Economics 404W
Suggested Answers, Homework 1

Note concerning notation.

There are several ways to represent growth in any variable, X: $\frac{\Delta X}{X}$ and $\Delta \ln X$. They are almost identical when growth rates are less than 10 percent, so they can be used interchangeably. I will use $\Delta \ln X$ for purposes of this answer key.

1a) 10pts From lecture and the textbook, the basic Harrod–Domar model yields: $\Delta \ln Y = \frac{s}{\theta_K} - \delta$. Manipulating this formula yields $s = (\Delta \ln Y + \delta) \cdot \theta_K$; s = (0.08 + 0.1) 2.5= 0.45

b) 15pts Manipulating the same equation, we have $\delta = \frac{s}{\theta_K} - \Delta \ln Y = \frac{0.2}{2.5} - 0.08 = 0$. It is highly unlikely that the depreciation rate will be zero; that is, it is highly unlikely that machines never wear out. Thus the basic Harrod-Domar growth equation does not seem to accurately describe Indonesia during the 1970s.

2a) 8pts Growth in any ratio is growth in the numerator less growth in the denominator, so $\Delta \ln\left(\frac{K}{L}\right) = \Delta \ln K - \Delta \ln L$ = 0.04 – 0.027 =0.013

b) 9pts As established in lecture 5,

$$\Delta \ln\left(\frac{Y}{N}\right) = \Delta \ln\left(\frac{Y}{L}\right) + \Delta \ln\left(\frac{L}{N}\right) = \left[\Delta \ln A + \alpha \Delta \ln\left(\frac{K}{L}\right)\right] + \Delta \ln\left(\frac{L}{N}\right)$$

where:

Y= output (GDP)
K = capital
N= population
A= factor productivity
α=elasticity of output with respect to capital. It can be measured as the fraction of national income that goes to capital, assuming perfect competition.

Everything but $\Delta \ln K$ and $\Delta \ln L$ remains constant so

(Y/P)'= 0.45(0.04 – 0.027)=0.00585

c) 8pts Isolating productivity in equation (1), we obtain:

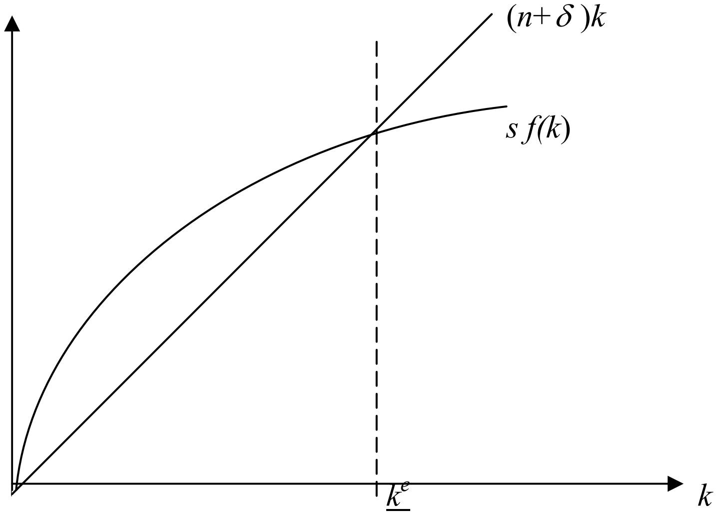
$$\Delta \ln A = \Delta \ln\left(\frac{Y}{N}\right) - \Delta \ln\left(\frac{L}{N}\right) - \alpha\Delta \ln\left(\frac{K}{L}\right) = \Delta \ln\left(\frac{Y}{N}\right) - \Delta \ln\left(\frac{L}{N}\right) - \alpha\left[\Delta \ln K - \Delta \ln L\right]$$

Substituting in for the various growth rates, we obtain

$$\Delta \ln A = 0.05 - 0.01 - 0.45(0.04 - 0.027) = 0.03415$$

3a) 9pts. From lecture 6, the Solow-Swan growth model *without* productivity growth implies that the long run capital labor ratio, k_e, satisfies: $sf(k) = (n+\delta)k$, where $f(k)$ gives output per worker as a function of capital per worker. In this problem, $f(k) = A\sqrt{k} \equiv Ak^{1/2}$. (Note that the elasticity of output with respect to capital is $\alpha = 1/2$ here.) Substituting in the productivity level ($A = 1$), the savings rate ($s = 0.20$), the rate of population growth ($n = 0.03$), and the rate of depreciation ($\delta = 0.07$), one can solve for k_e: $0.2\, k_e^{1/2} = (0.07 + 0.03)\, k_e$. One obtains $k_e = (0.2/0.1)^2 = 4$, which can be substituted back into the production function to obtain $y_e = f(k_e) = (4)^{1/2} = 2$.

3b) 8pts

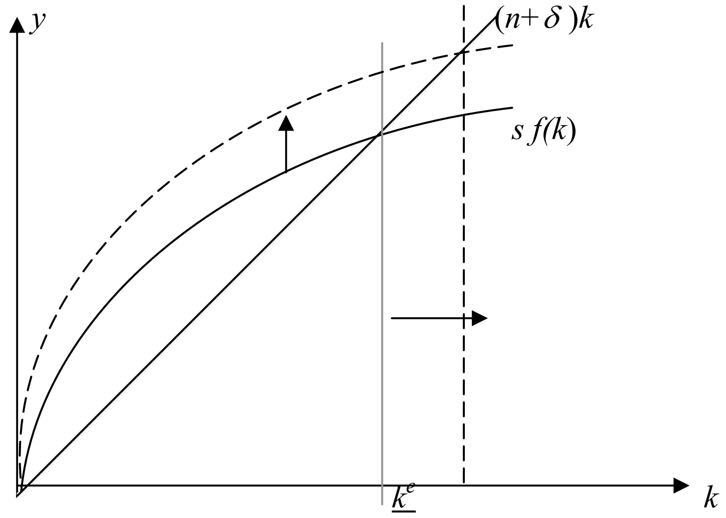


If $k < k_e$ then savings per worker, $sf(k)$, are greater than the amount that is needed to keep capital per worker constant $(\delta + n)\,k$. Hence k increases.

If $k > k_e$ then savings per worker, $sf(k)$, are less than the amount that is needed to keep capital per worker constant $(\delta + n)\,k$. Hence k must decrease.

This process yields the long run equilibrium k_e.

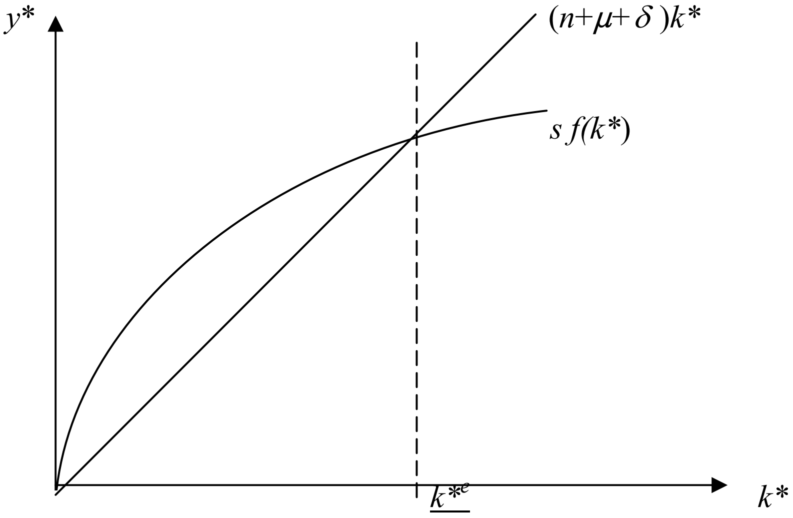
c) 8pts If A is growing, the function $f(k) = Ak^{½}$ is continually shifting upward at each k, inducing continual growth in the capital-labor ratio and output per worker.



As discussed in lecture 6, this growth process can be characterized by thinking of productivity growth as coming from growth in the efficiency of labor. Since students seemed to have trouble with this problem, I will restate the logic here, using the production function stated above rather than the more general notation of the lecture.

Writing the production function as $\frac{Y}{L} = A\left(\frac{K}{L}\right)^{1/2}$, or $\frac{Y}{A^2L} = \left(\frac{K}{A^2L}\right)^{1/2}$, one sees that productivity growth (growth in A) can be thought of as increasing the flow of labor services at any given number of workers (L). Or, defining the volume of labor services as $\lambda_L L$, where λ_L is an index of efficiency per worker, A^2 is equivalent to λ_L. Thus in terms of growth rates, $2\Delta\ln A = \Delta\ln\lambda_L$, and with productivity growing at the rate $\Delta\ln A = 0.02$, we may think of labor efficiency as growing at the rate $\mu \equiv \Delta\ln\lambda_L = 0.04$.

With this restatement of productivity growth in terms of labor efficiency growth, the logic behind the Solow-Swan model can be retraced, replacing output per worker with output per efficiency unit of labor ($y^* = \frac{Y}{\lambda_L L}$), and replacing capital per worker with capital per efficiency unit of labor ($k^* = \frac{K}{\lambda_L L}$). From above, y^* and k^* are related to each other by the production function $y^* = f(k^*) = \left(k^*\right)^{1/2}$, and the growth rate in efficiency units of labor is ($n + \Delta\ln\lambda_L = n + \mu = 0.03 +$ 0.04). The standard diagram, recast in terms of k^* and y^*, is reproduced below.



Since capital per efficiency units of labor (k^*) is constant in the long run, capital must be growing as rapid as labor efficiency units: $\Delta \ln K = n + \mu = 0.07$. Likewise, since y^* is constant in the long run, output is growing at the rate of growth in labor efficiency units: $\Delta \ln Y = n + \mu = 0.07$. But the number of workers is only growing at the rate $\Delta \ln L = n = 0.03$, so output per worker is growing at the rate $\Delta \ln Y - \Delta \ln L = \mu = 0.04$.

4) 4 pts per statistic + 1 free point.