Supplementary Notes on "Contracting for Information with Imperfect Commitment"

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Abstract

These notes derive the solution, in the uniform-quadratic case, for the optimal contract under full commitment. It supplements material contained in Appendix A of the paper.

0.1 Optimal contract with full commitment in the uniform-quadratic case

In the uniform-quadratic case, the Pontryagin conditions (26) to (29) (on page 29 of the paper) are also sufficient since the relevant convexity conditions are satisfied (see for instance, Seierestad and Sydsæter (1987)) and become:

$$\lambda_1' = -\frac{\partial L}{\partial t} = 1 - \mu \tag{1}$$

$$\lambda_2' = -\frac{\partial L}{\partial y} = 2\left(y - \theta\right) - 2\lambda_1 u \tag{2}$$

$$0 = \frac{\partial L}{\partial u} = \lambda_1 2\left(y - (\theta + b)\right) + \lambda_2 \tag{3}$$

$$0 = \mu t \tag{4}$$

The exact solution to (1) to (4) depends on whether $b \leq \frac{1}{3}$ or $b > \frac{1}{3}$.

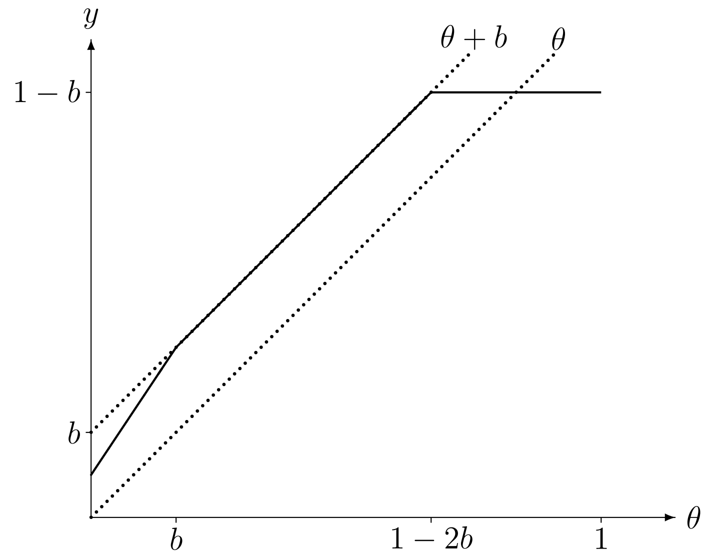

Figure 1: Optimal Contract with Perfect Commitment, $b \leq \frac{1}{3}$

Case 1: $b \leq \frac{1}{3}$. In this case the solution is:

	$0 \leq \theta \leq b$	$b \leq \theta \leq 1-2b$	$1-2b \leq \theta \leq 1$
$y(\theta)$	$\frac{3}{2}\theta + \frac{1}{2}b$	$\theta + b$	$1-b$
$t(\theta)$	$\frac{3}{4}(b-\theta)^2$	0	0
$u(\theta)$	$\frac{3}{2}$	1	0
$\lambda_1(\theta)$	θ	b	$\frac{1}{2}(1-\theta)$
$\lambda_2(\theta)$	$\theta(b-\theta)$	0	$(1-\theta)(\theta-(1-2b))$
$\mu(\theta)$	0	1	$\frac{3}{2}$

When the bias is low, that is, if $b \leq \frac{1}{3}$, the optimal contract has three separate pieces (see Figure 1). In low states, that is when $\theta \leq b$, the project $y(\theta) = \frac{3}{2}\theta + \frac{1}{2}b$ lies between that optimal for the principal ($y^*(\theta) = \theta$) and that optimal for the agent ($y^*(\theta, b) = \theta + b$). As θ increases, the project chosen tilts increasingly in favor of the agent, with a commensurate decrease in the transfer payments. For states between b and $1-2b$, the project that is best for the agent ($y^*(\theta, b) = \theta + b$) is played and no transfers are made. It is as if the project choice were delegated to the agent. The set of feasible projects is "capped" at $\bar{y} = 1-b$. For states above $1-2b$, the project is unresponsive to the state—that is, the agent always chooses project $\bar{y}$ and there is, effectively, pooling over this interval.

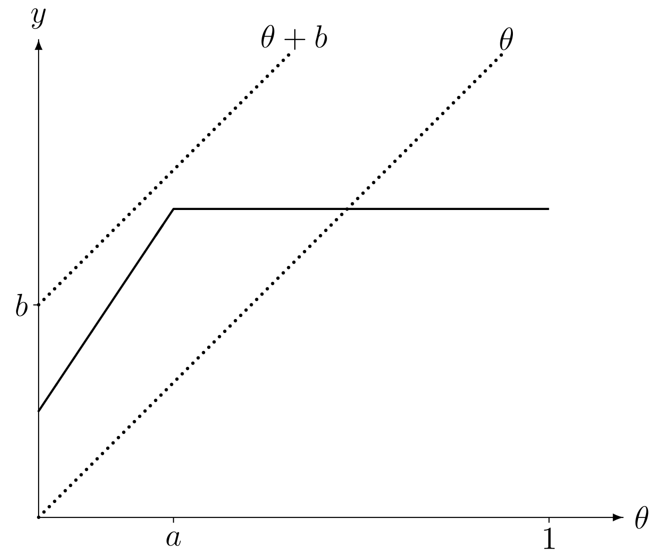

Figure 2: Optimal Contract with Perfect Commitment, $b > \frac{1}{3}$

Case 2: $\frac{1}{3} < b \leq 1$. In this case the solution is:

	$0 \leq \theta \leq a$	$a \leq \theta \leq 1 - 2b$
$y(\theta)$	$\frac{3}{2}\theta + \frac{1}{2}b$	$\frac{3}{2}a + \frac{1}{2}b$
$t(\theta)$	$\frac{3}{4}(b-\theta)^2 - \frac{3}{4}(b-a)^2$	0
$u(\theta)$	$\frac{3}{2}$	0
$\lambda_1(\theta)$	θ	$\frac{(1-\theta)(3a+b-\theta-1)}{3a-b-2\theta}$
$\lambda_2(\theta)$	$\theta(b-\theta)$	$(1-\theta)(1+\theta-b-3a)$
$\mu(\theta)$	0	$2\frac{3(\theta-a)(\theta-2a+b)-3a(1-a)+1-b}{(3a-b-2\theta)^2}$

where $a = \frac{1}{2} - \frac{1}{6}\sqrt{12b-3}$.

When the bias is high, that is, $\frac{1}{3} < b \leq 1$, the optimal contract consists of only two pieces (see Figure 2). In low states, the project again lies between the project ideal for the principal and that ideal for the agent. As in the case when the bias is low, the choice tilts in favor of the agent as the state increases with a corresponding decrease in the transfer payments. The set of feasible projects is again capped, but at a lower level. Indeed, as the agent becomes more biased, the cap decreases; that is, the agent becomes more constrained in her choice of projects. For high states, the agent always chooses the highest feasible project and there is, effectively, pooling over this interval. Unlike the case of low bias, there is no region in which the principal effectively delegates authority to the agent.

For very high biases, that is when $b > 1$, contracting is of no use—the optimal contract is no contract at all.