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Economics 507a: International Trade
Lecture 5

The HOS (factor content) theorem with many goods and factors

Last time: **Davis and Weinstein** (AER, 2001)

Start from standard HOS specification:

P1: $A^{c'}\underline{X}^{c}=\underline{V}^{c}\quad\forall c$; where $A^{c'}$ is a technology matrix common to all countries

T1: $A^{c'}\underline{T}^{c}=A^{c'}(\underline{X}^{c}-\underline{Y}^{c})=\underline{V}^{c}-s^{c}\underline{V}^{w}\quad\forall c$

Technology matrix exhibits country-specific measurement error: $\ln A^{c}=\ln A^{\mu}+\varepsilon^{c}$

P2: $A^{\mu}\underline{X}^{c}=\underline{V}^{c}\quad\forall c$

T2: $A^{\mu}\underline{T}^{c}=A^{\mu}(\underline{X}^{c}-\underline{Y}^{c})=\underline{V}^{c}-s^{c}\underline{V}^{w}\quad\forall c$

Cross-country variation in productivity, as in Trefler: $A^{c}=\lambda^{c}A^{\lambda}$, $\underline{V}^{cE}=\frac{1}{\lambda^{c}}\underline{V}^{c}$,

P3: $A^{\lambda}\underline{X}^{c}=\underline{V}^{cE}\quad\forall c$

T3: $A^{\lambda}\underline{T}^{c}=A^{\lambda}(\underline{X}^{c}-\underline{Y}^{c})=\underline{V}^{cE}-s^{c}\underline{V}^{wE}\quad\forall c$

Aggregation bias

P4: $A^{cDFS}\underline{X}^{c}=\underline{V}^{c}\quad\forall c$

T4: $A^{cDFS}\underline{X}^{c}-\left[A^{cDFS}\underline{Y}^{cc}+\sum_{c'\neq c}A^{c'DFS}\underline{M}^{cc'}\right]=\underline{V}^{c}-s^{c}\underline{V}^{w}\quad\forall c$

No factor price equalization

P5: $A^{cH}\underline{X}^{c}=\underline{V}^{c}$

T5: $A^{cHT}\underline{X}^{cT}-\left[A^{cHT}\underline{Y}^{ccT}+\sum_{c'\neq c}A^{c'HT}\underline{M}^{cc'}\right]=\underline{V}^{cT}-s^{c}\underline{V}^{wT}=(\underline{V}^{c}-s^{c}\underline{V}^{w})-(\underline{V}^{cN}-s^{c}\underline{V}^{wN})$

Non-homothetic demand with home bias: $\ln M_{i}^{cc'}=\alpha_{0i}+\alpha_{1i}\ln(s_{i}^{cT}X_{i}^{c'})+\delta_{i}d_{cc'}+\xi_{i}^{cc'}$

P7: $A^{cH}\underline{X}^{c}=\underline{V}^{c}$

T7: $A^{cH}\underline{X}^{c}-\left[A^{cH}\underline{Y}^{cc}+\sum_{c'\neq c}A^{c'H}\underline{M}^{cc'}\right]=V^{c}-\left[A^{cH}\underline{\hat{Y}}^{cc}+\sum_{c'\neq c}A^{c'H}\underline{\hat{M}}^{cc'}\right]$

New Material:

Data

IO tables, production by sector, primary factor usage and trade flows for 34 industries in 10 OECD countries. Also have less detailed information on 20 additional countries that are aggregated into the "rest of the world." Factors are limited to capital and aggregate labor. (No natural resources or skill categories.)

Econometric issues

1. Alternative ways to estimate the *A* elements:

- treating each country as a noisy observation on the same matrix (and allowing for heteroskedasticity).
- allowing the scale of *A* to vary across countries (i.e., estimating the "efficiency units" parameters)
- allowing the structure of *A* to depend upon capital labor ratios among traded goods.
- allowing the structure of *A* to depend upon capital-labor ratios among all goods in the same way, all sectors
- allowing the structure of *A* to depend upon capital-labor ratios among all goods in different ways for different sectors.

The most general specification is: $\ln a_{ji}^{c} = \theta^{c} + \beta_{ji} + \gamma_{ji} \ln\left(K^{c} / L^{c}\right) + \phi_{ji}^{c}$

where ϕ_{ji}^{c} is treated as a heteroskedastic disturbance. Feenstra (2004, chapter 3) provides a clear explanation of this specification.

2. Estimating the demand equation:

$$\ln M_{i}^{cc'} = \alpha_{0i} + \alpha_{1i} \ln(s_{i}^{cT} X_{i}^{c'}) + \delta_{i} d_{cc'} + \xi_{i}^{cc'}$$

Distance is measured simply as geographic distance, and distance to the compound ROW region is measured as the GDP-weighted distance between the ROW countries and country *c*.

Results

2 basic kinds of comparisons:

- predicted versus actual factor usage in production (P)
  - slope test: regression of measured factor usage on predicted usage
  - median error test: absolute value of deviation over actual factor usage

- predicted versus actual factor content of trade flows (T)
  - sign test: what fraction of net factor exports have predicted sign?
  - slope test: regress measured on predicted net factor exports
  - variance ratio test: variance in measured net factor exports over variance in predicted net factor exports

The benchmark model (T1, P1)

P1 fails, mainly because the ROW is so different from the OECD (figure 1). With ROW, slope coefficient is .24, without it, .67. Median prediction error is 34 percent for capital and 42 percent for labor.

T1 fails miserably, as usual. Signs correct 32 percent of time. Variance ratio .0005! Slope is –0.002. This isn't just a consequence of including ROW (see figure 2).

Using the average technology matrix (T2, P2)

With all data points included, P2 slope is .33; using only the OECD it is 1.27, reflecting the influence of high productivity in the U.S. Throwing out the U.S., the slope is .90—not too bad (figure 3).
T2 still fails miserably. Signs right 45 percent of the time. slope –0.006, variance ratio essentially zero.

Hicks-neutral, country-wide technology differences (P3, T3)

For P3 there are still prediction errors, especially for the ROW, but these are less important. The median error is down to about 7 percent.

One would expect, then that HOS fails partly because of the identical technology (or homogeneous factor) assumption. But T3 doesn't really improve. Variance ratio is .008, slope coefficient essentially zero, correct signs 50 percent of the time

Aggregation bias model with FPE (P4, T4)

P4 slope remains at 0.89, median prediction error falls sightly to 5 percent.

T4 improves more dramatically (figure 6). Correct signs rises to 86 percent; slope coefficient rises to 0.17; variance ratio rises to 0.07. This alternative way to parameterize *A* differences (compare to Trefler) seems useful.

No-FPE model: *The A matrix varies for non-traded goods too (P5,T5)*

P5 slope rises to .97, R^2 becomes .997; median prediction error is 3 percent.

T5: signs are correct 86 percent of the time; slope coefficient rises to 0.43; variance ratio rises to 19 percent. The assumption of FPE seems to matter.

Alternative imputation of ROW technology (T6)

Thus far the *A* matrix has been imputed from element by element regressions, plugging in the capital intensity of the ROW. One could alternatively force $A^{ROW} X^{ROW} = V^{ROW}$ somehow. (They don't say what they do to ensure this—the possibilities are infinite.)

This improves T5 performance: the slope becomes .59, the trade variance ratio doubles to .38. “A more realistic assessment of the labor intensity of the ROW production materially improves the results.”

Adding gravity to the HOS model (T7)

T7: the slope coefficient increases from .59 to .82, variance ratio rises to .7, percent of correct signs rises over 90 percent. The implication is that HOS fails in significant part because it neglects home market bias, due to distance, commercial policy, or preferences. (Trefler found this too but they neglect to give him any credit.)

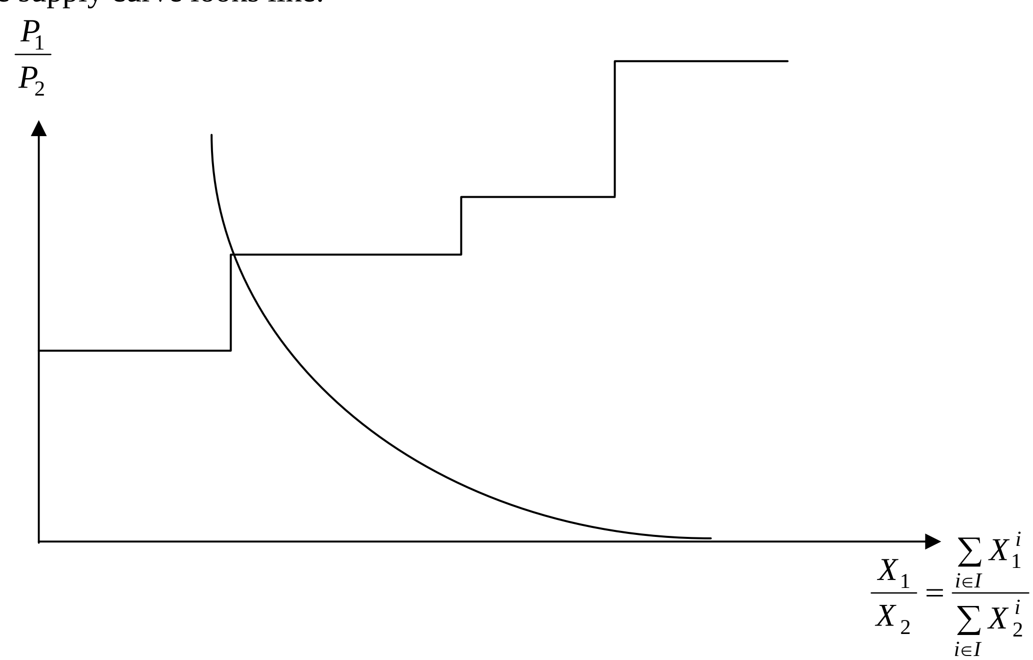
<u>punch line</u>: *Systematic variation in technologies (with capital intensity) and home market bias are the main causes of failure of the HOS theorem.*

2. RICARDIAN TRADE THEORY

Thus far, our models have assumed identical technologies everywhere, and explained trade flows with differences in factor endowments.

There is literature that began much earlier. It explains trade flows with cross country differences in technology.

Simple 2-good case: $X_j^i = L_j^i / a_j^i$, $L^i = \sum_{i=1}^{I} L_j^i$ where *i* indexes country, *j* indexes goods, a_j^i is the amount of labor needed to produce a unit of output *j* in country *i*, and $0 \le L_j^i \le L^i$ is the amount of labor devoted to the production of *j* in country *i*. The global relative supply curve looks like:



To see why, note that the revenue function for the i^{th} country can be written as

$r(p,L\mid a_1^i,a_2^i)=\max_{\alpha}\left\{\frac{P_1 L\alpha}{a_1^i}+\frac{P_2 L(1-\alpha)}{a_2^i}\right\}$. When $\frac{P_1}{a_1}>\frac{P_2}{a_2}$ the expression in brackets is a monotonic positive function of α, so revenue is maximized at α=1 (specialization in good 1). When the inequality is flipped, revenue is maximized at α=0 (specialization in good 2), and when it is an equality, revenue doesn't depend upon the product mix. Thus, if $\frac{P_1}{P_2}<\frac{a_1^i}{a_2^i}$, the i^{th} country will specialize in good 2 But as relative prices rise, it will eventually flip over to good 1. When countries differ in their technologies, more and more will be switched over to good 1 as relative prices rise.

Adding a demand curve, the relative price is determined. It divides countries into two groups—those that produce good 1, and those that produce good 2. (There may be one country that does both.)

The Ricardian trade model lost appeal as people started to wonder *why* technologies varied across countries. The dominant answer that emerged (in the mid 20^{th} century) was there are really multiple factors, and factor endowments differed across countries. This leads to differences in labor productivites (as well as diminishing returns to labor). This, of course, is the foundation for the HOV model.

But the Ricardian model has enjoyed newfound respect in the past 15 years because people have become interested in things other than factor proportions that influence sector-specific productivity—especially learning by doing and R&D. The *a*'s were made endogenous, and trade models with Ricardian feature began to appear again.

Dornbusch, Fischer and Samuelson (AER, 1977)

For that reason, we'll lay the groundwork for modern Ricardian analysis today, and return to it at various points in the course. The standard modern treatment of pure Ricardian trade is found in Dornbush, Fischer and Samuelson (1977). Rather than deal with 2 goods, they allow a continuum of goods, although they focus on the two country case.

Let variables with an asterisk (*) describe the foreign country and those without describe the home country. Then if there were *n* goods, they could be ranked in order declining relative efficiency from the home country's perspective: $\frac{a_1^*}{a_1}>\frac{a_2^*}{a_2}>\cdots>\frac{a_{n-1}^*}{a_{n-1}}>\frac{a_n^*}{a_n}$.

The home country would export the goods in the lower portion of the chain, while the foreign country would export the rest. The cut-off point would be determined by relative country sizes and world demand conditions.

To see how this works, its actually easier to generalize to a continuum of goods indexed on the [0,1] interval, and define the relative labor requirement function: $A(z) = \frac{a_z^*}{a_z}$, $0 \le z \le 1$, $A'(z) < 0$. Letting home and foreign wages be w and w^*, respectively, the commodity z will be produced at home iff $a(z)w \le a^*(z)w^*$, or $\omega \le A(z)$, where $\omega = \frac{w}{w^*}$. Once we know relative wages, we know the trade pattern. How solve?

Note first that each relative wage implies a cut-off value of z at which production shifts from country to the other. Define this cut-off value as $\tilde{z} = A^{-1}(\omega)$, where A^{-1} is a monotonic negative function.

Next, note that if good $z \le \tilde{z}$ is produced at home while good $z'' \ge \tilde{z}$ is produced abroad, competitive product markets ensure that $\frac{P(z)}{P(z'')} = \frac{wa(z)}{w^*a^*(z'')} = \omega \frac{a(z)}{a^*(z'')}$. Also, if goods z and $z' \le \tilde{z}$ are both produced at home, $\frac{P(z)}{P(z')} = \frac{a(z)}{a(z')}$.

To characterize the demand side, assume identical Cobb-Douglas preferences everywhere: $U = \int_0^1 b(z)\ln[C(z)]dz$, where $\int_0^1 b(z)dz = 1$. This implies that consumers spend the share $b(z)$ of their income on good z, regardless of prices: $b(z) = \frac{P(z)C(z)}{Y}$
That is, the elasticity of substitution among goods is unity, so a one percent increase in price leads to a one percent reduction in quantity consumed, leaving expenditure on the good unchanged. (Prove this as a homework exercise using control theory.)

With this simple structure, the fraction of income spent on home goods depends only on the cut-off good: $\upsilon(\tilde{z}) = \int_0^{\tilde{z}} b(z)dz$, where $\upsilon'(\tilde{z}) = b(\tilde{z}) > 0$.

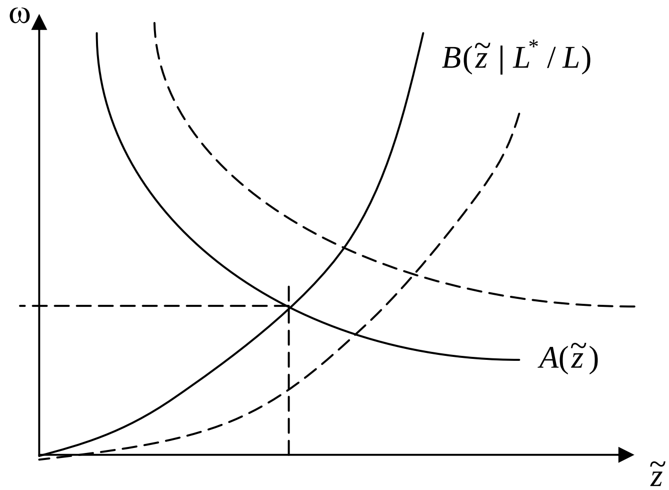
Now we can solve for relative wages. Note that home income must be the share $\upsilon(\tilde{z})$ of global income: $wL = \upsilon(\tilde{z})\left[wL + w^*L^*\right]$, so the relative wage must satisfy:

$$\omega = \left(\frac{\upsilon(\tilde{z})}{1-\upsilon(\tilde{z})}\right)\frac{L^*}{L} \equiv B(\tilde{z} \mid L^*/L)$$

which is a positive function of $\tilde{z}$, and a negative function of relative country size.

If you prefer, you can think of $wL = \upsilon(\tilde{z})\left[wL + w^*L^*\right]$ as a balanced trade condition. That is, the share of home income not spent on home goods (home export supply) must match the share of foreign income spent on home goods (foreign export demand): $wL\left(1-\upsilon(\tilde{z})\right) = \upsilon(\tilde{z})w^*L^*$.

Combining this with our supply side relationship between the relative wage and the cut-off good, equilibrium is determined:



With $\tilde{z}$ determined, all relative prices and trading patterns follow immediately.

<u>Consider:</u>

1. An increase in home country size (pivoting $B(\tilde{z} \mid L^* / L)$ downward). This creates an initial excess supply of home labor, drives down home wages and shifts $\tilde{z}$ to the right because more goods are produced at home at the lower wages. The share of the home country in global consumption must rise with $\tilde{z}$, so the fall in *w* is less than proportionate to the rise in *L*. The home country experiences an increase in real income but a decline in income per capita. The foreign country experiences an increase in both.

2. Neutral improvement in relative productivity at home. This reduces all *a*(*z*) by some fraction shifting the *A*( ) schedule upward, and increasing both the relative wage and the market share of the home country. Real income improves abroad because the purchasing power of foreign income in terms of domestically produced goods is: $w^* L^* / wa(z) = L^* / \overline{\omega} a(z)$ and in terms of home goods it is $w^* L^* / w^* a^*(z) = L^* / a^*(z)$. The former rises for all *a*(*z*) because the equilibrium relative wage rises less than proportionately to the fall in *a*(*z*), while the latter is unaffected. Real incomes improve at home because the purchasing power in terms of home goods is $wL / wa(z) = L / a(z)$, and the purchasing power in terms of foreign goods is $wL / w^* a^*(z) = L\overline{\omega} / a^*(z)$. The former rises as *a*(*z*) falls and the latter rises as $\overline{\omega}$ increases. However, the home country obviously does better than the foreign country because its relative wage increases.

Nontraded goods simply absorb a certain share (1-*k*) of income in each country, leading to a modified balanced trade condition:

$wL\left(1-\upsilon(\tilde{z})-(1-k)\right)=\upsilon(\tilde{z})w^{*}L^{*}$, or $\omega=\left(\dfrac{\upsilon(\tilde{z})}{k-\upsilon(\tilde{z})}\right)\dfrac{L^{*}}{L}\equiv B(\tilde{z}\mid k,L^{*}/L)$. Otherwise the analysis is the same.

Transport costs endogenously create a range of non-traded goods.

Eaton and Kortum (Econometrica, 2002)

Develop a more elaborate Ricardian model with multiple countries, intermediate inputs, and particular distributions for the country-specific productivity levels, good by good. Then they fit their model to data on country characteristics to try to determine the importance of productivity differences in driving trade, as well as transport costs and other factors.

The model focuses on the manufacturing sector, where products are differentiated, but embeds this sector in a very cursory general equilibrium model. We'll limit our discussion to manufacturing.

First, define:

$z_i(j)$ amount of good *j* tht a bundle of inputs can produce in country *i*

c_i cost of a unit bundle of inputs (made up of labor and materials)

$1/d_{ni}$ fraction of exports from *i* to *n* that arrives at *n*: iceburg transport costs. ($d_{ii}=1$)

Then in country *n*, the cost of obtaining good *j* from country *i* is: $p_{ni}(j)=c_i d_{ni}/z_i(j)$

Assuming global markets are competitive, each country will use the lost cost source for each good it consumes, inclusive of shipping expense. The price observed in country *n* will be $p_n(j)=\min\left\{p_{nj}(j);i=1,\ldots,N\right\}$.

In keeping with the micro literature on product/process innovations, let the technology for good *j* in country *i* be a realization on the random variable $Z_i(j)$. Then the probability that country *n* buys good *j* from country *i*, π_{ni} will be the probability that $\dfrac{Z_i(j)}{c_i d_{ni}}=\max\left\{\dfrac{Z_s(j)}{c_s d_{ns}};s=1,\cdots,N\right\}$. This probability has a closed form expression if all $Z_i(j)$ have the Frechet (type II extreme value) distribution:

$$P[Z_i(j) \leq z] = e^{-T_i z^{-\theta}}$$

This might seem an arbitrary assumption, but there are reasons to believe that it is consistent with the innovative processes that determine productivity growth (Kortum, 1997). Note that as *T* increases the probability of a low *Z* realization declines. So T_i is an index of the technological sophistication of country *i*. Also, the rate at which the absolute value of the exponent falls with *z* depends positively on θ. So high θ values lead to steep cumulative distribution functions, or relatively low variance in productivity realizations within countries. (Note that the distribution is common to all goods within a country.)

With this assumption on the distribution of prices, it follows that

$$P[P_{ni}(j) \leq p] \equiv G_{ni}(p) = P\left[p \geq \frac{c_i d_{ni}}{z_i}\right] = P\left[z \geq \frac{c_i d_{ni}}{p}\right]$$
$$= 1 - P\left[z \leq \frac{c_i d_{ni}}{p}\right] = 1 - \exp\left[-T_i \left(\frac{c_i d_{ni}}{p}\right)^{-\theta}\right] = 1 - \exp\left[-\phi_i d_{ni}{}^{-\theta} p^{\theta}\right]$$

where $\phi_i = T_i c_i{}^{-\theta}$ summarizes the productive efficiency of country *i*. Accordingly, the distribution of prices for good *j* faced by country *i* is given by

$$P[P_n(j) \leq p] = G_n(p) = 1 - \prod_{i=1}^{N}\left(1 - G_{ni}(p)\right) = 1 - \exp\left(-\Phi_n p^{\theta}\right)$$

where $\Phi_n \equiv \sum_{i=1}^{N} \phi_i d_{ni}^{-\theta} = \sum_{i=1}^{N} T_i \left(c_i d_{ni}\right)^{-\theta}$. Note that the more countries there are, the bigger is Φ_n and thus the lower the price of good *n*. This is because increases in the number of countries increases the number of draws we compare when finding the cost-minimizing source.