Prof. Tybout

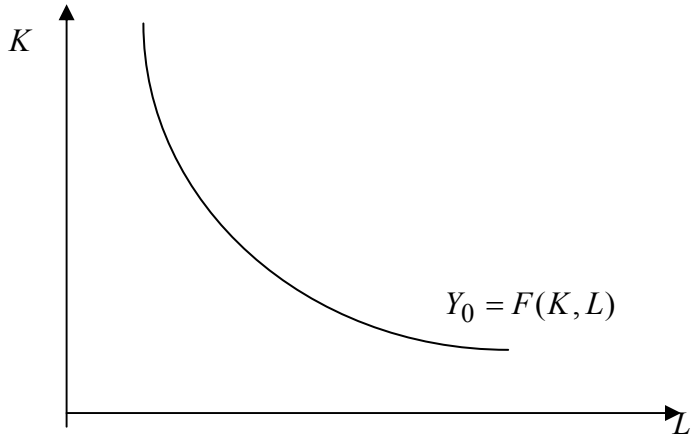
Economics 404W
Lecture 6
January 26, 2006

Note: first homework assignment posted; due Feb 7

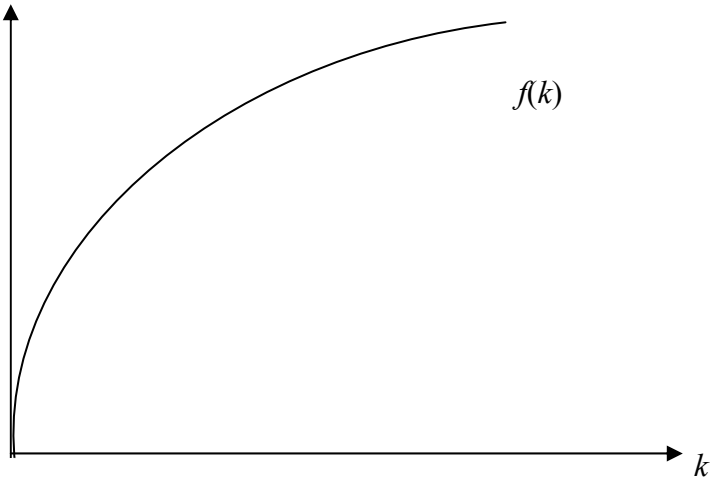
B. The Solow-Swan Model, continued

1) Basic mechanics

Define $Y = F(K,L)$, as a standard constant returns neoclassical production function. (Draw isoquants)



Converting to output per worker, $Y/L = F(K,L)/L = F(K/L, 1)$. Or, $y = f(k)$. Graphically:



As capital per worker increases, output per worker goes up, but at a diminishing rate. This reflects diminishing returns to capital.

What about growth? It is determined solely by the rate of growth in k, which can be expressed as the rate of growth in capital minus the rate of growth in labor. More precisely, since growth rates are approximated by changes in logs, the rate of growth in k

can be written as $\Delta \ln k = \Delta \ln(K/L) = \Delta(\ln K - \ln L) = \Delta \ln K - \Delta \ln L$. Or writing growth rates as percentage changes, $\Delta k / k = \Delta K / K - \Delta L / L$.

What determines each component of $\Delta k / k$? As before, assume that labor grows at some exogenous rate, $\Delta L/L = n$, reflecting human behavior and the state of medicine. (Later we'll consider endogenous pop. growth.) Also, capital growth is governed by the savings rate:

$$\Delta K = sY - \delta K$$

$$\frac{\Delta K}{L} = sf(k) - \delta k$$

$$\frac{\Delta K}{K} = [sf(k) - \delta k] / k = \frac{sf(k)}{k} - \delta$$

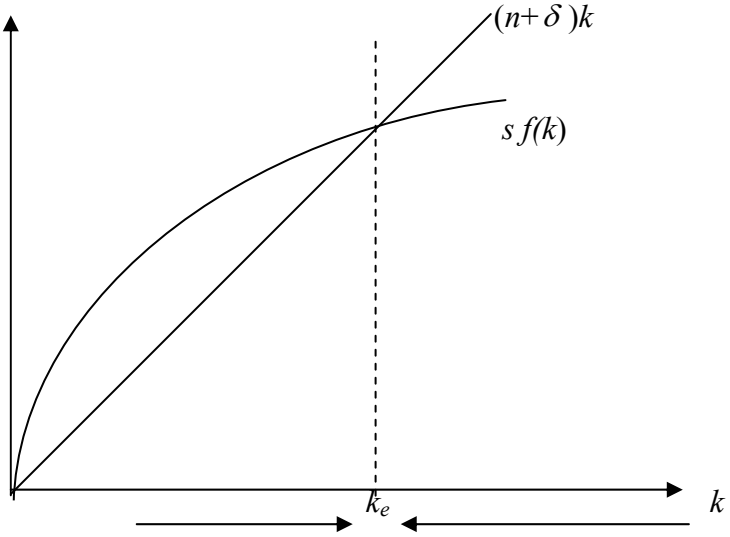
Whenever capital is growing more rapidly than labor, output per worker is growing, and the country is developing. On the other hand, if labor is growing more rapidly than capital, the country is become poorer. So the critical question is whether

$$\frac{\Delta K}{K} > \frac{\Delta L}{L}, \quad or \quad \frac{sf(k)}{k} - \delta > n$$

Multiplying both sides by the capital-labor ratio, we obtain an expression that is easier to interpret. Output per worker grows whenever savings per worker exceeds the amount need to keep pace with labor force growth, after depreciation is accounted for:

$$sf(k) > (n + \delta)k$$

Graphically, we have:



Notice that k grows whenever it is below k_e, and shrinks whenever it is above. Intuitively, at low k, each unit of capital generates a lot of output because the marginal product of capital is high. Also, the small capital stock means that each unit of savings adds a lot *percentage-wise* to the capital stock. So at low k, capital stocks are likely to grow faster than the rate of population growth.

But as the capital stock increases, savings per worker increase less than proportionately because of diminishing returns. Yet, to keep capital growing as fast as labor, the savings needs per worker remain proportional to capital per worker. Equilibrium occurs where $k = k^e$, and deviations from this equilibrium are temporary. At this equilibrium, $sf(k) = (n + \delta)k$.

Aside: Cobb-Douglas case: $y = Y/L = f(k) = k^{\alpha}$. In equilibrium, $sk^{\alpha} = (n+\delta)k$, so solving for k we have $k = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$ and $y = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$.

2) Implications of the Model

a) **Factors will tend toward full employment**

Seems right. We don't see secular tendencies toward excess factor stocks.

b) **Population growth and savings rates don't affect long run growth rates. Shocks to any of these variables have a transitory effect on growth**

These seem to fit the data better than HD. (Refer to graphs on savings rates, population growth and per capita income growth.)

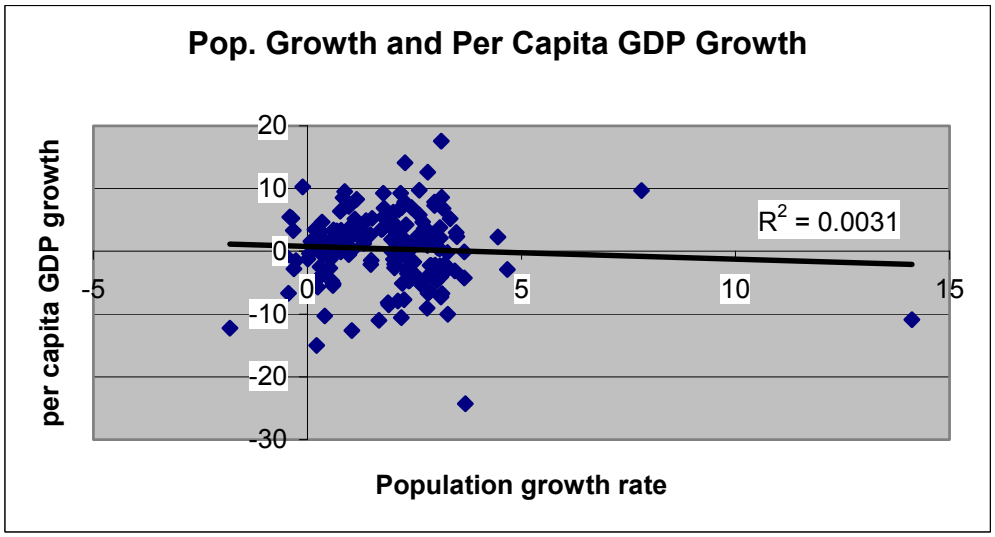
c) **Countries with similar savings rates, population growth rates and depreciation rates should converge toward similar levels of per capita GDP.**

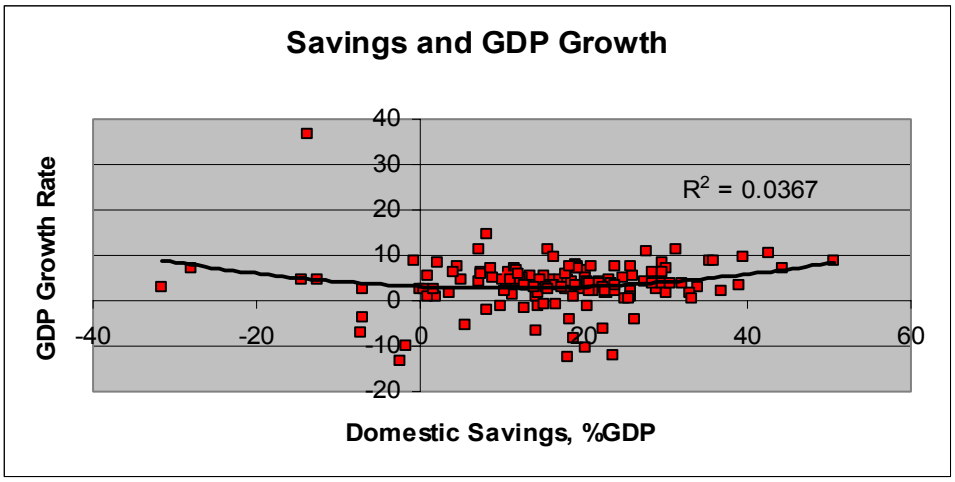
Although countries overall don't exhibit unconditional convergence, those with similar savings rates and population rates do weakly tend toward similar income levels.

d) **<u>Levels</u> of output per worker will tend to be high in countries with:**
- high savings rates (true?)
- low population growth rates (true?)
- low depreciation rates (true?)

e) **<u>Levels</u> of output per worker will tend to be high in countries with:**
- high savings rates (true?)
- low population growth rates (true?)
- low depreciation rates

These predictions also seem to fit the stylized facts. *Growth* in per capita GDP is not strongly associated with either population growth rates or savings rates:





But *levels* of GDP per capita are associated with savings (or investment) rates and population growth rates in the predicted way: