Estimating a System of Simultaneous Probit Models by Nonlinear GMM

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Abstract

In this paper I will show how to formulate a system of simultaneous Probit models as a nonlinear GMM model. This paper is part of the guided tour on user-defined nonlinear GMM in *EasyReg International.*

1 Simultaneous Probit models

1.1 Introduction

Consider the following system of simultaneous Probit models:

$$y_1 = I(y_1^* > 0), \text{ where } y_1^* = \alpha' x_1 + \gamma y_2 + u_1, \tag{1}$$

$$y_2 = I(y_2^* > 0), \text{ where } y_2^* = \beta' x_2 + u_2, \tag{2}$$

where $I(.)$ is the indicator function: $I(true) = 1$, $I(false) = 0$, and

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \sim N_2\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right], \ \rho \neq 0.$$

The latent variables y_1^* and y_2^* are not observed. The errors $u_1$and u_2 are independent of the vectors $x_1$and x_2 of exogenous variables.

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*This paper is a revision of a previous version which contained an error. Thanks to Dubravko Radic for pointing this error out to me.

Equation (2) implies a Probit model:

$$P(y_2 = 1|x_1, x_2) = F(\beta' x_2), \tag{3}$$

where

$$F(z) = \int_{-\infty}^{z} f(v)dv, \text{ with } f(v) = \frac{\exp\left(-v^2/2\right)}{\sqrt{2\pi}}. \tag{4}$$

If $\rho = 0$, then u_1 and u_2 are independent, so that y_2 in model (1) may be treated as an exogenous variable, which yields a Probit model

$$P[y_1 = 1|x_1, y_2] = F\left(\alpha' x_1 + \gamma y_2\right). \tag{5}$$

However, if $\rho \neq 0$ then (5) is incorrect, as follows from the following argument.

Let

$$u_1 = \rho u_2 + \sqrt{1-\rho^2}u. \tag{6}$$

By construction, $cov(u_1, u) = 0$, hence u_2 and u are independent, and therefore u and y_2 are independent. Moreover, $var(u_1 - \rho u_2) = (1 - \rho^2)$, hence $var(u) = 1$.

Now substitute (6) in (1):

$$y_1 = I\left(u > -\frac{\alpha' x_1 + \gamma y_2 + \rho u_2}{\sqrt{1-\rho^2}}\right). \tag{7}$$

Then

$$\begin{aligned} P(y_1 &= 1|u_2, x_1, x_2) = F\left[\frac{\alpha' x_1 + \gamma y_2 + \rho u_2}{\sqrt{1-\rho^2}}\right] \\ &= I(u_2 > -\beta' x_2)F\left[\frac{\alpha' x_1 + \gamma + \rho u_2}{\sqrt{1-\rho^2}}\right] \\ &\quad + I(u_2 \leq -\beta' x_2)F\left[\frac{\alpha' x_1 + \rho u_2}{\sqrt{1-\rho^2}}\right]. \end{aligned} \tag{8}$$

Integrating u_2 out in (8) yields:

$$\begin{aligned} P\left[y_1 = 1|x_1, x_2\right] &= \int_{-\beta' x_2}^{\infty} F\left[\frac{\alpha' x_1 + \gamma + \rho v}{\sqrt{1-\rho^2}}\right] f(v)dv \\ &\quad + \int_{-\infty}^{-\beta' x_2} F\left[\frac{\alpha' x_1 + \rho v}{\sqrt{1-\rho^2}}\right] f(v)dv. \end{aligned} \tag{9}$$

Moreover, since y_2 is completely determined by u_2 and x_2 we can split up (8) into

$$\begin{aligned}
&P\left[y_1 = 1, y_2 = 1 | u_2, x_1, x_2\right] \\
&= E\left[y_1 y_2 | u_2, x_1, x_2\right] = y_2 E\left[y_1 y_2 | u_2, x_1, x_2\right] \\
&= I(u_2 > -\beta' x_2) F\left[\frac{\alpha' x_1 + \gamma + \rho u_2}{\sqrt{1-\rho^2}}\right], \\
&P\left[y_1 = 1, y_2 = 0 | u_2, x_1, x_2\right] \\
&= E\left[y_1(1 - y_2) | u_2, x_1, x_2\right] = (1 - y_2) E\left[y_1 | u_2, x_1, x_2\right] \\
&= I(u_2 \leq -\beta' x_2) F\left[\frac{\alpha' x_1 + \rho u_2}{\sqrt{1-\rho^2}}\right],
\end{aligned}$$

hence

$$P\left[y_1 = 1, y_2 = 1 | x_1, x_2\right] = \int_{-\beta' x_2}^{\infty} F\left[\frac{\alpha' x_1 + \gamma + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv, \tag{10}$$

$$P\left[y_1 = 1, y_2 = 0 | x_1, x_2\right] = \int_{-\infty}^{-\beta' x_2} F\left[\frac{\alpha' x_1 + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv. \tag{11}$$

Furthermore, it follows from (10) and (11) that

$$\begin{aligned}
&P\left[y_1 = 1 | y_2 = 1, x_1, x_2\right] \\
&= \frac{P\left[y_1 = 1, y_2 = 1 | x_1, x_2\right]}{P\left[y_2 = 1 | x_1, x_2\right]} = \frac{\int_{-\beta' x_2}^{\infty} F\left[\frac{\alpha' x_1 + \gamma + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv}{F(\beta' x_2)}
\end{aligned}$$

and

$$\begin{aligned}
&P\left[y_1 = 1 | y_2 = 0, x_1, x_2\right] \\
&= \frac{P\left[y_1 = 1, y_2 = 0 | x_1, x_2\right]}{P\left[y_2 = 0 | x_1, x_2\right]} = \frac{\int_{-\infty}^{-\beta' x_2} F\left[\frac{\alpha' x_1 + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv}{1 - F(\beta' x_2)},
\end{aligned}$$

respectively, hence for $j = 0, 1$,

$$\begin{aligned}
&P\left[y_1 = 1 | y_2 = j, x_1, x_2\right] \\
&= \frac{j \int_{-\beta' x_2}^{\infty} F\left[\frac{\alpha' x_1 + \gamma j + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv + (1 - j) \int_{-\infty}^{-\beta' x_2} F\left[\frac{\alpha' x_1 + \gamma j + \rho v}{\sqrt{1-\rho^2}}\right] f(v) dv}{j F(\beta' x_2) + (1 - j)\left(1 - F(\beta' x_2)\right)}.
\end{aligned}$$

Consequently,

$$
\begin{aligned}
& P\left[y_1=1|y_2,x_1,x_2\right] \qquad (12)\\
&= \frac{y_2\int_{-\beta' x_2}^{\infty} F\left[\left(\alpha' x_1+\gamma y_2+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv}{y_2 F(\beta' x_2)+\left(1-y_2\right)\left(1-F(\beta' x_2)\right)}\\
&+\frac{\left(1-y_2\right)\int_{-\infty}^{-\beta' x_2} F\left[\left(\alpha' x_1+\gamma y_2+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv}{y_2 F(\beta' x_2)+\left(1-y_2\right)\left(1-F(\beta' x_2)\right)}.
\end{aligned}
$$

Comparing (12) with (5) we see that indeed (5) is wrong if $\rho \neq 0$.

1.2 Conditional moments

The two conditional probabilities (12) and (3) determine the joint distribution of (y_1, y_2), conditional on x_1 and x_2, so that the model can be estimated by maximum likelihood, using the conditional probability function[1]

$$
\begin{aligned}
& p(y_1,y_2|x_1,x_2,\alpha,\beta,\gamma,\rho)\\
&= \left(1-y_1\right)\left[y_2 F(\beta' x_2)+\left(1-y_2\right)\left(1-F(\beta' x_2)\right)\right]\\
&+\left(2y_1-1\right)y_2\int_{-\beta' x_2}^{\infty} F\left[\left(\alpha' x_1+\gamma y_2+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv\\
&+\left(2y_1-1\right)\left(1-y_2\right)\int_{-\infty}^{-\beta' x_2} F\left[\left(\alpha' x_1+\gamma y_2+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv.
\end{aligned}
$$

However, it is easier to estimate the model by GMM, as follows.

Observe from (10) and (11) that

$$
E\left[y_1 y_2|x_1,x_2\right]=\int_{-\beta' x_2}^{\infty} F\left[\left(\alpha' x_1+\gamma+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv,
$$

$$
E\left[y_1(1-y_2)|x_1,x_2\right]=\int_{-\infty}^{-\beta' x_2} F\left[\left(\alpha' x_1+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv.
$$

whereas (3) implies

$$
E\left[y_2|x_1,x_2\right]=F(\beta' x_2).
$$

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[1] It is not too hard to verify from (10) and (11) that for $i, j = 0, 1$,

$$
P\left[y_1=i,y_2=j|x_1,x_2\right]=p(i,j|x_1,x_2,\alpha,\beta,\gamma,\rho).
$$

Therefore, defining

$$m_1\left(\alpha,\beta,\gamma,\rho|y_1,y_2,x_1,x_2\right) \tag{13}$$
$$= y_1y_2 - \int_{-\beta'x_2}^{\infty} F\left[\left(\alpha'x_1+\gamma+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv$$

$$m_2\left(\alpha,\beta,\rho|y_1,y_2,x_1,x_2\right) \tag{14}$$
$$= y_1(1-y_2) - \int_{-\infty}^{-\beta'x_2} F\left[\left(\alpha'x_1+\rho v\right)/\sqrt{1-\rho^2}\right] f(v)dv$$

$$m_3\left(\beta|y_2,x_2\right) = y_2 - F(\beta'x_2), \tag{15}$$

we have the conditional moment restrictions

$$E\left[\left. m_1\left(\alpha,\beta,\gamma,\rho|y_1,y_2,x_1,x_2\right)\right| x_1,x_2\right] = 0 \tag{16}$$

$$E\left[\left. m_2\left(\alpha,\beta,\rho|y_1,y_2,x_1,x_2\right)\right| x_1,x_2\right] = 0 \tag{17}$$

$$E\left[\left. m_3\left(\beta|y_2,x_2\right)\right| x_2\right] = 0 \tag{18}$$

1.3 The choice of instruments

First, we determine an instrumental variable vector z_3 such that

$$E\left[m_3\left(\beta^*|y_2,x_2\right)z_3\right] = 0$$

if and only if $\beta^* = \beta$. It follows from the mean value theorem that for a $\beta^* \neq \beta$ there exists a $\lambda \in [0,1]$ such that

$$\begin{aligned} m_3\left(\beta^*|y_2,x_2\right) &= m_3\left(\beta|y_2,x_2\right) + \left.\frac{\partial m_3\left(\widetilde{\beta}|y_2,x_2\right)}{\partial\widetilde{\beta}}\right|_{\widetilde{\beta}=\beta+\lambda(\beta^*-\beta)} \left(\beta^*-\beta\right) \\ &= m_3\left(\beta|y_2,x_2\right) - f\left(\beta'x_2+\lambda(\beta^*-\beta)'x_2\right)x_2'\left(\beta^*-\beta\right) \end{aligned}$$

The second equality follows from $\partial m_3\left(\beta|y_2,x_2\right)/\partial\beta' = -f(\beta'x_2)x_2$. Hence

$$\begin{aligned} E\left[m_3\left(\beta^*|y_2,x_2\right)z_3\right] &= E\left[m_3\left(\beta|y_2,x_2\right)z_3\right] \\ &\quad -E\left[f\left(\beta'x_2+\lambda(\beta^*-\beta)'x_2\right)z_3x_2'\right]\left(\beta^*-\beta\right) \end{aligned}$$

Therefore, z_3 should be chosen such that $E\left[m_3\left(\beta|y_2,x_2\right)z_3\right] = 0$ and the matrix $E\left[f\left(\beta'x_2+\lambda(\beta^*-\beta)'x_2\right)z_3x_2'\right]$ is of full column rank. These conditions are satisfied if we choose $z_3 = x_2$, or $z_3 = (x_1', x_2')'$ in order to achieve over-identification.

Given this choice z_3 of instruments, β is now pinned down. Therefore, it suffices to determine a vector z_2 of instrumental variables such that $E\left[m_2\left(\alpha^*, \beta, \rho^* | y_1, y_2, x_1, x_2\right) z_2\right] = 0$ if and only $\alpha^* = \alpha$ and $\rho^* = \rho$.

Similarly to the choice of z_3, the vector z_2 should be chosen such that $E\left[m_2\left(\alpha, \beta, \rho | y_1, y_2, x_1, x_2\right) z_2\right] = 0$, and for any $(\widetilde{\alpha}', \widetilde{\rho})'$ on the line piece between $(\alpha', \rho)'$ and $(\alpha^{*\prime}, \rho^*)'$ the matrix

$$
\begin{aligned}
& E\left[\frac{\partial}{\partial\left(\widetilde{\alpha}', \widetilde{\rho}\right)'} \int_{-\infty}^{-\beta' x_2} F\left[\left(\widetilde{\alpha}' x_1 + \widetilde{\rho} v\right) / \sqrt{1-\widetilde{\rho}^2}\right] f(v) dv . z_2\right] \\
& = E\left[\frac{1}{\sqrt{1-\widetilde{\rho}^2}} \int_{-\infty}^{-\beta' x_2} f\left[\left(\widetilde{\alpha}' x_1 + \widetilde{\rho} v\right) / \sqrt{1-\widetilde{\rho}^2}\right] f(v) dv\right. \\
& \left. \times\, z_2\left(x_1', 1 + \widetilde{\rho}^2 / \sqrt{1-\widetilde{\rho}^2}\right)\right]
\end{aligned}
$$

is of full column rank. Since x_1 does not contain a constant, a suitable choice is $z_2 = (x_1', 1)'$, which may be augmented with x_2, $z_2 = (x_1', x_2', 1)'$ if also x_2 does not contain a constant, or $z_2 = (x_1', x_2')$ if it does, in order to achieve over-identification.

Finally, along the same lines it is now easy to see that z_2 is also a suitable vector of instrumental variables for (13).

Summarizing, assume that x_2 does not contain a constant. Let $z = (x_1', x_2', 1)'$ if also x_2 does not contain a constant, or $z = (x_1', x_2')$ if it does. Then the model parameters are identified by the unconditional moment restrictions

$$
\begin{aligned}
E\left[m_1\left(\alpha, \beta, \gamma, \rho | y_1, y_2, x_1, x_2\right) z\right] &= 0 \\
E\left[m_2\left(\alpha, \beta, \rho | y_1, y_2, x_1, x_2\right) z\right] &= 0 \\
E\left[m_3\left(\beta | y_2, x_2\right) z\right] &= 0
\end{aligned}
$$

2 Estimation of the simultaneous Probit model by EasyReg

2.1 Transformations

The integrals in (13) and (14) are of the form

$$\int_c^{\infty} F(a + b.v) f(v) dv,$$

$$\int_{-\infty}^{c} F(a+b.v)f(v)dv,$$

respectively, for which we do not have a closed form expression. However, they are available in EasyReg as

$$\begin{aligned}\text{NormalIntegralUp}(a,b,c) &= \int_{c}^{\infty} F(a+b.v)f(v)dv \\ \text{NormalIntegralDown}(a,b,c) &= \int_{-\infty}^{c} F(a+b.v)f(v)dv\end{aligned}$$

Thus,

$$\int_{-\beta' x_2}^{\infty} F\left(\frac{\alpha' x_1+\gamma+\rho v}{\sqrt{1-\rho^2}}\right) f(v)dv = \text{NormalIntegralUp}(a,b,c)$$

with

$$a=\frac{\alpha' x_1+\gamma}{\sqrt{1-\rho^2}},\ b=\frac{\rho}{\sqrt{1-\rho^2}},\ c=-\beta' x_2,$$

and

$$\int_{-\infty}^{-\beta' x_2} F\left(\frac{\alpha' x_1+\rho v}{\sqrt{1-\rho^2}}\right) f(v)dv = \text{NormalIntegralDown}(a,b,c)$$

with

$$a=\frac{\alpha' x_1}{\sqrt{1-\rho^2}},\ b=\frac{\rho}{\sqrt{1-\rho^2}},\ c=-\beta' x_2$$

Moreover, the function $F(.)$ in (4) is the EasyReg Probit transformation:

$$F(\beta' x_2) = \text{Probit}(\beta' x_2)$$

2.2 Recursive build-up of the conditional moments

The conditional moment functions (13), (14) and (15) have to be build up recursively, starting from an initial set of variables, for example

$$\begin{aligned}\text{X(1)} &= \text{Y1} \\ \text{X(2)} &= \text{X1} \\ \text{X(3)} &= \text{Y2} \\ \text{X(4)} &= \text{X2} \\ \text{X(5)} &= 1\end{aligned}$$

where y_1 = Y1 and y_2 = Y2 are dependent dummy variables in models (1) and (2), respectively, and X1 and X2 are the independent variables. You only have to select Y1, X1, Y2, X2. EasyReg automatically adds the constant 1 to the list.

Suppose that in model (1), x_1 =X1, and in model (2), $x_2 = (1,\text{X2})'$.

The first transformations are the linear combinations

$$\begin{aligned}
\text{X}(6) &= \text{b}(1)\text{X}(2) \ = \alpha' x_1 \\
\text{X}(7) &= \text{b}(2)\text{X}(5)\text{+b}(3)\text{X}(4) \ = \beta' x_2 \\
\text{X}(8) &= \text{b}(4)\text{X}(5) \ = \gamma \\
\text{X}(9) &= \text{b}(5)\text{X}(5) \ = \rho
\end{aligned}$$

Next, make $\sqrt{1-\rho^2}$, in three steps

$$\begin{aligned}
\text{X}(10) &= \text{X}(9)\hat{\ }2 \text{ (Transformation: z}\hat{\ }2) \\
\text{X}(11) &= \text{X}(5) - \text{X}(10) \text{ (Transformation: Subtract)} \\
\text{X}(12) &= \text{SQR(X(11))} \ = \sqrt{\text{X}(11)} = \sqrt{1-\rho^2}
\end{aligned}$$

Create $\alpha' x_1 + \gamma$ by the Add up transformation:

$$\text{X(13)=X(6)+X(8)} \ = \alpha' x_1 + \gamma$$

and then create $(\alpha' x_1 + \gamma)/\sqrt{1-\rho^2}$, $\alpha' x_1/\sqrt{1-\rho^2}$, and $\rho/\sqrt{1-\rho^2}$ by using the Ratio transformation three times in a row:

$$\begin{aligned}
\text{X}(14) &= \text{X}(13)/\text{X}(12) \ = (\alpha' x_1 + \gamma)/\sqrt{1-\rho^2} \\
\text{X}(15) &= \text{X}(6)/\text{X}(12) \ = \alpha' x_1/\sqrt{1-\rho^2} \\
\text{X}(16) &= \text{X}(9)/\text{X}(12) \ = \rho/\sqrt{1-\rho^2}
\end{aligned}$$

Create $-\beta' x_2$ by the Negative transformation

$$\text{X(17)=} - \text{X}(7) \ = -\beta' x_2$$

Create

$$\begin{aligned}
\text{X}(18) &= \text{NormalIntegralUp}(\text{X}(14), \text{X}(16), \text{X}(17)) \\
&= \int_{-\beta' x_2}^{\infty} F\left(\frac{\alpha' x_1 + \gamma + \rho v}{\sqrt{1-\rho^2}}\right) f(v) dv
\end{aligned}$$

$$\begin{aligned} \text{X(19)} &= \text{NormalIntegralDown(X(15), X(16), X(17))} \\ &= \int_{-\infty}^{-\beta' x_2} F\left(\frac{\alpha' x_1 + \rho v}{\sqrt{1-\rho^2}}\right) f(v) dv \end{aligned}$$

and

$$\text{X(20)=Probit(X(7))} = F(\beta' x_2)$$

Next, make $1 - y_2$, using the Subtract transformation:

$$\text{X(21)=X(5)} - \text{X(3)} \ = 1 - y_2$$

and then create $y_1 y_2$ and $y_1(1 - y_2)$, using the Multiply transformation:

$$\text{X(22)=X(1)X(3)} = y_1 y_2$$

$$\text{X(23)=X(1)X(21)} = y_1(1 - y_2)$$

We now have all the building blocks for the conditional moment functions (13), (14) and (15). Using the Subtract transformation, the moment functions can be created by

$$\begin{aligned} \text{X(24)} &= \text{X(22)} - \text{X(18)} = m_1(\alpha, \beta, \gamma, \rho | y_1, y_2, x_1, x_2) \\ \text{X(25)} &= \text{X(23)} - \text{X(19)} = m_2(\alpha, \beta, \rho | y_1, y_2, x_1, x_2) \\ \text{X(26)} &= \text{X(3)} - \text{X(20)} = m_3(\beta | y_2, x_2) \end{aligned}$$

2.3 Summary

The complete EasyReg program for building up the moment functions (13), (14) and (15) consists of the following lines of code:

$$\begin{aligned} \text{X(1)} &= \text{Y1} = y_1 \ \text{(selected by the user)} \\ \text{X(2)} &= \text{X1} \ \text{(selected by the user)} \\ \text{X(3)} &= \text{Y2} = y_2 \ \text{(selected by the user)} \\ \text{X(4)} &= \text{X2} \ \text{(selected by the user)} \\ \text{X(5)} &= 1 \ \text{(selected by EasyReg)} \\ \text{X(6)} &= \text{b(1)X(2)} \ = \alpha' x_1 = \alpha' x_1 = \alpha \text{X1} \\ \text{X(7)} &= \text{b(2)X(5)+b(3)X(4)} \ = \beta' x_2 = \beta_0 + \beta_1 \text{X2} \\ \text{X(8)} &= \text{b(4)X(5)} \ = \gamma \end{aligned}$$

$$
\begin{aligned}
\text{X}(9) &= \text{b}(5)(\text{X}5) \ = \rho \\
\text{X}(10) &= \text{X}(9)\hat{}2 \ = \rho^2 \\
\text{X}(11) &= \text{X}(5) - \text{X}(10) \ = 1 - \rho^2 \\
\text{X}(12) &= \text{SQR}(\text{X}(11)) \ = \sqrt{1-\rho^2} \\
\text{X}(13) &= \text{X}(6){+}\text{X}(8) \ = \alpha' x_1 + \gamma \\
\text{X}(14) &= \text{X}(13)/\text{X}(12) \ = (\alpha' x_1 + \gamma)/\sqrt{1-\rho^2} \\
\text{X}(15) &= \text{X}(6)/\text{X}(12) \ = \alpha' x_1/\sqrt{1-\rho^2} \\
\text{X}(16) &= \text{X}(9)/\text{X}(12) \ = \rho/\sqrt{1-\rho^2} \\
\text{X}(17) &= -\text{X}(7) \ = -\beta' x_2 \\
\text{X}(18) &= \text{NormalIntegralUp}(\text{X}(14), \text{X}(16), \text{X}(17)) \\
&= \int_{-\beta' x_2}^{\infty} F\left(\frac{\alpha' x_1 + \gamma + \rho v}{\sqrt{1-\rho^2}}\right) f(v) dv \\
\text{X}(19) &= \text{NormalIntegralDown}(\text{X}(15), \text{X}(16), \text{X}(17)) \\
&= \int_{-\infty}^{-\beta' x_2} F\left(\frac{\alpha' x_1 + \rho v}{\sqrt{1-\rho^2}}\right) f(v) dv \\
\text{X}(20) &= \text{Probit}(\text{X}(7)) = F(\beta' x_2) \\
\text{X}(21) &= \text{X}(5) - \text{X}(3) \ = 1 - y_2 \\
\text{X}(22) &= \text{X}(1)\text{X}(3) = y_1 y_2 \\
\text{X}(23) &= \text{X}(1)\text{X}(21) = y_1(1-y_2) \\
\text{X}(24) &= \text{X}(22) - \text{X}(18) = m_1\left(\alpha, \beta, \gamma, \rho | y_1, y_2, x_1, x_2\right) \\
\text{X}(25) &= \text{X}(23) - \text{X}(19) = m_2\left(\alpha, \beta, \rho | y_1, y_2, x_1, x_2\right) \\
\text{X}(26) &= \text{X}(3) - \text{X}(20) = m_3\left(\beta | y_2, x_2\right)
\end{aligned}
$$

Here the conditional moment functions are the last three X variables, but that is not necessary. Only the last X variable should be a moment function, but other moment functions may appear anywhere in this list as long as they use parameters. Of course, you need to verify that the moment functions together use all the parameters, and are not homogenous in the parameters, as otherwise the parameters are not identified.

After you have entered the code for the conditional moment functions, EasyReg will prompt you to specify the vector z of instrumental variables for each moment function. As argued before, $z = (\text{X1,X2,1})'$ is a suitable specification for all three moment functions.