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Economics 507a: International Trade
Lecture 3

II. The simple mechanics of the 2 x 2 Hecksher-Ohlin model

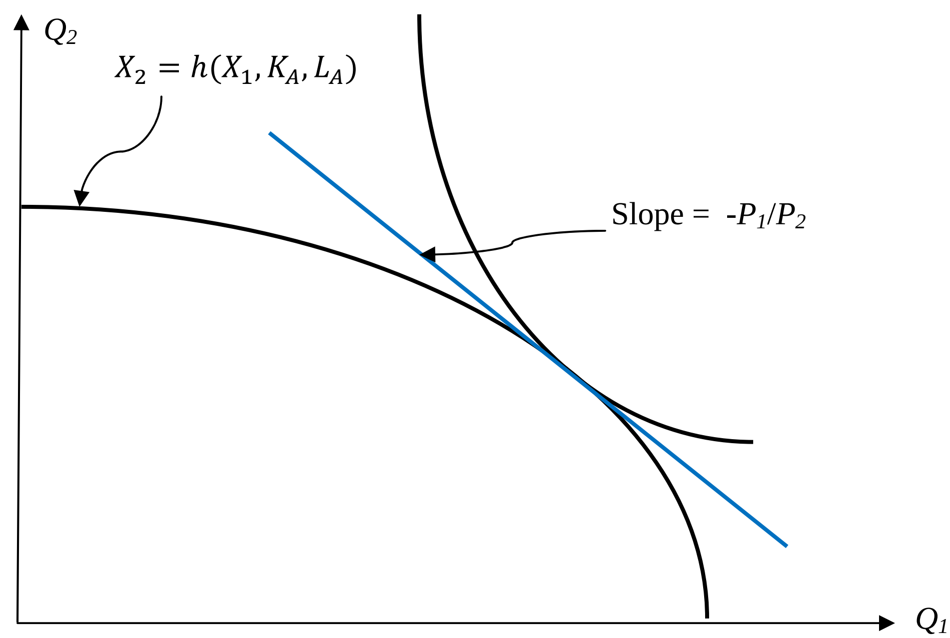
<u>Last time</u>:

<u>The production possibility frontier</u>
A competitive economy get maximum amount of one good, given any particular amount of the other good and resource endowments: $X_2 = h(X_1, K, L)$ Further, entrepreneurs choose to produce the mix of output that has maximum market value:

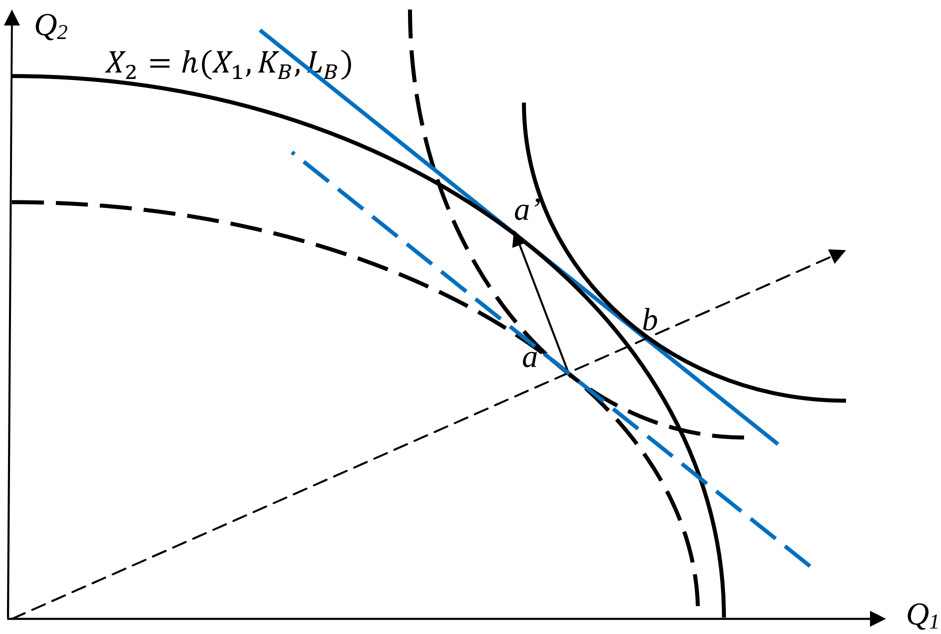
$max_{X_1}(P_1X_1 + P_2X_2)$ subject to $X_2 = h(X_1, K, L)$ implies: $\frac{P_1}{P_2} = \frac{-\partial h}{\partial X_1} = \frac{-\partial X_2}{\partial X_1}$.

<u>Consumer behavior</u>
Because we have assumed identical homothetic preferences, we can hereafter represent consumer choices with a utility function (and associated set of indifference curves) for a representative consumer, and we can act as if this single consumer has a budget that matches national income.

<u>Autarky prices</u> (that is, prices in the absence of trade) must therefore match the slope of the PPF at a point of tangency with an indifference curve for this representative consumer. For country A, this equilibrium might look like:

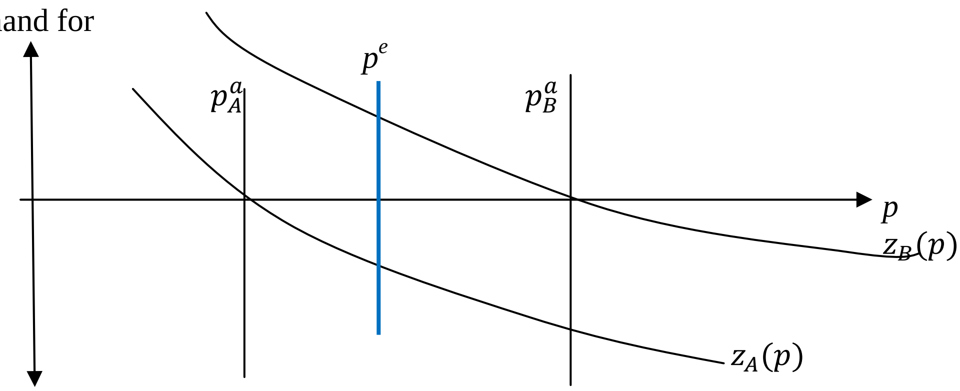


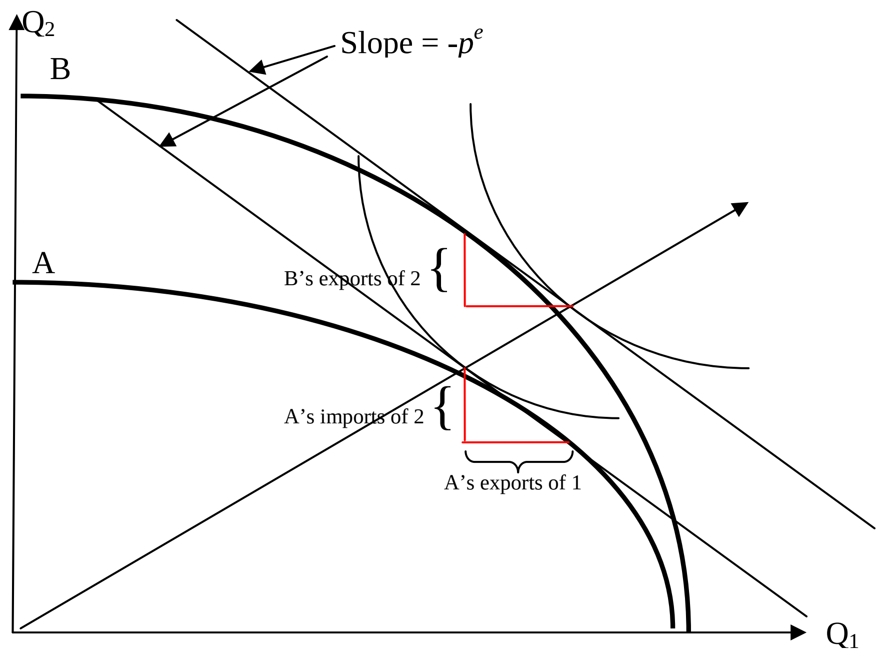
If another country, B, has more capital its autarky equilibrium will be at a higher relative price for the labor-intensive good. (Does it matter whether this country is larger or smaller?)



Prices with free trade

There must be an equilibrium world price ratio, p^e, between the two autarky price ratios at which global markets clear.





If the global market for good 1 clears, then the global market for good 2 must also clear.

New material

Hecksher-Ohlin Theorem (law of comparative advantage)

Version 1: ***A country will export that good which is intensive in its abundant factor, where abundance is determined relative to global endowment proportions.***

Version 2: ***A country will export that good which is relatively cheap in autarky, compared to world prices.***

Version 2 is nearly impossible to test because we don't have many experiments where a country goes from autarky to free trade. In any case, in the context of the model we have laid out, they amount to the same prediction. Version 1 generalizes to many factors.

III. Generalizing to many goods and factors.

Suppose i =1,… N goods and j = 1, …M factors. Let the production function for the i^{th} good be $X_i = f_i(\underline{v}_i)$ where $\underline{v}_i = (v_{i1}, v_{i2}, \ldots, v_{iM})$ is the vector of factor inputs. (Underscores denote vectors.) Assume $f(\,)$ is positive, increasing, concave and homogeneous of degree 1 for all $\underline{v}_i \geq 0$. Let the associated unit cost function be $g_i(\underline{w}) = \min_{v_i \geq 0} \{\underline{w}'\underline{v}_i \mid f_i(\underline{v}_i) \geq 1\}$, where $\underline{w}$ is the vector of factor prices. $g_i(\underline{w})$ will have the same properties as the production function. Perfect competition implies

$$p_i \leq g_i(\underline{w}) \quad i = 1, \cdots, N \tag{1}$$

with equality if good *i* is produced.

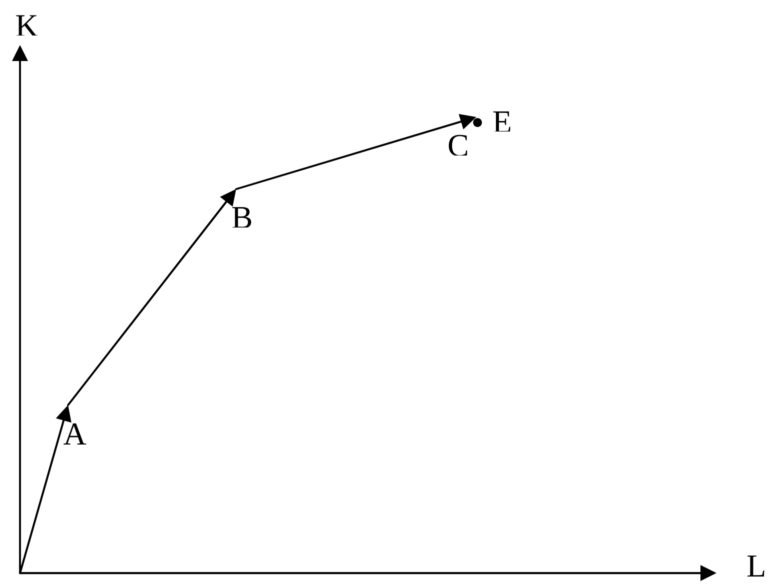
Perfect competition also implies full employment of resources. More precisely, by the envelope theorem, the cost-minimizing vector of input requirements for a unit of output is $\frac{\partial g_i}{\partial \underline{w}} = \underline{a}_i(\underline{w})$, where $\underline{a}_i(\underline{w}) = \left(a_{i1}(\underline{w}), a_{i2}(\underline{w}), \ldots, a_{iM}(\underline{w})\right)'$. Thus the full employment condition is:

$$\sum_i a_{ij}(\underline{w}) X_i = V_j \quad j = 1, \ldots, M, \tag{2}$$

where V_j is the economy-wide stock of factor *j* and y_j is the amount of good *j* produced. Using matrix notation these *M* equations can be written compactly as:

$$A\underline{X} = \underline{V} \tag{2'}$$

In the case of two factors the full employment condition can be depicted graphically as the requirement that the sum of the factor employment rays yields the endowment point. Note that with more goods than factors, there is an infinite number of possible ways the full employment condition might be satisfied at a *given* set of factor prices:



Finally, given that consumers everywhere have identical homothetic tastes, the share of their income they spend on good *i* depends only on the price vector, $\alpha_i(p)$, *i* = 1, … *n*.

We are now prepared to say something about the way that production vectors, Y_i, vary across countries with factor endowments, V_i. This will set the stage for a statement about the relationship between trade flows and factor endowments.

The key observation is that equation 2' describes the link between output vectors and endowment vectors for all countries, so long as all countries share the same factor prices. We thus want to know when this will occur.

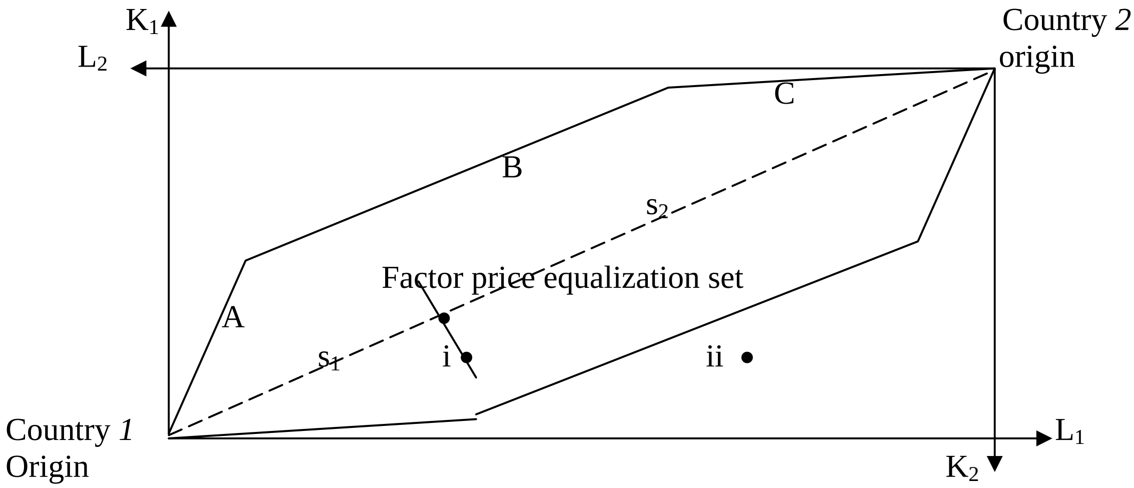
Suppose the world is comprised of countries indexed by $c = 1, \ldots, C$. Also, assume that free world trade ensures that the same product prices prevail everywhere. Then if the set of n equations (1) holds with equality for every country, and if these equations imply a one-to-one mapping from factor price vectors to output price vectors, the same set of factor prices will prevail everywhere (factor price equalization). Further, these factor prices will induce the same factor intensities everywhere. That is, all countries will share the same technology matrix, A.

When does this occur? Think of an *integrated* equilibrium (IE) in which we pool global resources into one giant economy. (Equivalently, think of all factors as perfectly mobile across borders.) There is some vector of market clearing product prices in this equilibrium, an associated vector of factor prices, and a matrix of factor intensities, A.

Now, starting from this IE, imagine that we divide up the world's factor endowment among countries and we prohibit international factor movements. If the division is done in such a way that all countries' factor endowment vectors are identical up to a scalar multiple, then the IE need not be disturbed. Because production technologies are homogeneous of degree 1, a country getting some fraction λ_c of the global endowment, $\underline{V}_c = \lambda_c \underline{V}_W$, could simply produce the same fraction of the global output vector. There would be no effect on global supplies of goods, so no adjustment in prices would be necessary. Thus, so long as the equilibrium vector of output prices implied a unique vector of factor prices, the integrated equilibrium (IE) would be replicated. (Would there be any need for trade?)

More interestingly, when dividing up the world's endowment, it is possible to preserve the IE even if we deviate from proportionate endowments. But if endowments get *too* far from proportionality, it will be necessary for countries to deviate from the IE factor intensities in order to use all of their factor stocks, causing factor prices to deviate too.

Consider the example depicted below with 3 goods (A,B,C), 2 countries (1,2), and 2 factors (K,L). Here the line segments A, B and C represent factor usage in each good when the IE obtains. Any point enclosed by the convex hull represents an endowment division that preserves the IE, because this set of points allows the two countries to replicate the production vectors A,B and C between them.

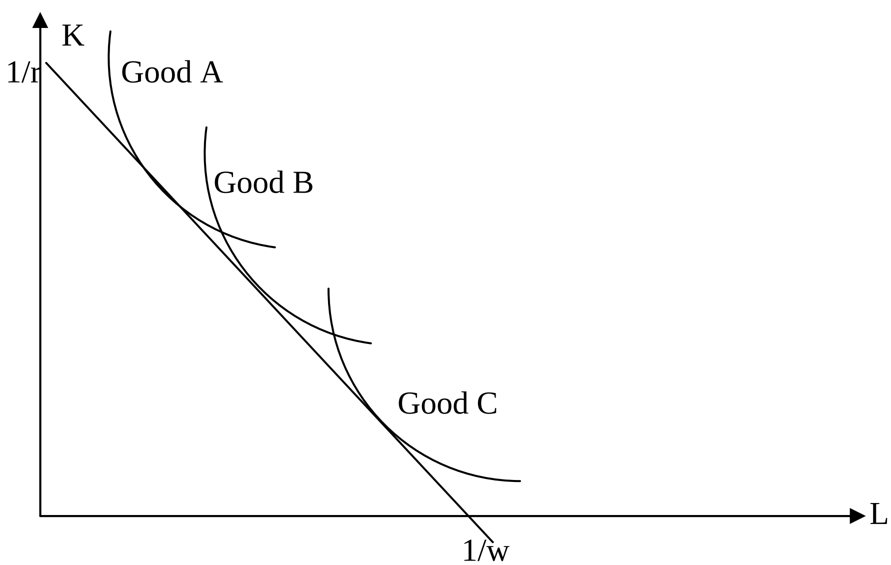


At endowment point (i), is the production mix determinant?
Moving toward (ii), what happens to A's production mix?

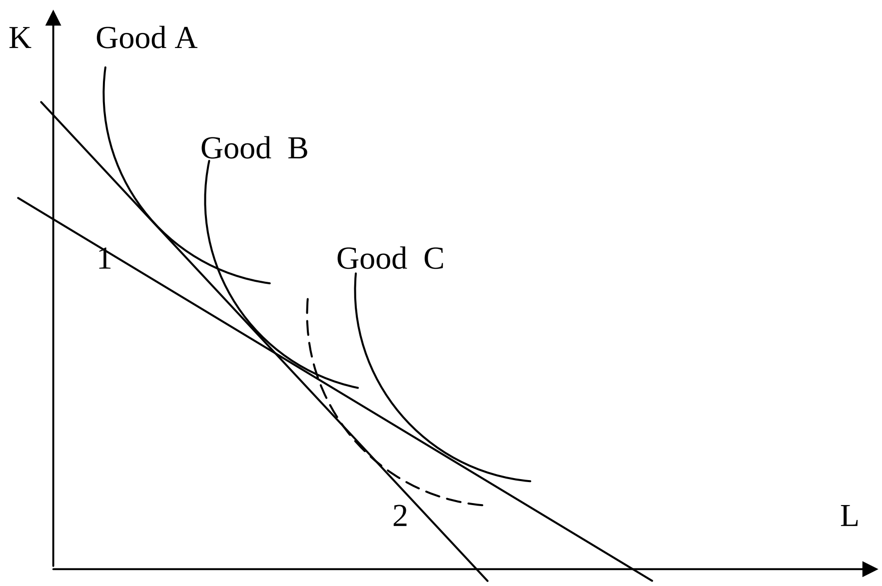
<u>Answer</u>: even if 1 supplies *all* of the most labor-intensive goods demanded, it can't absorb its factor stocks at (ii) using the factor intensities of the integrated equilibrium. Factor prices and output prices will have to adjust, so the integrated equilibrium won't be replicated. Some countries won't be producing the entire set of goods, and the same good will be produced using different factor intensities in different countries.

In short, some major simplifications emerge so long as if countries don't differ too much in their relative factor abundance.

<u>A producers' eye view</u>: In the integrated equilibrium (IE), the same factor prices prevail everywhere, all goods are producible at zero profit in each country, and each uses the same factor intensities:



But suppose that at IE prices, country 1 wants to supply too much of good C to clear the market. Then the relative price of C will adjust until a new equilibrium obtains, and the unit value isoquant will shift, perhaps as below. Now country 1 produces only B and C while 2 produces only A and B, and FPE fails. Note that different countries use different factor intensities to produce the same good (B).



Formally, Helpman and Krugman (1985, chapter 1) describe the set of factor endowments that lead to FPE (with no non-traded goods) as:

$$FPE = \left\{ (\underline{V}_1, \underline{V}_2, \ldots, \underline{V}_C) \mid \exists \lambda_{ic} \geq 0, \sum_{c \in C} \lambda_{ic} = 1 \; \forall i \in I \ni \underline{V}_c = \sum_{i \in I} \lambda_{ic} \underline{V}_{iW} \;\; \forall c \in C \right\}$$

Any factor allocation across *C* countries that leads to FPE must be consistent with non-negative outputs of each good in each country, when (1) global output levels of each good match those of the integrated equilibrium, and (2) each country exhausts its endowments when using production techniques that reflect the integrated equilibrium factor intensities. (V_{iW} is the vector of global factor usage for production of good *i* in the IE, and λ_{ic} is the share of global *i* production done in country *c*.)

Sufficient conditions for FPE are:

i) all *C* countries share the same technology (production functions);
ii) there are at least as many goods as factors;
iii) there are no factor intensity reversals;
iv) at equilibrium prices, *all* goods can be produced in *each* country at zero profit.

The HOS (factor content) theorem with many goods and factors
By assuming identical homothetic tastes everywhere, the HOS model ensures that when identical prices prevail everywhere, all consumers consume goods in the same proportion. And since global markets must clear, this implies that each country consumes goods in the proportions that they are produced globally, with its consumption level determined by its share in global income.

Define
F_{jc} = net export of factor *j* embodied in trade by country *c*

V_{jc} = country *c*'s endowment of factor *j*.

s_c = country *c*'s share in global expenditures.

If factor price equalization obtains, the same factor intensities prevail everywhere, and

Production satisfies: $A\underline{X}_c = \underline{V}_c$
The factor content of consumption is $A\underline{Y}_c = As_c X_W = s_c\underline{V}_W$
The factor content of net exports is thus $A\underline{T}_c = A(\underline{X}_c - \underline{Y}_c) = \underline{V}_c - s_c\underline{V}_W$

where *W* subscripts refer to the entire world. (Is *A* invertible?) The HOS theorem simply states that when factor price equalization obtains, the factor content of trade, $\underline{F}_c = A\underline{T}_c$, is:

$$F_{jc} = V_{jc} - s_c V_{jW} \tag{1}$$

Does this actually hold? Trefler (1995) uses data from 1983 on 33 countries that account for 76 percent of world exports and 79 percent of world trade. (Is it necessary to have all factors?) He defines 9 categories of factors (capital, cropland, pasture, labor—prof. and tech., clerical, sales, service, agric., prod. and transport). To calculate factor content, he uses the U.S. I.O. table from 1983, and data on factor usage by industry from various industry surveys and censuses.

Note that he must calculate the amount of primary factor usage embodied in intermediate goods—this is the role of the I.O. table. Let $B = \{b_{mi}\}$ represent the amount of good *m* needed as an intermediate input to produce one unit of good *i*. Then total intermediate demand for good *m* is $\sum_i b_{mi}G_i$ where G_i is gross production of good *i*. In matrix notation, therefore, the total amount of production that gets consumed in intermediate good production is $B\underline{G}$, and the amount of production left over for final good use is $\underline{Y} = [I - B]\underline{G}$. Or, to produce any vector of final goods for export, $\underline{T}$, one needs $[I - B]^{-1}\underline{T}$ *gross* units of production. If each unit of gross production in sector *i* requires a^*_{ji} units of primary factor *j*, then $A^*[I - B]^{-1}\underline{T} \equiv A\underline{T}$ gives the vector of primary factor inputs used to produce with the final output vector *T*, where $A^* = \{a^*_{ji}\}$.

Trefler (and everyone else except Davis and Weinstein) calculates the factor content of trade in this way, assuming that the U.S. I.O. table and primary factor usage information are equally valid for everyone.

At this point (1) becomes testable. Clearly, because of measurement error and the many abstractions embodied in the theory, (1) will not hold exactly. Call the discrepancy between predicted and calculated net factor content $\varepsilon_{jc} = F_{jc} - (V_{jc} - s_c V_{jW})$. To render these errors roughly homoskedastic across factors, he divides each observation by the square root of $\sigma_j^2 = \sum_c \left(\varepsilon_{jc} - \bar{\varepsilon}_j\right)^2 /(C-1)$. He also assumes that cross-country intra-factor variation in the errors is proportional to the size of the country, and divides by the square root of s_c. So ultimately each observation is scaled by $1/\left(\sigma_j \sqrt{s_c}\right)$.

It is now a straightforward matter to check whether the factor content of trade flows is accurately predicted by the HOS theorem. Three exercises are reported:

1. What fraction of variation in factor contents is "explained" by the HOS theorem? (correlation coefficient is .28)

2. Sign test: What fraction of the data exhibit matching signs on the left-hand and right-hand side? (49.8 percent—exactly random)

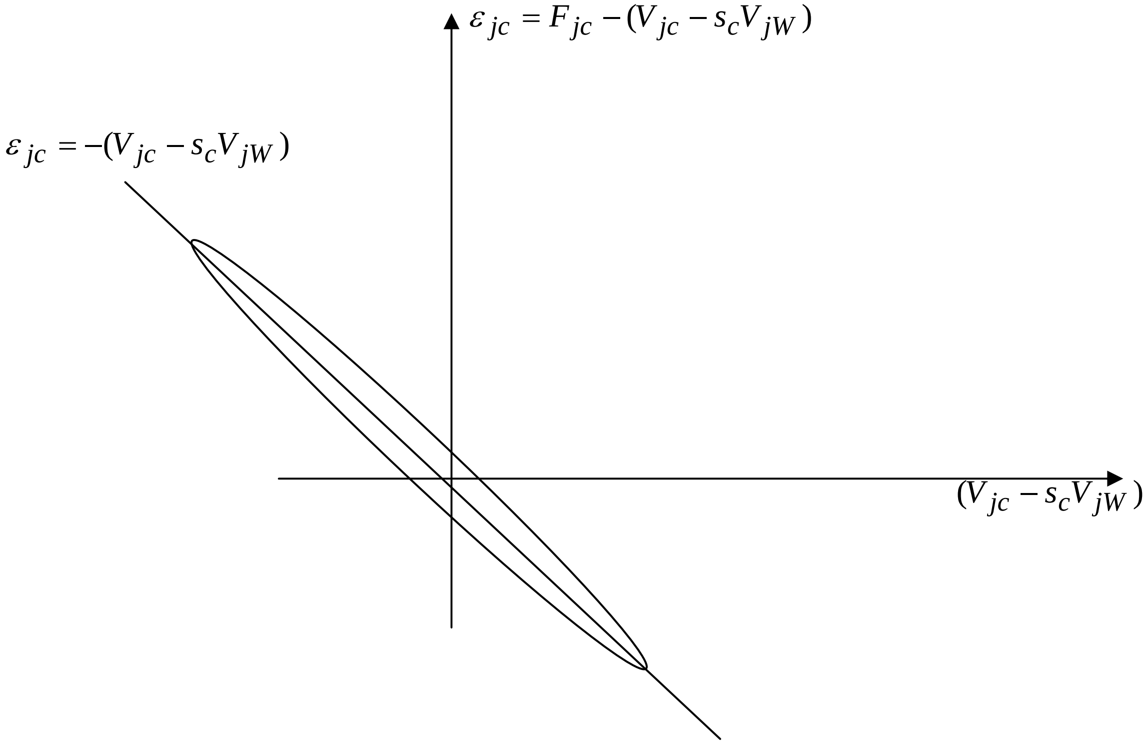
3. After weighting by deviation from zero factor content, $\left|F_{jc}\right| / \sum_{j,c}\left|F_{jc}\right|$, what fraction of signs match? (71 percent)

These results are well known; Trefler only reports them as a point of departure.

<u>An historical note</u>: The first test of factor content was done by Leontief (1953), who pioneered input output analysis. He calculated that exports from the U.S. were more labor intensive than imports, even though the U.S. was (presumably) a relatively capital intensive country. This became known as "Leontief's paradox." We won't worry about it much because there were some problems with "Leontief's methodology (see Leamer, 1980), and he only considered the U.S.

Trefler's (1995) main contribution is to document and analyze a further shortcoming of the theory: the volume of trade is far less than one would predict on the basis of the simple HOS equations.

Plot the residual $\varepsilon_{jc} = F_{jc} - (V_{jc} - s_c V_{jW})$ against the predicted factor content $(V_{jc} - s_c V_{jW})$. One would expect no clear relationship. But in fact, there is a very strong one, with $\varepsilon_{jc} \approx -(V_{jc} - s_c V_{jW})$. (Refer to the diagonal line.) That is, the factor content of trade F_{jc} is very close to zero. "The case of the missing trade."



Almost all residuals lie within the elipse.

In sum the HOS theory grossly over-predicts implicit trade in primary factors, and the flows it predicts are often in the wrong direction. Others have found the same thing, although the "missing trade" point is Trefler's. The question is, why doesn't HOS work?

Trefler added cross-country productivity variations to the model (but not intra-country cross industry).

One possibility is that factor productivity levels differ freely across countries, so that effective factor services flowing from the j^{th} factor in country *c* would be $V^*_{jc} = \pi_{jc} V_{jc}$. Then in terms of effective factor services, the factor content theorem becomes: $F^*_{jc} = \pi_{jc} V_{jc} - s_c \sum_{k=1,C} \pi_{jk} V_{jk}$, where F^*_{jc} is a typical element of $F^*_c = \tilde{A} T_c$ and $\tilde{A}$ is the common technology matrix for all countries when their factors are measured in effective units. This formulation introduces *J*(*C*-1) unknown relative productivity parameters for the system of *JC* equations, but since net factor flows sum to zero across countries for each factor, there are *J*(*C*-1) independent equations. The unknown factor productivities can generally be found that exactly account for trade. Earlier, Trefler (1993) performed this exercise and argued that the resulting productivity parameters are reasonable. But this isn't really a test of the HOS prediction. In Trefler (1995) he imposes the constraint that $\pi_{jc} = \delta_c, \forall c$ (Normalizing U.S. productivity parameters to unity implies $\delta_c A_c = A_{US}$.) The resulting factor content prediction in terms of effective factor services is:

$$F^*_{jc} = \delta_c V_{jc} - s_c \sum_{k=1,C} \delta_k V_{jk} \qquad c = 1, C; \; j = 1, J$$

This won't fit perfectly, so it is necessary to make some assumption about the source of the noise. Trefler assumes measurement error in factor stocks that are *iid* normal, and estimates the system of equations above. (Return to Feenstra table 2.5.) Using neutral technology differences raises the ratio of predicted to actual variance from 0.03 to 0.48. (Not surprising; this version of the model goes part way toward the perfect fit version.)

He also allows for home market bias, due to preferences or transport costs. The result is a model that fits the data better, and predicted factor contents are the right order of magnitude. But as Davis and Weinstein (2003) note, home market bias simply reduces the volume of trade, so obviously it will look better.

Next: Davis and Weinstein (2003)