

**TOPICS IN ADVANCED ECONOMETRICS:
ESTIMATION, TESTING, AND SPECIFICATION
OF CROSS-SECTION AND TIME
SERIES MODELS**

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This book, *Topics in Advanced Econometrics*, is written primarily as a textbook for an advanced graduate econometrics course. The topics covered include consistent model specification testing, unit roots and cointegration, and nonparametric regression estimation; they are mainly the topics in which Professor Bierens has made significant contributions to the literature over the last 15 years. This book is unusual as a textbook in the sense that it treats both cross-sectional and time series (i.e., both subscript i and t) issues in econometrics at an advanced level. Most of the results given, other than those available in standard econometrics or statistics textbooks, are drawn from the published work of the author. The book is very useful because it puts together a number of important current issues that have been treated separately in the literature and presents them systematically using a well-organized set of statistical tools. Another advantage of this book is that the materials given are almost self-contained, making this book suitable for self-tuition. This book thus ideally suits students who need tools for independent research especially in the area of nonlinear and nonparametric models and time series analysis. It will also be useful to more advanced researchers who are interested in a thorough understanding of some of the author's original and influential work in these areas.

This book is presented in a theorem-proof style, which enables the author to make a clear and rigorous presentation but also tends to make this book a bit too dry for use as a sole textbook. There is also insufficient discussion of the results and motivation, and there are very few examples. Most of the results discussed are first-order asymptotic; no higher-order, exact, or simulation results are given.

This book consists of 10 chapters. Each chapter contains exercises that are directly related to the materials in the preceding text. The content of this book can be classified into six parts: (1) the basic concepts and statistical tools required for the discussion of the remaining parts (Chapters 1–3); (2) nonlinear parametric regression analysis and maximum likelihood theory (Chapter 4); (3) tests of model specification (Chapter 5); (4) linear and nonlinear time series models (Chapters

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6–8); (5) unit roots and cointegration (Chapter 9); and (6) nonparametric kernel regression estimation (Chapter 10). I now briefly discuss each of these parts and comment on the results contained therein.

The first part, Chapters 1–3, deals with the background materials for the remaining sections. Chapter 1 introduces measure-theoretic probability tools including the concept of probability spaces, Borel fields, independence of random variables, Borel measurable random functions, mathematical expectation with respect to a general measure, and characteristic functions. Chapter 2 presents the asymptotic theory regarding convergence of sequences of random variables (and random functions indexed by a finite dimensional parameter), pointwise and uniform laws of large numbers, and central limit theorems. The results given in this chapter apply mostly to independent and (non)identically distributed random variables (and functions). Chapter 3 gives the definition of a conditional expectation relative to a random vector and summarizes some of the basic properties of conditional expectations.

Much of the material in Chapters 1–3 is standard and could be found in, for example, Billingsley (1979) or Chung (1974), though the level of presentation is less technical than that of these works. Proofs are given for most of the results presented, and they are generally quite rigorous, taking into account the thorny issues of measurability, and are also very instructive. On the other hand, there are some unique results in this book that do not appear in standard textbooks; for example, the uniqueness of the conditional expectation is proved using the results of Bierens (1982) rather than using the usual Radon–Nikodym theorem. Also, there are few results in Chapters 1–3 that are not referred to in the later parts of this book; that is, the selection of the basic material is very efficient for the purpose of this book. A consequence of this efficiency is that some concepts, such as outer measure and stochastic equicontinuity, that have much use in the recent nonlinear and semiparametric econometric literature (see Andrews, 1994a, 1994b; Newey, 1991) are not discussed. The reader who needs a more extensive review of recent statistical tools may refer to other textbooks such as Davidson (1994). Some results seem to be unnecessarily specific; for example, to verify a multivariate convergence in distribution using a univariate convergence result, the Cramer–Wold device might have been presented instead of Theorem 2.3.7, which applies only when the asymptotic distribution is normal. Also, some results are derived using concepts defined later on; for example, the concept of “null set” is used in Chapter 1 (p. 17) but is defined in Chapter 2 (Definition 2.1.2). Finally, Section 3.3 consists of material that is not basic and is more relevant to the specification testing results; it might have more appropriately been placed somewhere in Chapter 5.

The second part, Chapter 4, discusses the asymptotic properties of nonlinear least squares and maximum likelihood estimators, along with those of the trinity of tests of (non)linear restrictions—Wald, likelihood ratio, and Lagrange multiplier (LM) tests. It starts with the definition and motivation of nonlinear regression models and proves weak and strong consistency and asymptotic normality of an

estimator that minimizes a smooth criterion function under a set of high-level assumptions. It then gives more primitive assumptions for the special cases where the criterion functions are those of nonlinear least squares and maximum likelihood estimators, respectively. The arguments used in the proofs are standard, but they are rigorous and instructive.

The results given, however, are restrictive as a result of the assumption of a twice continuously differentiable criterion function; it rules out a number of important estimators in econometrics such as, for example, least absolute deviation (LAD) estimator, censored LAD estimator (see Powell, 1984), and Huber M-estimators (see Huber, 1982). The results also do not cover most semiparametric estimators. In these cases, there are general results for the asymptotic properties of extremum estimators that minimize a criterion function that is not smooth and/or depends on an estimate of an infinite-dimensional nuisance parameter. Such results include Andrews (1994a) and Newey and McFadden (1994), among others. On the other hand, no local power and optimality results of the trinity of the tests are discussed.

The third part, Chapter 5, is about tests of model specification. The tests considered are those of the orthogonality condition that the conditional expectation of the error relative to the regressors in the nonlinear regression model equals zero, without a well-specified alternative. Specifically, Hausman tests, M-tests of Newey (1985) and Tauchen (1985), and several versions of consistent tests of Bierens (1982, 1991) are considered, with a particular emphasis on the consistent tests, which is not surprising given the author's interest and expertise in this subject.

There is, however, little general discussion of specification tests; the general framework of Newey (1985) and Tauchen (1985) (hereafter N&T) is introduced only to motivate the consistent conditional M-tests discussed later. A framework more general than that of N&T is also available; Whang and Andrews (1993), among others, extended the results of N&T by allowing the criterion function to be possibly nondifferentiable and indexed by an infinite-dimensional nuisance parameter. The latter framework, therefore, justifies a number of specification tests in both parametric and nonparametric models. The consistent specification tests (discussed on pp. 96–109), on the other hand, assume that the probability limits of the estimator θ_n of the parameter of interest θ_0 are the same under both the null and alternative hypotheses (see, e.g., Assumption 5.2.3 and Theorem 5.2.4). This assumption does not hold in general, because θ_n may be inconsistent for θ_0 if the model is misspecified, and it is not strictly necessary for the consistency of the tests. (For a general treatment of this case, see Whang and Andrews, 1993.)

The fourth part, Chapters 6–8, deals with some of the (stationary) time series issues. Chapter 6 contains basic materials for Chapters 7 and 8 and extends some of the results in Chapters 2 and 3 to dependent data. Notably, the concept of ν -stability due to McLeish (1975) and Bierens (1983) is defined and assumed frequently to control the amount of dependence of functions of mixing random variables. The related concept of functions of mixing processes or near epoch

dependency (see Billingsley, 1968; Gallant and White, 1988; Andrews, 1988) is not introduced, although it is common in the literature. It is also surprising that this chapter does not contain the ergodic theorem.

The focus of Chapter 7 is modeling the conditional expectation of a time series based on its entire past history. A linear AMRA(X) time series model is justified using the Wold decomposition (Theorem 7.2.1) and nonlinear ARMAX models are briefly discussed. Most of the discussion in this chapter, however, is given to the ARMA memory index model suggested in the author's earlier work (see Bierens, 1988). The reader might be referred to Ploberger and Deistler (1988) and Sims (1988) for a further discussion of this model.

Chapter 8 starts with estimation of linear and nonlinear ARMAX models and proves consistency and asymptotic normality of such estimators. It also presents the consistent test of Bierens (1987a) of parametric time series models that has the standard normal limit distribution. The ARMA memory index modeling theory of Chapter 7 plays a key role in justifying this test. The last section of this chapter discusses an autocorrelation test of the errors of nonlinear ARMAX models. The test, however, is consistent against the alternative of autocorrelation of an order fixed a priori. Given the author's particular interest in consistent tests, however, it is worth noting that tests of autocorrelation that are powerful against more general alternatives are also available in the literature. Examples of such tests include Hong (1996) and Andrews and Ploberger (1996), among others.

The fifth part, Chapter 9, is on unit roots and cointegration. The content of this chapter is now quite standard: the concept of weak convergence, the functional central limit theorem, the unit root tests of Dickey and Fuller (1979), Phillips (1987), and Phillips and Perron (1988), the concept of cointegration, error correction models, and inference of cointegrated system using the approach of Engle and Granger (1987) and Johansen (1988, 1991). It is surprising that this book has no discussion about spurious regression. Given the explosion of the literature on nonstationary time series over the last decade, the coverage of this chapter is far from extensive. In this respect, the reader might refer to Phillips (1995, 1997) and Stock (1994), among others, for a more up-to-date survey and to Hamilton (1994) for a textbook treatment that has more extensive results and is more oriented toward applied workers.

The last part, Chapter 10, reviews the asymptotic properties of the Nadaraya-Watson kernel estimator of a nonparametric regression function. Most of the results given are those in the influential work of Bierens (1987b), with an exception that a proof of strong consistency is now added (see Theorem 10.1.1). The asymptotic results given are mostly based on a fixed sequence of bandwidth parameter; no general discussion is provided on the theory of optimal data-dependent bandwidth parameters, which is important for a good performance of nonparametric estimators. A generalization of the consistency results given in this chapter was made by Andrews (1995), among others, who allowed data-dependent bandwidth parameters and relaxed several other assumptions. This chapter also does not discuss nonparametric estimators other than the kernel type;

for a more extensive review of nonparametric estimators, the reader might refer to Härdle (1990) and Eubank (1988).

In conclusion, this book aims, and largely succeeds, at providing the student with tools useful for independent (theoretical) research in some recent econometric areas. The choice of subjects is such that this book might not be appropriate as the stand-alone textbook for most advanced topics courses; many parts of this book, however, could certainly be effectively used as a supplementary text. As a whole, this book is remarkable; it will be a very useful source for those that are interested in nonlinear, nonparametric regression and time series analysis.

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