Prof. Tybout

Economics 404W
Lecture 8
February 2, 2006

Note: first homework assignment posted; due Feb 7

B. The Solow-Swan Model, continued

To recap: Using an aggregate production function, $Y = AF(K,L)$, we decomposed growth in output per worker into two terms—productivity growth and capital deepening:

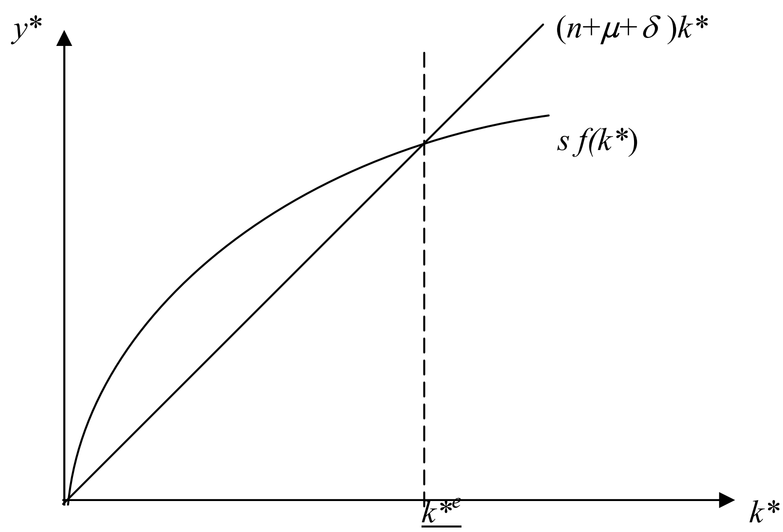
$$\Delta \ln(Y / L) = \Delta \ln A + \alpha \cdot \Delta \ln(K / L) \text{, or}$$

$$\Delta \ln y = \Delta \ln A + \alpha \cdot \Delta \ln k$$

Evidence suggests that growth in output per worker is attributable to both productivity growth and capital deepening (i.e., growth in $k = K/L$). So we set aside productivity growth for the time being, and we embarked upon a quest to understand capital deepening. This led to the Harrod-Domar and Solow-Swan model.

The Solow-Swan model seemed to provide a reasonable characterization of capital accumulation in a number of respects. *But* contrary to evidence, it predicted that there should be no long run growth in y or k. It also drastically under-predicts the amount of cross country variation in y.

To deal with these shortcomings, we generalized the model by allowing the efficiency of labor to grow over time. Expressing labor in efficiency unit s (L^*), we wrote $L^* = L\lambda_L$, and $Y = F(K, L^*)$, with $\Delta \ln L^* = \Delta \ln L + \Delta \ln \lambda_L = n + \mu$ .This generated a new characterization of capital deepening:



Now $\Delta \ln Y = \Delta \ln K = \Delta \ln L^* = n + \mu > \Delta L = n$, so per capita income grows at the rate μ.

<u>What is the relationship between μ and A?</u>
Productivity growth—that is, growth in A—might reflect many things. One possibility is that growth in A reflects growth in the quality of the labor force. Suppose that this is the *only* source of productivity gain in an economy. Then, returning to the Cobb-Douglas case, and measuring labor in efficiency units, we have:

$$Y = K^{\alpha}(L^*)^{1-\alpha} = \left(\lambda_L^{1-\alpha}\right) K^{\alpha} L^{1-\alpha}.$$

which takes the same form as $Y = A \cdot F(K, L)$ So one can think of $\lambda_L^{1-\alpha}$ as playing the role of A. That is, variation in labor quality is one reason that output might vary, holding capital and the number of workers constant. Or put differently, when productivity growth is due exclusively to improvements in the quality of the labor force, *growth* in A is the same as (1-α) times growth in labor efficiency (λ_L): $\Delta \ln A = (1-\alpha)\Delta \ln \lambda_L = (1-\alpha)\mu$.

<u>Example</u>
Suppose $\Delta\ln L = 0.02$, $\Delta\ln K = 0.04$, $\Delta\ln y = 0.02$, and capital's share of income is 0.4. The growth in output per capita could be decomposed into growth in labor force participation rates, capital deepening and productivity growth.

$$\Delta \ln y = \Delta \ln A + \alpha \cdot \Delta \ln k \text{, or } \Delta \ln y = \Delta \ln A + \alpha \cdot (\Delta \ln K - \Delta \ln L)$$

Substituting into this equation, and recalling that α is equal to capital's share of income,

$$0.02 = \Delta\ln A + 0.4\cdot(0.04 - 0.02)$$

So we can solve for $\Delta\ln A = 0.03 - 0.4\cdot(0.04 - 0.02) - 0.01 = 0.012$. That is, these figures imply 1.2 percent growth in productivity. Further, if all productivity growth is due to improvements in the efficiency of labor, then $\Delta \ln A = (1-\alpha)\Delta \ln \lambda_L$, so labor productivity must be growing at 0.012 / (1 – 0.4) = 0.02, or two percent per annum.

Even if education doesn't affect growth rates much, might not it still account for the productivity differences across countries?

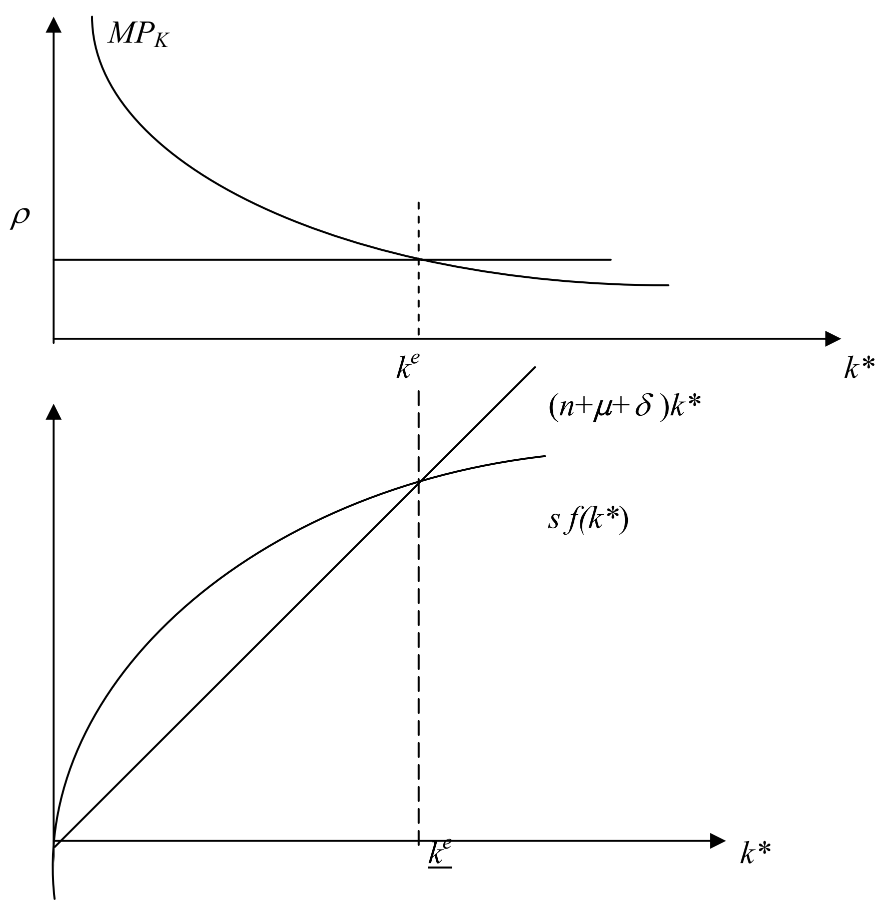
Adjusting for this certainly helps, but there is a big unexplained productivity differential left over. Much of the remainder of the course will be about these other factors.

Other limitations of the model: It says nothing about the determinants of s, n or μ.

Revisions of the model that allow policy to influence productivity growth are sufficiently radical that they aren't really considered the Solow-Swan model anymore. The same is true for endogenous population growth rates. We won't get into these now. We will, however, allow for endogenous savings behavior.

Endogenous savings In the 1960s, Ramsey, Cass and Koopmans showed that the Solow-Swan model could be modified to treat the savings rate as an endogenous variable. Here's the intuition (without depreciation, which complicates things).

Suppose people discount the future at rate ρ, so they are indifferent between a unit of consumption today and 1+ρ units next year. Then whenever they can earn a rate of return above ρ by saving, they will do so. Or, they will put their income into saving (and hence accumulate capital) up to the point where the return on capital —which is simply the marginal product of capital—no longer exceeds the discount rate. (See Easterly article, figure 5, first panel.) Savings rates adjust to fix the equilibrium *k:*



Questions to discuss

- What is the effect of an increase in the discount rate on the savings rate and the equilibrium per capita income?

- What happens to savings rates when the rate of population growth rises? (raises the marginal product of capital, savings rise to restore same k unless the discount rate changes).

- Since higher savings rates imply higher per capita income, is it possible to save too much?

Our next objective is to investigate: What might be causing A to grow besides improvements in the quality of the labor force?