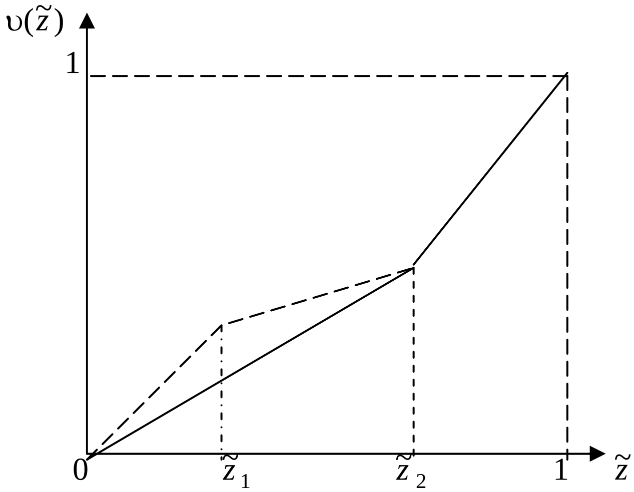
Economics 507A (International Trade)
Second Problem Set
Suggested answers

1. In the Dornbusch, Fischer and Samuelson (1977) model, suppose the $\upsilon(\tilde{z})$ function changes for all consumers from the solid curve to the broken curve below. (Recall that this function indicates the fraction of total income spent on goods with index $z \leq \tilde{z}$.)



a) At unchanged prices, for which goods, if any, has demand increased and for which goods has demand decreased.

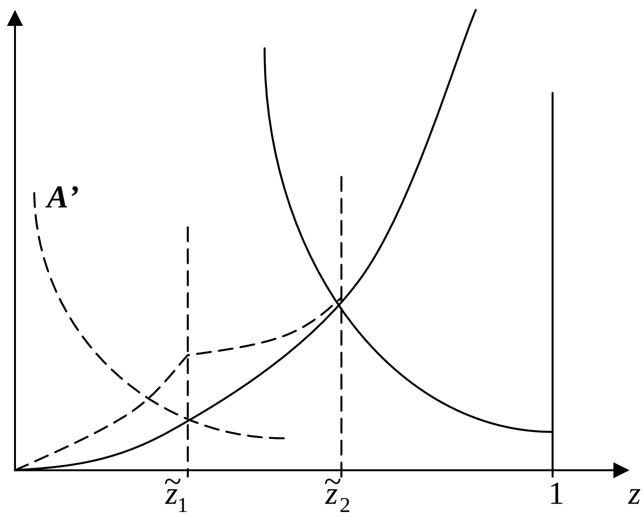
The demand for goods in any interval $[z_1, z_2]$ is $\upsilon(z_2) - \upsilon(z_1)$, so the change in the shape increases demand for goods on the interval $[z_0, z_1]$ interval, reduces demand for goods on the $[z_1, z_2]$ interval, and does not affect demand for goods on the interval $[z_2, 1]$

b) Determine the effect of this change on free trade equilibrium prices of all goods relative to labor in both countries and on the pattern of specialization and trade. Assume, in the initial equilibrium, that $\tilde{z}_2$ marks the division between exports and imports.

The function $B(z) = \left(\frac{\upsilon(z)}{1-\upsilon(z)}\right)\frac{L^*}{L}$ shifts up wherever $\upsilon(z)$ does, although the transformation is non-linear. Thus the equilibrium point is unaffected if $\tilde{z}_2$ marks the division between imports and exports. Graphically, the result is represented below by the dotted lines departures from the original *B* schedule:

ω

A ***B***



c) Repeat part (b), letting $\tilde{z}_1$ be the initial division between exports and imports.

Clearly, if the original intersection of *A* and *B* had been above $\tilde{z}_1$, the new equilibrium would be at a higher ω and a lower $\tilde{z}$. Such a demand shift toward home country goods would have caused the home country to produce fewer goods but collect higher relative wages. (Refer to the intersection of *A’* with the new *B* schedule.) Prices of the home goods would have remained the same in terms of home country wages because technologies didn’t change, but the increase in ω would have meant that prices of the goods below $\tilde{z}_1$ were more expensive relative to foreign wages. As for goods above $\tilde{z}_1$, these would have remained the same price in terms of foreign wages but become cheaper in terms of home country wages.

2. Prove that consumers who choose consumption to maximize $U = \int_0^1 b(z)\ln[C(z)]dz$, subject to $Y = \int_0^1 P(z)C(z)dz$, spend the fraction $b(z)$ of their income on good z, regardless of prices (Assume that $b(z) \geq 0$ and $\int_0^1 b(z)dz = 1$.)

This maximization problem can be restated as unconstrained optimization using a LaGrangian:

$$L = \int_0^1 b(z)\ln C(z)dz + \lambda\left(Y - \int_0^1 P(z)C(z)dz\right) =$$
$$\int_0^1 \left[b(z)\ln C(z) + \lambda\left(Y - P(z)C(z)\right)\right]dz$$

In this form, the problem is simply a degenerate case of a control problem in which there are no equations of motion and the Hamiltonian is simply:

$$H = b(z)\ln C(z) + \lambda\left(Y - P(z)C(z)\right)$$

First-order conditions require $\frac{\partial H}{\partial C(z)} = 0 \quad \forall z$ **, so** $\frac{b(z)}{C(z)} = \lambda P(z) \quad \forall z$ **, or consumers spend** $\frac{b(z)}{\lambda} = P(z)C(z)$ **on each good. To find** λ **, integrate both sides over the unit interval to obtain** $\int_0^1 \frac{b(z)}{\lambda} dz = \int_0^1 P(z)C(z)dz = Y$ **, or using** $\int_0^1 b(z)dz = 1$, $\lambda = Y^{-1}$ **. Thus the share of income spent on good z is simply** $b(z) = \frac{P(z)C(z)}{Y}$ **.**

3. Prove that consumers who choose the consumption bundle that maximizes $U = \left(\int_0^1 C_j^{\rho} dj\right)^{\frac{1}{\rho}}$ subject to $Y = \int_0^1 P_j C_j dj$ can be characterized by the demand system $C_j = \frac{P_j^{-\sigma} Y}{\int_0^1 P_i^{1-\sigma} di}$, where $\sigma = \frac{1}{\rho - 1}$.

   Maximizing *U* is the same thing as maximizing U^{ρ} **so we will do the latter. By the same logic used in question 6, the LaGrangean for this optimization problem is:**

   $$L = \int_0^1 C_j^{\rho} dj + \lambda\left(Y - \int_0^1 P_j C_j dz\right) = \int_0^1 \left[C_j^{\rho} + \lambda\left(Y - P_j C_j\right)\right] dz$$

   The Hamiltonian is $C_j^{\rho} + \lambda\left(Y - P_j C_j\right)$ **so the first-order condition is** $\frac{\partial H}{\partial C_j} = \rho C_j^{\rho - 1} - \lambda P_j = 0 \quad \forall z$ **, or solving for** C_j **,** $C_j = \left(\frac{\lambda P_j}{\rho}\right)^{1/(\rho-1)}$ **. To find** $\left(\frac{\lambda}{\rho}\right)^{1/(\rho-1)}$ **, multiply the first-order condition by** C_j **and integrate:** $\rho \int_0^1 C_j^{\rho} dj = \lambda \int_0^1 C_j P_j dj = \lambda Y$ **, then substitute for** C_j **in terms of** λ **and prices:**

   $\left(\frac{\lambda}{\rho}\right)^{1/(\rho-1)} \int_0^1 P_j^{\rho/(\rho-1)} dj = Y$ **, or** $\left(\frac{\lambda}{\rho}\right)^{1/(\rho-1)} = Y / \left(\int_0^1 P_j^{\rho/(\rho-1)} dj\right)$ **. Substituting this into our expression for** C_j **, and using** $\sigma = \frac{1}{\rho - 1}$ **yields** $C_j = \frac{P_j^{-\sigma} Y}{\int_0^1 P_i^{1-\sigma} di}$.

4. In their multi-country Ricardian model, Eaton and Kortum (2001) assume that production requires only labor and intermediate inputs. How might capital be added to the model without changing its basic features?

A trivial way to introduce capital would be to assume that primary factor services are included in the bundle of inputs that all sectors use. The unit cost function would become homogeneous of degree 1 in the three types of factor prices, for example: $c_i = w^{\beta_1} r^{\beta_2} P_i^{1-\beta_1-\beta_2}$ **. This really adds nothing conceptually to the model, but it changes the calculations of production costs in a way that could, in principle, affect the measurement of productivity. (Note that, barring international capital movements, capital-abundant countries would have relatively cheap unit input bundles, *ceteris paribus*, so their high market shares would no longer have to be attributed to superior technology.) It also raises the question of how capital accumulates, which is something the authors do not wish to deal with. Allowing for differences in factor intensities across sectors, as in the HOS model would make the model intractable. A continuum of goods distinguished in two dimensions (technological sophistication and capital intensity) is unlikely to yield closed-form solutions.**