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Economics 507a: International Trade
Lecture 4

The HOS (factor content) theorem with many goods and factors

Last time
By assuming identical homothetic tastes everywhere, the HOS model ensures that when identical prices prevail everywhere, all consumers consume goods in the same proportion. And since global markets must clear, this implies that each country consumes goods in the proportions that they are produced globally, with its consumption level determined by its share in global income.

Define
F_{jc} = net export of factor *j* embodied in trade by country *c*

V_{jc} = country *c*'s endowment of factor *j*.

s_c = country *c*'s share in global expenditures.

If factor price equalization obtains, the same factor intensities prevail everywhere, and

Production satisfies: $A\underline{X}_c = \underline{V}_c$
The factor content of consumption is $A\underline{Y}_c = As_c X_W = s_c\underline{V}_W$
The factor content of net exports is thus $A\underline{T}_c = A(\underline{X}_c - \underline{Y}_c) = \underline{V}_c - s_c\underline{V}_W$

where *W* subscripts refer to the entire world. (Is *A* invertible?) The HOS theorem simply states that when factor price equalization obtains, the factor content of trade, $\underline{F}_c = A\underline{T}_c$, is:

$$F_{jc} = V_{jc} - s_c V_{jW} \qquad (1)$$

Does this actually hold? Trefler (1995) uses data from 1983 on 33 countries that account for 76 percent of world exports and 79 percent of world trade. (Is it necessary to have all factors?) He defines 9 categories of factors (capital, cropland, pasture, labor—prof. and tech., clerical, sales, service, agric., prod. and transport). To calculate factor content, he uses the U.S. I.O. table from 1983, and data on factor usage by industry from various industry surveys and censuses.

Note that he must calculate the amount of primary factor usage embodied in intermediate goods—this is the role of the I.O. table. Let $B = \{b_{mi}\}$ represent the amount of good *m* needed as an intermediate input to produce one unit of good *i*. Then total intermediate demand for good *m* is $\sum_i b_{mi}G_i$ where G_i is gross production of good *i*. In matrix notation, therefore, the total amount of production that gets consumed in intermediate good production is $B\underline{G}$, and the amount of production left over for final good use is

$\underline{X}$ = $[I - B]\underline{G}$. Or, to produce any vector of final goods for export, $\underline{T}$, one needs $[I - B]^{-1}\underline{T}$ *gross* units of production. If each unit of gross production in sector *i* requires a^*_{ji} units of primary factor *j*, then $A^*[I - B]^{-1}\underline{T} \equiv A\underline{T}$ gives the vector of primary factor inputs used to produce with the final output vector *T*, where $A^* = \{a^*_{ji}\}$.

Trefler (and everyone else except Davis and Weinstein) calculates the factor content of trade in this way, assuming that the U.S. I.O. table and primary factor usage information are equally valid for everyone.

New material

Once we have calculated the factor content of trade for each country, (1) becomes testable. Clearly, because of measurement error and the many abstractions embodied in the theory, (1) will not hold exactly. Call the discrepancy between actual and predicted net factor content $\varepsilon_{jc} = F_{jc} - (V_{jc} - s_c V_{jW})$. To render these errors roughly homoskedastic across factors, he divides each observation by the square root of $\sigma_j^2 = \sum_c \left(\varepsilon_{jc} - \bar{\varepsilon}_j\right)^2 /(C-1)$. He also assumes that cross-country intra-factor variation in the errors is proportional to the size of the country, and divides by the square root of s_c. So ultimately each observation is scaled by $1/\left(\sigma_j \sqrt{s_c}\right)$.

It is now a straightforward matter to check whether the factor content of trade flows is accurately predicted by the HOS theorem. Three exercises are reported:

1. What fraction of variation in factor contents is "explained" by the HOS theorem? (correlation coefficient is .28)

2. Sign test: What fraction of the data exhibit matching signs on the left-hand and right-hand side? (49.8 percent—exactly random)

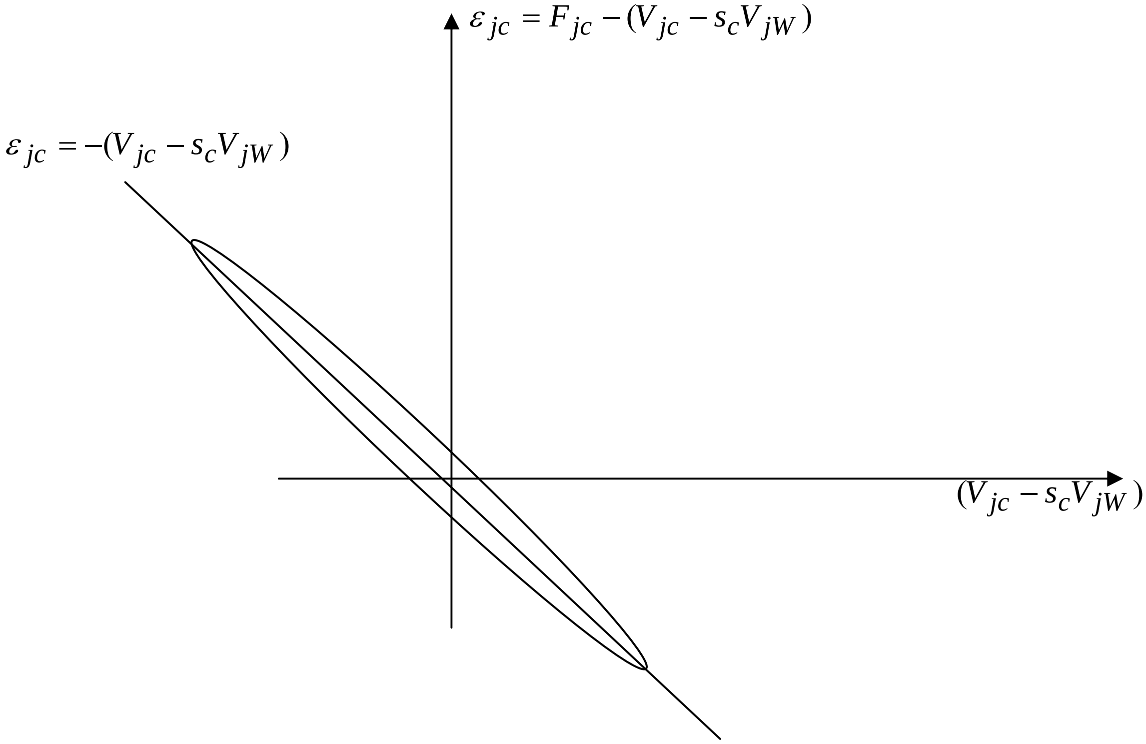
3. After weighting by deviation from zero factor content, $\left|F_{jc}\right| / \sum_{j,c}\left|F_{jc}\right|$, what fraction of signs match? (71 percent)

These results are well known; Trefler only reports them as a point of departure.

> An historical note: The first test of factor content was done by Leontief (1953), who pioneered input output analysis. He calculated that exports from the U.S. were more labor intensive than imports, even though the U.S. was (presumably) a relatively capital intensive country. This became known as "Leontief's paradox." We won't worry about it much because there were some problems with "Leontief's methodology (see Leamer, 1980), and he only considered the U.S.

Trefler's (1995) main contribution is to document and analyze a further shortcoming of the theory: the volume of trade is far less than one would predict on the basis of the simple HOS equations.

Plot the residual $\varepsilon_{jc} = F_{jc} - (V_{jc} - s_c V_{jW})$ against the predicted factor content $(V_{jc} - s_c V_{jW})$. One would expect no clear relationship. But in fact, there is a very strong one, with $\varepsilon_{jc} \approx -(V_{jc} - s_c V_{jW})$. (Refer to the diagonal line.) That is, the factor content of trade F_{jc} is very close to zero. "The case of the missing trade."



Almost all residuals lie within the elipse.

In sum the HOS theory grossly over-predicts implicit trade in primary factors, and the flows it predicts are often in the wrong direction. Others have found the same thing, although the "missing trade" point is Trefler's. The question is, why doesn't HOS work?

Trefler added cross-country productivity variations to the model (but not intra-country cross industry).

One possibility is that factor productivity levels differ freely across countries, so that effective factor services flowing from the j^{th} factor in country c would be $V_{jc}^* = \pi_{jc} V_{jc}$. Then in terms of effective factor services, the factor content theorem becomes: $F_{jc}^* = \pi_{jc} V_{jc} - s_c \sum_{k=1,C} \pi_{jk} V_{jk}$, where F_{jc}^* is a typical element of $F_c^* = \widetilde{A} T_c$ and $\widetilde{A}$ is the common technology matrix for all countries when their factors are measured in effective units. This formulation introduces $J(C\text{-}1)$ unknown relative productivity parameters for the system of JC equations, but since net factor flows sum to zero across countries for each factor, there are $J(C\text{-}1)$ independent equations. The unknown factor productivities can generally be found that exactly account for trade. Earlier, Trefler (1993) performed this exercise and argued that the resulting productivity parameters are reasonable. But this isn't really a test of the HOS prediction. In Trefler (1995) he imposes the constraint that $\pi_{jc} = \delta_c, \forall c$ (Normalizing U.S. productivity parameters to unity implies $\delta_c A_c = A_{US}$.) The resulting factor content prediction in terms of effective factor services is:

$$F^*_{jc} = \delta_c V_{jc} - s_c \sum_{k=1,C} \delta_k V_{jk} \qquad c = 1, C;\ j = 1, J$$

This won't fit perfectly, so it is necessary to make some assumption about the source of the noise. Trefler assumes measurement error in factor stocks that are *iid* normal, and estimates the system of equations above. (Return to Feenstra table 2.5.) Using neutral technology differences raises the ratio of predicted to actual variance from 0.03 to 0.48. (Not surprising; this version of the model goes part way toward the perfect fit version.)

He also allows for home market bias, due to preferences or transport costs. The result is a model that fits the data better, and predicted factor contents are the right order of magnitude. But as Davis and Weinstein (2003) note, home market bias simply reduces the volume of trade, so obviously it will look better.

Davis and Weinstein (2001) go beyond Trefler by:

- allowing the matrix of primary good usage to vary across countries in more general ways (due variously to measurement error, aggregation bias, and/or absence of factor price equalization);

- introducing distance explicitly;

- isolating and quantifying the sources of failure in the HOS theorem with step-wise relaxation of assumptions.

Start from standard HOS specification:

P1: $A^{c'} \underline{X}^c = \underline{V}^c \quad \forall c$; where $A^{c'}$ is a technology matrix common to all countries

T1: $A^{c'} \underline{T}^c = A^{c'} (\underline{X}^c - \underline{Y}^c) = \underline{V}^c - s^c \underline{V}^w \quad \forall c$

Now let the technology matrix exhibit country-specific measurement error:

$\ln A^c = \ln A^\mu + \varepsilon^c$, so that

P2: $A^\mu \underline{X}^c = \underline{V}^c \quad \forall c$

T2: $A^\mu \underline{T}^c = A^\mu (\underline{X}^c - \underline{Y}^c) = \underline{V}^c - s^c \underline{V}^w \quad \forall c$

Next allow for cross-country variation in productivity, as in Trefler:

$A^c = \lambda^c A^\lambda$, where λ^c is a country-specific scalar—the bigger λ^c, the *less* efficient the country.(Thus $\lambda^c = 1/\delta_c$., where δ_c is Trefler's efficiency measure.) This allows us to write endowments in terms of efficiency units, $\underline{V}^{cE} = \frac{1}{\lambda^c}\underline{V}^c$, so the model becomes

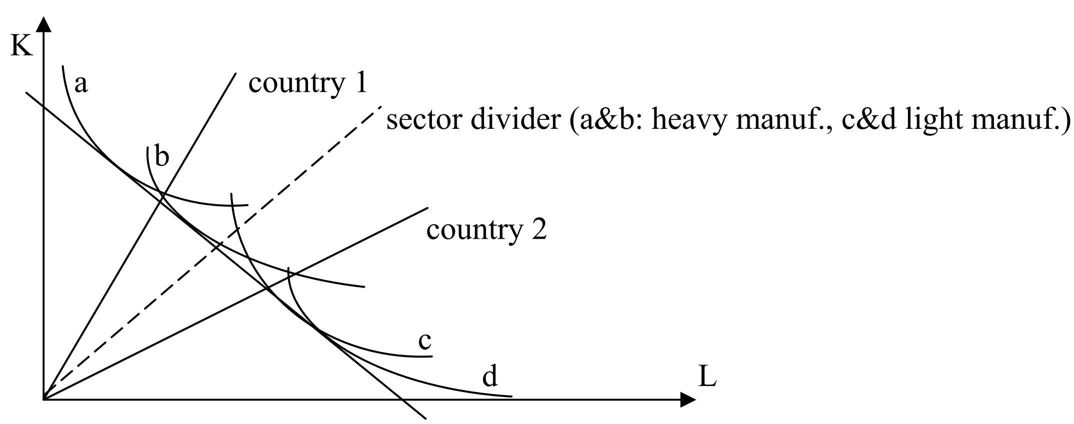
P3: $A^\lambda \underline{X}^c = \underline{V}^{cE} \quad \forall c$

T3: $A^\lambda \underline{T}^c = A^\lambda(\underline{X}^c - \underline{Y}^c) = \underline{V}^{cE} - s^c \underline{V}^{wE} \quad \forall c$

Now recognize intra-industry heterogeneity

Product classifications include many products with distinct factor intensities. Suppose a K-intensive country tends to export goods in K-intensive sector a, import goods in L-intensive sector d, and not trade some goods of intermediate intensity in sectors b and c. Also, suppose that the available data combine a and b into one sector, and c and d into another. Then the *A* matrix of input coefficients for sector 1 is likely to understate the capital intensity of exports because it is a weighted average of K-intensive export goods and less K-intensive non-exported goods. Likewise the matrix of input coefficients for sector 2 will understate the L-intensity of imported goods because it is a weighted average of imported and non-traded goods. This induces a downward-bias in the calculations of the factor content of this country's trade flows. Applying the same *A* matrix to this country's trading partner will also result in a downward bias in net factor trade.

The weighted average input requirements will vary with the country's *K*-intensity because this affects the mix of goods in the tradeable goods sectors, *even* when factor prices equalize. Countries with higher *K*-intensity will tend to emphasize production of more *K*-intensive goods, so even with factor price equalization, *A* varies across countries.



Under these conditions, the correct matrix of input requirements for trade flows is specific to the exporting country. Hence:

P4: $A^{cDFS}\underline{X}^{c} = \underline{V}^{c} \quad \forall c$

T4: $A^{cDFS}\underline{X}^{c} - \left[A^{cDFS}\underline{Y}^{cc} + \sum_{c' \neq c} A^{c'DFS}\underline{M}^{cc'} \right] = \underline{V}^{c} - s^{c}\underline{V}^{w} \quad \forall c$

Here $Y^{cc'}$ is the vector of consumption by country *c* of goods from country *c'*, and with identical homothetic tastes it can be calculated as $s_c X^{c'T}$. It is equal to imports from country *c*' when *c* differs from *c*' ($Y^{cc'} = M^{cc'}$). DFS stands for Dornbush-Fischer-Samuelson because of the notion of a continuum of goods exploited here. (More on DFS later in the course.)

To complete this theory of factor content we need an explicit representation of the way that A^{cDFS} matrices vary across countries. We'll return to this shortly.

Note that the aggregation problem won't occur among non-tradeables because, with FPE, the same techniques are used everywhere and the same output prices will prevail everywhere, so homothetic tastes imply that the non-traded sectors will produce the same bundle everywhere.

<u>Next, suppose *no* factor price equalization</u>

This will lead to country-specific *A* matrices for *all* goods. Partition the output vector into traded and non-traded goods, so that factor usage among the former is the total factor stock, less factor usage among the latter: $A^{cHT}\underline{X}^{cT} = \underline{V}^{c} - A^{cHN}\underline{X}^{cN} = \underline{V}^{cT}$. Further, assume Cobb-Douglas preferences between tradeables and non-tradeables goods, common to all countries, so that s^c is not only country *c*'s share in total expenditures, it is its share in expenditures on tradeables. The model is then:

P5: $A^{cH}\underline{X}^{c} = \underline{V}^{c}$

T5: $A^{cHT}\underline{X}^{cT} - \left[A^{cHT}\underline{Y}^{ccT} + \sum_{c' \neq c} A^{c'HT}\underline{M}^{cc'} \right] = \underline{V}^{cT} - s^{c}\underline{V}^{wT} = (\underline{V}^{c} - s^{c}\underline{V}^{w}) - (\underline{V}^{cN} - s^{c}\underline{V}^{wN})$

where $\underline{V}^{wT} = \sum_{c'} \underline{V}^{c'T}$. That is, we now allow for departures from HOS due to differences in factor usage among non-traded goods. Like version 4, this is not a full theory of trade because it doesn't really tell us how the A^{cH} matrices are determined. We do expect, however, that countries with higher capital intensity will use more capital-intensive techniques for all of their goods.

How should we think about factor intensities when FPE fails? A relatively labor-abundant country will have relatively low wages and will use relatively labor-intensive techniques for its exports. This suggests:

$$\ln a_{ji}^{c} = \theta^{c} + \beta_{ji} + \gamma_{ji} \ln\left(K^{c} / L^{c}\right) + \phi_{ji}^{c}$$

<u>Finally, relax the simple homothetic demand and frictionless trade specification</u>
Suppose demand for imports takes the Cobb-Douglas form:

$$\ln M_i^{cc'} = \alpha_{0i} + \alpha_{1i} \ln(s_i^{cT} Y_i^{c'}) + \delta_i d_{cc'} + \xi_i^{cc'}$$

where $Y_i^{c'}$ is *gross* production of good *i* in country *c*' and $d_{cc'}$ is the distance between countries *c* and *c'*. This is clearly an ad-hoc specification, but Davis and Weinstein argue that it is a useful way to quantify the importance of distance and home market bias, and to gauge how much further gain can be expected from a more complete model. Substituting the fitted values from this expression:

P7: $A^{cH} \underline{X}^{c} = \underline{V}^{c}$

T7: $A^{cH} \underline{X}^{c} - \left[A^{cH} \underline{Y}^{cc} + \sum_{c' \neq c} A^{c'H} \underline{M}^{cc'} \right] = V^{c} - \left[A^{cH} \underline{\hat{Y}}^{cc} + \sum_{c' \neq c} A^{c'H} \underline{\hat{M}}^{cc'} \right]$

<u>Data</u>
IO tables, production by sector, primary factor usage and trade flows for 34 industries in 10 OECD countries. Also have less detailed information on 20 additional countries that are aggregated into the "rest of the world." Factors are limited to capital and aggregate labor. (No natural resources or skill categories.)

<u>Econometric issues</u>
1. Alternative ways to estimate the *A* elements:
   - treating each country as a noisy observation on the same matrix (and allowing for heteroskedasticity).
   - allowing the scale of *A* to vary across countries (i.e., estimating the "efficiency units" parameters)
   - allowing the structure of *A* to depend upon capital labor ratios among traded goods.
   - allowing the structure of *A* to depend upon capital-labor ratios among all goods in the same way, all sectors
   - allowing the structure of *A* to depend upon capital-labor ratios among all goods in different ways for different sectors.

   The most general specification is: $\ln a_{ji}^{c} = \theta^{c} + \beta_{ji} + \gamma_{ji} \ln\left(K^{c} / L^{c}\right) + \phi_{ji}^{c}$

where ϕ_{ji}^{c} is treated as a heteroskedastic disturbance. Feenstra (2004, chapter 3) provides a clear explanation of this specification.

2. Estimating the demand equation:

$$\ln M_i^{cc'} = \alpha_{0i} + \alpha_{1i} \ln(s_i^{cT} I_i^{c'}) + \delta_i d_{cc'} + \xi_i^{cc'}$$

Distance is measured simply as geographic distance, and distance to the compound ROW region is measured as the GDP-weighted distance between the ROW countries and country *c*.

Results
2 basic kinds of comparisons:

- predicted versus actual factor usage in production (P)
  slope test: regression of measured factor usage on predicted usage
  median error test: absolute value of deviation over actual factor usage

- predicted versus actual factor content of trade flows (T)
  sign test: what fraction of net factor exports have predicted sign?
  slope test: regress measured on predicted net factor exports
  variance ratio test: variance in measured net factor exports over variance in predicted net factor exports

The benchmark model (T1, P1)

P1 fails, mainly because the ROW is so different from the OECD (figure 1). With ROW, slope coefficient is .24, without it, .67. Median prediction error is 34 percent for capital and 42 percent for labor.

T1 fails miserably, as usual. Signs correct 32 percent of time. Variance ratio .0005! Slope is –0.002. This isn't just a consequence of including ROW (see figure 2).

Using the average technology matrix (T2, P2)

With all data points included, P2 slope is .33; using only the OECD it is 1.27, reflecting the influence of high productivity in the U.S. Throwing out the U.S., the slope is .90—not too bad (figure 3).
T2 still fails miserably. Signs right 45 percent of the time. slope –0.006, variance ratio essentially zero.

Hicks-neutral, country-wide technology differences (P3, T3)

For P3 there are still prediction errors, especially for the ROW, but these are less important. The median error is down to about 7 percent.

One would expect, then that HOS fails partly because of the identical technology (or homogeneous factor) assumption. But T3 doesn't really improve. Variance ratio is .008, slope coefficient essentially zero, correct signs 50 percent of the time

Aggregation bias model with FPE (P4, T4)

P4 slope remains at 0.89, median prediction error falls sightly to 5 percent.

T4 improves more dramatically (figure 6). Correct signs rises to 86 percent; slope coefficient rises to 0.17; variance ratio rises to 0.07. This alternative way to parameter *A* differences (compare to Trefler and to Harrigan) seems useful.

No-FPE model: *The A matrix varies for non-traded goods too (P5,T5)*

P5 slope rises to .97, R^2 becomes .997; median prediction error is 3 percent.

T5: signs are correct 86 percent of the time; slope coefficient rises to 0.43; variance ratio rises to 19 percent. The assumption of FPE seems to matter.

Alternative imputation of ROW technology (T6)

Thus far the *A* matrix has been imputed from element by element regressions, plugging in the capital intensity of the ROW. One could alternatively force $A^{ROW} X^{ROW} = V^{ROW}$ somehow. (They don't say what they do to ensure this—the possibilities are infinite.) This improves T5 performance: the slope becomes .59, the trade variance ratio doubles to .38. "A more realistic assessment of the labor intensity of the ROW production materially improves the results."

Adding gravity to the HOS model (T7)

T7: the slope coefficient increases from .59 to .82, variance ratio rises to .7, percent of correct signs rises over 90 percent. The implication is that HOS fails in significant part because it neglects home market bias, due to distance, commercial policy, or preferences. (Trefler found this too but they neglect to give him any credit.)

<u>punch line</u>: *Systematic variation in technologies (with capital intensity) and home market bias are the main causes of failure of the HOS theorem.*