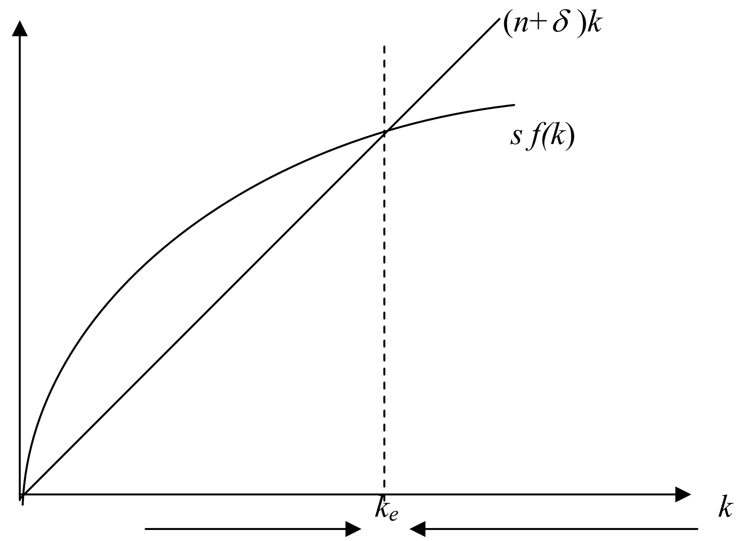
Prof. Tybout

Economics 404W
Lecture 7
January 31, 2006

Note: first homework assignment posted; due Feb 7

B. The Solow-Swan Model, continued



At this equilibrium, $sf(k) = (n+\delta)k$.

> Aside: Cobb-Douglas case: $y = Y/L = f(k) = k^{\alpha}$. In equilibrium, $sk^{\alpha} = (n+\delta)k$, so solving for k we have $k = \left(\dfrac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$ and $y = \left(\dfrac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$.

2) Implications of the Model

a) Factors will tend toward full employment

b) Population growth and savings rates don't affect long run growth rates. Shocks to any of these variables have a transitory effect on growth

c) Countries with similar savings rates, population growth rates and depreciation rates should converge toward similar levels of per capita GDP.

d) Levels of output per worker will tend to be high in countries with:

3. These predictions also seem to fit the stylized facts. But the S-S model has two obvious limitations:

- It predicts no growth in steady state.
- It predicts cross-country differences in per capita income that aren't large enough.

  Recall from the Cobb-Douglas example (a reasonable first-order approximation):

  $y = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$ Suppose δ = .05. A very poor country might have n = .03, s = .10. For simplicity, let α = .5 (This is actually a little low for LDCs.) Then $y = \left(\frac{.10}{.08}\right) = 1.25$. Alternatively, for an industrialized country, suppose n = 0, s = .30, so that $y = \left(\frac{.30}{.05}\right) = 6$. Way too similar. For example, U.S. GDP per capita was about 15 times the Indian GDP per capita in the 1980s.

One way to get more cross-country heterogeneity, *and* to get output per capita growing in steady state, is by adding technological progress. Suppose technological progress or human capital accumulation makes the flow of labor services larger for each stock of workers, $L^* = \lambda_L L$, where λ_L measures worker productivity. The production function becomes $Y = F(K,L^*)$, and output per unit of labor *services* becomes

$$y^* = \frac{Y}{L^*} = \frac{F(K,L^*)}{L^*} = F(\frac{K}{L^*},1) = f(k^*)$$

Also, *growth in labor services* takes place at the rate

$$\frac{\Delta L^*}{L^*} = \frac{\Delta \lambda_L}{\lambda_L} + \frac{\Delta L}{L} = \mu + n \text{ or using logs, } \Delta \ln L^* = \Delta \ln L + \Delta \ln \lambda_L = \mu + n$$

Then the solution to the model proceeds exactly as before, but $n+\mu$ replaces n. We end up with capital and output growing at the rate of labor force growth *plus* the rate of growth in labor efficiency, μ.. Hence, since the labor force is growing only at the rate n, output growth exceeds labor force growth by μ, and per capita income rises at this rate in the steady state:

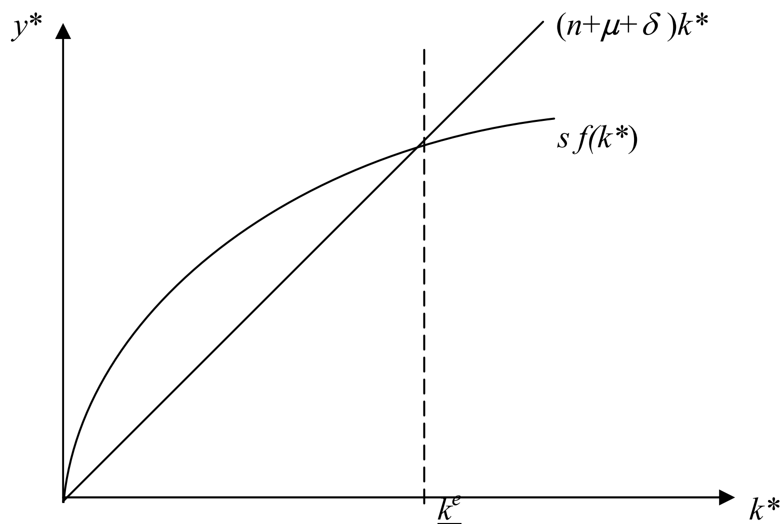
$$\frac{\Delta L^*}{L^*} = \frac{\Delta K}{K} = \frac{\Delta Y}{Y} = n + \mu; \quad \frac{\Delta L}{L} = n$$

or equivalently, using logs,

$$\Delta \ln L^* = \Delta \ln K = \Delta \ln Y = n + \mu; \quad \Delta \ln L = n$$

Solow himself proposed this fix-up of the model, treating the rate of growth in labor efficiency as an exogenous parameter. (It is what he considered himself to be capturing with his "Solow residual" measure of productivity growth.)

What is the relationship between μ and A? Returning to the Cobb-Douglas case, $Y = F(K, L^*) = K^{\alpha}(L^*)^{1-\alpha} = \left(\lambda_L^{1-\alpha}\right)K^{\alpha}L^{1-\alpha}$. So *growth* in A is the same as (1-α) times growth in labor efficiency (λ_L), μ.



One interpretation of λ_L is that it reflects investment in education. But does education affect the *level* of labor productivity, or the rate of growth in labor productivity?

Many people believe that it affects the rate of growth in labor productivity, and that countries that invest a lot in education grow relatively rapidly. But as Easterly notes, the evidence suggests otherwise: