|$$
$$= \exp(-x^2/2)\exp(\mathrm{Re}[r(x)])| \leq \exp\left(-\tfrac{1}{2}x^2 + |x|^3\right) \leq 1 \ \ if\ |x| \leq 1/2$$

so that

$$\left|\exp\left(-(\xi^2/2)(1/n)\sum_{t=1}^{n}X_t^2\right)\exp\left(\sum_{t=1}^{n}r\left(\xi X_t/\sqrt{n}\right)\right)\right| \leq \prod_{t=1}^{n}\left|\exp\left(-\tfrac{1}{2}\left(\xi.X_t/\sqrt{n}\right)^2 + r\left(\xi X_t/\sqrt{n}\right)\right)\right|$$
$$\leq \sup_{|x| \leq |\xi|.\max_{1\leq t\leq n}|X_t|/\sqrt{n}} |\exp(-x^2/2 + r(x))|^n \leq 1$$

if $|\xi|.\max_{1\leq t\leq n}|X_t|/\sqrt{n} \leq 1/2$. Condition (7.38) implies that the latter event has probability converging to 1, hence pointwise in ξ,

$$\lim_{n\to\infty}P[|Z_n(\xi)| \leq 2] = 1. \qquad (7.50)$$

Therefore, it follows from (7.49), (7.50) and the dominated convergence theorem that

Page 196, equation (7.58): Replace α^2 with σ^2

Page 202, lines 14-16 from top: Delete the sentence starting with "In particular,..."

Page 203, line 1-3 from bottom: Replace these lines with the following statements:

In general, $\hat{X}_t$ is defined as the limit of the projection $\hat{X}_{t,n}$ of X_t on the subspace spanned by $X_{t-1}, X_{t-2}, \ldots, X_{t-n}$, in the sense that $\lim_{n\to\infty} E[(\hat{X}_t - \hat{X}_{t,n})^2] = 0$, where $\hat{X}_{t,n}$ takes the form $\hat{X}_{t,n} = \sum_{j=1}^{n} \beta_{n,j} X_{t-j}$. Note that due to the covariance stationarity condition the coefficients $\beta_{n,j}$ do not depend on *t*. However, there is no guarantee that $\lim_{n\to\infty}\beta_{n,j}$ exists. Nevertheless, in the following it will be assumed that $\lim_{n\to\infty}\beta_{n,j} = \beta_j$ for each j, and that $\hat{X}_t$ takes the form $\hat{X}_t = \sum_{j=1}^{\infty}\beta_j X_{t-j}$.

Page 204, line 1 from top: Replace this line with:

Then the β_j's are the solutions of the normal equations

Page 204, bottom: Add the following paragraph:

Remark: The general proof of the Wold decomposition is given in:
http://econ.la.psu.edu/~hbierens/WOLD.PDF.

Page 217: Replace the whole Assumption 8.2 with:

Assumption 8.2: $\text{plim}_{n\to\infty}\sup_{\theta\in\Theta}|n^{-1}\ln(\hat{L}_n(\theta)) - E[n^{-1}\ln(\hat{L}_n(\theta))]| = 0$ *and* $\lim_{n\to\infty}\sup_{\theta\in\Theta}|n^{-1} E[\ln(\hat{L}_n(\theta)/\hat{L}_n(\theta_0))] - \ell(\theta|\theta_0)| = 0$, *where* $\ell(\theta|\theta_0)$ *is a continuous function of* θ *such that for arbitrarily small* $\delta > 0$, $\sup_{\theta\in\Theta:\ \|\theta-\theta_0\|\geq\delta}\ell(\theta|\theta_0) < 0$.

Page 244, line 16 from top: Replace $P_{i,j}P_{j,i} = 1$ with $P_{i,j}P_{j,i} = I$

Page 253, line 1 of Section I.8: Replace "Theorem I.9" with "Theorem I.11"

Page 254, line 3 of Th. I.12: Replace "Theorem I.9" with "Theorem I.11"

Page 254, line 3 of Th. I.13: Replace "Theorem I.9" with "Theorem I.11"

Page 256, last line of Section I.8: Replace "Theorem I.5" with "Theorem I.15"

Page 265, lines 5-8 from bottom: This statement is incorrect. Replace these lines with:

Taking the transpose of a square matrix does not affect the determinant:

Page 265, lines 1-3 from bottom: Replace these lines with:

This result follows straightforwardly from Theorem I.20 and the following argument.

Page 266, line 3 from bottom: Replace det(A) with det(AB)

Page 268, line 2 from bottom: Replace "block-diagonal" with "block-triangular"

Page 269, line 2 from top: Replace $k \times m$ with $m \times k$

Page 286, line 19 from top: Replace $A \subset B$ with $A \supset C$