

SUPPLEMENTARY NOTES for “A Model of Expertise”

Vijay Krishna and John Morgan

May 2001

Abstract

These notes contain some details that were omitted from the body of the paper, “A Model of Expertise,” *Quarterly Journal of Economics*, **116** (2001), pp. 747-775. They also contain a minor correction to the construction in section V.1.

1 Like Biases

1.1 Strategies and Beliefs for Examples

In the body of the paper we had omitted the detailed strategies and beliefs underlying the equilibrium outcome functions in the various examples. These are provided below. The arguments that these constitute equilibria are outlined in the paper.

Example 1 (p. 754) The biases are $b_1 = \frac{1}{40}$ and $b_2 = \frac{1}{9}$. Let $a_0 = 0, a_1 = \frac{1}{180}, a_2 = \frac{22}{180}, a_3 = \frac{61}{180}, a_4 = 1$ and let $y_1 = \frac{1}{360}, y_2 = \frac{23}{360}, y_3 = \frac{83}{360}, y_4 = \frac{241}{360}$. Notice that $y_n = \frac{1}{2}(a_{n-1} + a_n)$ so that y_n is optimal given that $\theta \in [a_{n-1}, a_n]$.

- The two experts’ strategies are:

$$\mu_1(\theta) = \begin{cases} y_1 & \text{if } \theta \in [0, a_1) \\ y_2 & \text{if } \theta \in [a_1, a_2) \\ y_3 & \text{if } \theta \in [a_2, 1] \end{cases}$$
$$\mu_2(\theta, m_1) = \begin{cases} y_1 & \text{if } \theta \in [0, a_1) \text{ and } m_1 = y_1 \\ y_2 & \text{if } \theta \in [a_1, a_2) \text{ and } m_1 = y_2 \\ y_3 & \text{if } \theta \in [a_2, a_3) \text{ and } m_1 = y_3 \\ y_4 & \text{if } \theta \in [a_3, 1] \text{ and } m_1 = y_3 \\ 1 & \text{if } \theta \in [a_1, a_2) \text{ and } m_1 = y_1 \\ y_2 & \text{otherwise} \end{cases}$$

- The posterior beliefs of the decision maker are

$$P(\cdot | m_1, m_2) = \begin{cases} \mathcal{U}[0, a_1] & \text{if } m_1 = m_2 = y_1 \\ \mathcal{U}[a_1, a_2] & \text{if } m_1 = y_2 \\ \mathcal{U}[a_2, a_3] & \text{if } m_1 = m_2 = y_3 \text{ or if } m_1 = y_1 \text{ and } m_2 = 1 \\ \mathcal{U}[a_3, 1] & \text{if } m_1 = y_3 \text{ and } m_2 = y_4 \\ \mathcal{U}[a_1, a_2] & \text{if } m_1 \notin \{y_1, y_3\} \\ \mathcal{U}[0, a_1] & \text{otherwise} \end{cases}$$

where $\mathcal{U}[x, y]$ denotes the uniform distribution over the interval $[x, y]$.

- The decision maker's strategy is:

$$y(m_1, m_2) = \begin{cases} y_1 & \text{if } m_1 = m_2 = y_1 \\ y_2 & \text{if } m_1 = y_2 \\ y_3 & \text{if } m_1 = m_2 = y_3 \text{ or if } m_1 = y_1 \text{ and } m_2 = 1 \\ y_4 & \text{if } m_1 = y_3 \text{ and } m_2 = y_4 \\ y_2 & \text{if } m_1 \notin \{y_1, y_3\} \\ y_1 & \text{otherwise} \end{cases}$$

Example 2 (p. 757) The biases are the same as in Example 1: $b_1 = \frac{1}{40}$ and $b_2 = \frac{1}{9}$. Let $a_1 = \frac{1}{72}$, $a_2 = \frac{23}{180}$ and $a_3 = \frac{41}{120}$. Let $y_1 = \frac{1}{144}$, $y_2 = \frac{17}{240}$, $y_3 = \frac{169}{720}$ and $y_4 = \frac{161}{240}$. Again $y_n = \frac{1}{2}(a_{n-1} + a_n)$ is optimal given that $\theta \in [a_{n-1}, a_n]$.

- The experts' strategies are:

$$\mu_1(\theta) = \begin{cases} y_1 & \text{if } \theta \in [0, a_1) \\ y_2 & \text{if } \theta \in [a_1, a_2) \\ y_3 & \text{if } \theta \in [a_2, 1] \end{cases}$$

$$\mu_2(\theta, m_1) = \begin{cases} y_3 & \text{if } \theta \in [0, a_3) \\ y_4 & \text{if } \theta \in [a_3, 1] \end{cases}$$

- The decision maker's beliefs are:

$$P(\cdot | m_1, m_2) = \begin{cases} \mathcal{U}[0, a_1) & \text{if } m_1 = y_1 \\ \mathcal{U}[a_1, a_2) & \text{if } m_1 = y_2 \\ \mathcal{U}[a_2, a_3) & \text{if } m_1 \notin \{y_1, y_2\} \text{ and } m_2 = y_3 \\ \mathcal{U}[a_3, 1] & \text{if } m_1 \notin \{y_1, y_2\} \text{ and } m_2 = y_4 \\ \mathcal{U}[0, a_1) & \text{otherwise} \end{cases}$$

- The decision maker's strategy is:

$$y(m_1, m_2) = \begin{cases} y_1 & \text{if } m_1 = y_1 \\ y_2 & \text{if } m_1 = y_2 \\ y_3 & \text{if } m_1 \notin \{y_1, y_2\} \text{ and } m_2 = y_3 \\ y_4 & \text{if } m_1 \notin \{y_1, y_2\} \text{ and } m_2 = y_4 \\ y_1 & \text{otherwise} \end{cases}$$

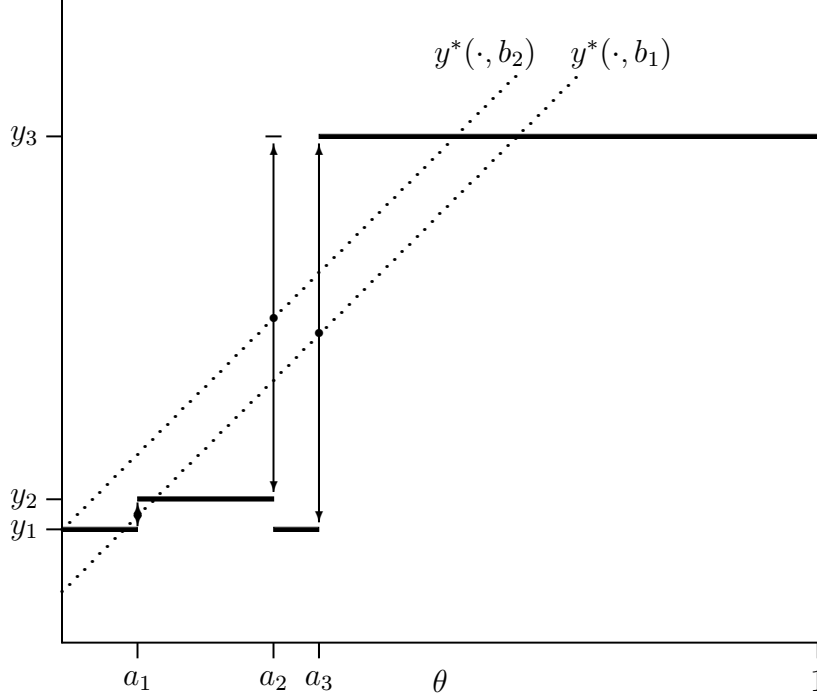


Figure 1: A Non-monotonic PBE

1.2 Example of Non-monotonic Equilibria

Consider the uniform quadratic case with $b_1 = \frac{11}{160}$ and $b_2 = \frac{3}{20}$. A PBE for this game is depicted in Figure 1 below, where the states $a_1 = .1$, $a_2 = .28$, $a_3 = .34$ and the actions $y_1 = .1475$, $y_2 = .19$ and $y_3 = .67$.

The outcome function Y associated with this equilibrium is depicted above and is clearly non-monotonic. Notice that in state a_1 expert 1 is indifferent between y_1 and y_2 ; hence, for all $\theta > a_1$, expert 1 prefers y_2 to y_1 . Likewise, in state a_3 , expert 1 is indifferent between y_1 and y_3 . Finally, in state a_2 expert 2 is indifferent between y_2 and y_3 .

To induce action y_2 , expert 1 must suggest the action $m_1 = y_2$, and expert 2 must agree. If, on the other hand, expert 2 disagrees, action y_3 is induced. Since expert 2 (weakly) prefers y_2 to y_3 if and only if $\theta \leq a_2$, then when expert 1 suggests y_2 , expert 2 will “agree” only if $\theta \leq a_2$. Thus, despite the fact that both experts prefer y_2 to y_1 for $\theta \in (a_2, a_3)$, expert 1 cannot obtain y_2 since expert 2 will then “disagree” and induce the higher action y_3 that he prefers to y_2 . It is the threat of “overshooting” by the more biased expert 2 that sustains the downward jump in the outcome function despite the fact that both experts and the decision maker prefer the higher action y_2 to y_1 when $\theta > a_2$.

Let $y_1 = .1475$, $y_2 = .19$, $y_3 = .67$. Now $y_2 = \frac{1}{2}(a_1 + a_2)$ is optimal given that $\theta \in [a_1, a_2]$. Similarly, $y_3 = \frac{1}{2}(a_3 + 1)$ is optimal given that $\theta \in [a_3, 1]$. The action y_1 is optimal given that $\theta \in [0, a_1] \cup [a_2, a_3]$.

- The experts' strategies are:

$$\mu_1(\theta) = \begin{cases} y_1 & \text{if } \theta \in [0, a_1] \cup [a_2, a_3] \\ y_2 & \text{if } \theta \in [a_1, a_2] \\ y_3 & \text{if } \theta \in [a_3, 1] \end{cases}$$

$$\mu_2(\theta, m_1) = \begin{cases} y_1 & \text{if } m_1 = y_1 \\ y_2 & \text{if } \theta \in [a_1, a_2] \text{ and } m_1 = y_2 \\ y_3 & \text{if } m_1 = y_3 \\ 1 & \text{if } m_1 = y_2 \text{ and } \theta \in [a_2, 1] \\ y_1 & \text{otherwise} \end{cases}$$

- The decision maker's beliefs are:

$$P(\cdot | m_1, m_2) = \begin{cases} \mathcal{U}([0, a_1] \cup [a_2, a_3]) & \text{if } m_1 = y_1 \\ \mathcal{U}[a_1, a_2] & \text{if } m_1 = m_2 = y_2 \\ \mathcal{U}[a_3, 1] & \text{if } m_1 = y_3 \text{ or if } m_1 = y_2 \text{ and } m_2 = 1 \\ \mathcal{U}([0, a_1] \cup [a_2, a_3]) & \text{otherwise} \end{cases}$$

- The decision maker's strategy is:

$$y(m_1, m_2) = \begin{cases} y_1 & \text{if } m_1 = y_1 \\ y_2 & \text{if } m_1 = m_2 = y_2 \\ y_3 & \text{if } m_1 = y_3 \text{ or if } m_1 = y_2 \text{ and } m_2 = 1 \\ y_1 & \text{otherwise} \end{cases}$$

1.3 Sufficient Conditions for Monotonicity with Like Biases

We now turn to establishing sufficient conditions for equilibria to be monotonic when experts have like biases.

Lemma 1 *There exists a $\bar{\theta} < 1$ such that $Y(\cdot)$ is monotone over $[\bar{\theta}, 1]$.*

Proof. Observe that since $U_1(y^*(1), 1) = 0$, and $U_{13} > 0$, $U_1(y^*(1), 1, b_i) > 0$. Let $\bar{\theta} = \inf \{\theta : U_1(y^*(1), \theta, \min\{b_1, b_2\}) > 0\}$. Since $U_1(y^*(1), 1, b_i) > 0$ for $i = 1, 2$, it follows that $\bar{\theta} < 1$. Then for all $\theta > \bar{\theta}$ and all $y \leq y^*(1)$, $U_1(y, \theta, b_i) > 0$ for $i = 1, 2$.

Suppose there exist states θ', θ'' satisfying $\bar{\theta} < \theta' < \theta''$ such that $Y(\theta') = y' > Y(\theta'') = y''$. Suppose (m'_1, m'_2) is sent in state θ' and $y' = y(m'_1, m'_2)$. Similarly, suppose (m''_1, m''_2) is sent in state θ'' and $y'' = y(m''_1, m''_2)$. Observe that since y' and y'' are equilibrium actions $y' \leq y^*(1)$ and $y'' \leq y^*(1)$.

Now suppose that expert 1 deviates and sends message m'_1 in θ'' . Now if expert 2 sends m'_2 , then action y' occurs and, since $\theta'' > \bar{\theta}$, it follows that

$$U(y'', \theta'', b_i) < U(y', \theta'', b_i)$$

for $i = 1, 2$ since $y' \leq y^*(1)$. Thus expert 2's best response to m'_1 in state θ'' must yield him at least $U(y', \theta'', b_2)$. Since $U_1(y', \theta'', b_2) > 0$ this best response, say m''_2 , cannot result in an action $y(m'_1, m''_2) < y'$. Thus $y(m'_1, m''_2) > y'$ and since $U_1(y', \theta'', b_1) > 0$,

$$U(y(m'_1, m''_2), \theta'', b_1) > U(y', \theta'', b_1).$$

Thus it is profitable for 1 to deviate to m'_1 in state θ'' and we have obtained a contradiction. ■

When $b_1 \geq b_2 > 0$, the conclusion of Lemma 3 obtains without the assumption of monotonicity.

Lemma 2 *If $b_1 \geq b_2 > 0$, there are a finite number of equilibrium actions in any PBE.*

Proof. From our earlier results, we know that if $Y(\theta)$ is monotonic, there can be only a finite number of equilibrium actions. We now establish that even if $Y(\theta)$ is not monotonic, the same result obtains.

Let the rightmost interval where $Y(\cdot)$ is continuous be given by $[\theta', \theta'']$ and let the actions in this interval be $[y', y'']$. Suppose that $Y(\cdot)$ is increasing in this interval. Then for these actions to be rationalizable, it must be that for all $\theta \in (\theta', \theta'')$, $Y(\theta) \leq y^*(\theta)$.

Consider a states θ lying in the interval $[\theta', \theta'']$. Suppose that expert 1 deviates in this state and sends $\mu_1(\theta + \varepsilon)$. Since $Y(\theta) < Y(\theta + \varepsilon) < y^*(\theta, b_2) < y^*(\theta, b_1)$ then any response by 2 will lead to some action y such that $U(y, \theta, b_2) \geq U(Y(\theta + \varepsilon), \theta, b_2)$. Since $b_1 \geq b_2$, for all such actions, $U(y, \theta, b_1) \geq U(Y(\theta + \varepsilon), \theta, b_1) > U(Y(\theta), \theta, b_1)$. Hence, this is a profitable deviation for 1 for all states in the interval $[\theta', \theta'']$.

Next, suppose that the rightmost interval where $Y(\cdot)$ is continuous is such that $Y(\cdot)$ is decreasing. Let this interval be $[\theta'', \theta']$. Let $Y([\theta'', \theta']) = [y'', y']$. There are two cases to consider.

(1) First, suppose that for all $\theta \notin [\theta'', \theta']$, $Y(\theta) \notin [y'', y']$. Then, in the interval $[\theta'', \theta']$ the state is fully revealed to the receiver and decreasing actions are not a best response by the receiver.

(2) Second, suppose that there exists an interval $[\bar{\theta}', \bar{\theta}'']$ satisfying $\bar{\theta}'' < \theta''$ over which $Y(\cdot)$ is continuous and increasing and $Y([\bar{\theta}', \bar{\theta}'']) = [y'', y']$ also. For all $\theta \in [\theta'', \theta']$, it must be the case that $Y(\theta) \leq y^*(\theta)$. This is because (almost) all states in this interval are the highest states for which this action is induced. Hence, for all

$\theta \in (\theta'', \theta')$, $U(Y(\theta), \theta, b_i) < U(Y(\theta - \varepsilon), \theta, b_i)$ for $i = 1, 2$. Suppose that expert 1 deviates by sending $\mu_1(\theta - \varepsilon)$ in state θ , then expert 2 will induce an action y such that $U(y, \theta, b_2) \geq U(Y(\theta - \varepsilon), \theta, b_2)$. Since $b_1 \geq b_2$, for all such actions, $U(y, \theta, b_1) \geq U(Y(\theta - \varepsilon), \theta, b_1) > U(Y(\theta), \theta, b_1)$. Hence, this is a profitable deviation for expert 1 for all states in the interval (θ'', θ') . ■

Definition 1 *Preferences are said to be symmetric if*

$$U(y^*(\theta, b_i) - k, \theta, b_i) = U(y^*(\theta, b_i) + k, \theta, b_i)$$

Alternatively, if we can write:

$$U(y, \theta, b_i) = -L(|y^*(\theta, b_i) - y|)$$

where L is increasing.

Definition 2 *U satisfies non-decreasing divergence if $b_i > 0$ implies $y^*(\theta, b_i) - y^*(\theta)$ is non-decreasing in θ .*

Lemma 3 *Suppose preferences are symmetric and satisfy non-decreasing divergence. If $b_1 \geq b_2 > 0$ then all PBE are monotonic.*

Proof. Let θ be the largest state at which there is a downward jump in Y . Such a θ exists since Y is eventually monotone (Lemma 1) and there are only a finite number of equilibrium actions (Lemma 2).

Suppose $b_1 \geq b_2$. As before, in what follows, we will denote $\theta - \varepsilon$ by θ^- and $\theta + \varepsilon$ by θ^+ .

We now argue that we cannot have a “trench,” that is, a situation in which Y has a downward jump at θ and an upward jump somewhere to the right of θ .

First, observe that there does not exist a state $\sigma > \theta$ such that $Y(\sigma) = y^-$. To see this note that if $\tau = \sup\{t : Y(t) = y^+\}$ then by the necessary conditions (3) and (4) in Lemma 1 of the paper, $U(y^+, \tau, b_1) \leq U(y^-, \tau, b_1)$ (where we use the fact that the conclusion of Lemma 1 of the paper holds as long as Y is monotonic on the interval $[\theta, 1]$). Also, by the definition of τ , $y^+ \leq y^*(\tau)$ since otherwise there are no beliefs that the decision maker could hold that would result in the action y^+ .

There are two sub-cases to consider.

If $U(y^+, \tau, b_1) < U(y^-, \tau, b_1)$, then suppose that in state τ^- he sends the message $\mu_1(\theta^-)$. Following the message $\mu_1(\theta^-)$, for all $z < y^-$ that expert 2 could possibly induce, it must be that $U(z, \theta^-, b_2) \leq U(y^-, \theta^-, b_2)$. Hence, $U(z, \tau^-, b_2) < U(y^-, \tau^-, b_2)$. Consequently, in state τ^- following message $\mu_1(\theta^-)$, expert 2 will wish to induce any action below y^- . Thus $\mu_1(\theta^-)$ is a profitable deviation for expert 1 in state τ^- .

If $U(y^+, \tau, b_1) = U(y^-, \tau, b_1)$, then $U(y^+, \theta, b_1) > U(y^-, \theta, b_1)$. Hence we also have $U(y^+, \theta, b_2) > U(y^-, \theta, b_2)$. Suppose that in state θ^- , expert 1 sends the message

$\mu_1(\theta^+)$. Then there must be an action z (say) that is inducible by 2 following $\mu_1(\theta^+)$ such that $U(z, \theta^-, b_1) < U(y^-, \theta^-, b_1)$ and in the limit as ε goes to zero we have $U(z, \theta, b_1) \leq U(y^-, \theta, b_1)$. With symmetric preferences this is the same as

$$\begin{aligned} y^*(\theta, b_1) - z &\geq y^- - y^*(\theta, b_1) \\ z &\leq 2y^*(\theta, b_1) - y^- \end{aligned}$$

On the other hand, in state θ^+ , expert 2 must weakly prefer the outcome y^+ to z and hence we must have $U(y^+, \theta^+, b_2) \geq U(z, \theta^+, b_2)$ and by continuity, $U(y^+, \theta, b_2) \geq U(z, \theta, b_2)$. Also, in state θ^- expert 2 must weakly prefer z to y^+ , that is, $U(y^+, \theta^-, b_2) \leq U(z, \theta^-, b_2)$ and again by continuity, $U(y^+, \theta, b_2) \leq U(z, \theta, b_2)$. Thus we have $U(y^+, \theta, b_2) = U(z, \theta, b_2)$. With symmetric preferences this is the same as

$$\begin{aligned} y^*(\theta, b_2) - z &= y^+ - y^*(\theta, b_2) \\ z &= 2y^*(\theta, b_2) - y^+ \end{aligned}$$

Substituting into the previous inequality, we obtain

$$\begin{aligned} y^- - y^+ &\leq 2y^*(\theta, b_1) - 2y^*(\theta, b_2) & (1) \\ &< 2(y^*(\theta, b_1) - y^*(\theta)) & (2) \end{aligned}$$

Notice that $\tau < 1$ and there exists a $\sigma > \tau$ such that $Y(\sigma) = y^-$. Since $U(y^-, \tau, b_1) = U(y^+, \tau, b_1)$, $y^*(\tau) \geq y^+$ and thus $y^*(\tau, b_1) > y^+$, we have

$$y^- - y^+ > 2(y^*(\tau, b_1) - y^*(\tau))$$

But since $y^*(\tau, b_1) - y^*(\tau) > y^*(\theta, b_1) - y^*(\theta)$ this contradicts (1) and thus Y has no ‘‘trenches.’’

This means that the only possibility that remains is that Y has a ‘‘mesa,’’ that is, a situation in which Y jumps up to the left of θ and then has a downward jump at θ . Once again let θ be the largest state at which Y has a downward jump. Since this is mesa, $y^*(\theta) > y^-$. Thus $U(y^-, \theta^+, b_1) > U(y^+, \theta^+, b_1)$. Now suppose expert 1 deviates in state θ^+ and sends the message $\mu_1(\theta^-)$. Then expert 2 will induce some action $z \geq y^+$ and this is a profitable deviation for 1. ■

2 Opposing Biases

2.0.1 Semi-Revealing PBE (p. 763)

Erratum: The construction of the semi-revealing equilibrium requires an additional condition that is not stated in the paper, that is, $|b_1| \geq |b_2|$. The addition of this condition does not affect the conclusion reached in Proposition 3.¹

Specifically, line 8 from the bottom on page 763 should read:

¹We are grateful to Bart Lipman for pointing out the error in the paper.

“... quadratic loss functions with $b_1 < -b_2 < 0 < b_2 < \frac{1}{2}$.”

Similarly, the second full paragraph on page 765 should read:

“Finally, observe that the strategies of the semi-revealing equilibrium are valid for all $b_1 < -b_2$ as long as $0 < b_2 < \frac{1}{2}$.”

Strategies and Beliefs for the Semi-Revealing Equilibrium Consider the case of quadratic loss functions with $|b_1| \geq |b_2|$ and $|b_2| < \frac{1}{2}$. Without loss of generality, suppose $b_1 < 0 < b_2$. Thus we have that $b_1 \leq -b_2$.

- The experts’ strategies are:

$$\begin{aligned} \mu_1(\theta) &= \begin{cases} \theta & \text{if } \theta < 1 - 2b_2 \\ 1 - b_2 & \text{if } \theta \geq 1 - 2b_2 \end{cases} \\ \mu_2(\theta, m_1) &= \begin{cases} m_1 & \text{if } m_1 \geq \theta \\ \max\{m_1 + 2b_2, \theta + b_2\} & \text{if } m_1 < \theta \end{cases} \end{aligned}$$

- The decision maker’s beliefs are:

$$P(\cdot | m_1, m_2) = \begin{cases} m_1 & \text{if } m_1 \leq 1 - 2b_2 \text{ and } m_2 < m_1 + 2b_2 \\ \min\{m_2, 1\} & \text{if } m_1 \leq 1 - 2b_2 \text{ and } m_2 \geq m_1 + 2b_2 \\ \mathcal{U}[1 - 2b_2, 1] & \text{if } m_1 > 1 - 2b_2 \end{cases}$$

- The decision maker’s strategy is:

$$y(m_1, m_2) = \begin{cases} m_1 & \text{if } m_1 \leq 1 - 2b_2 \text{ and } m_2 < m_1 + 2b_2 \\ \min\{m_2, 1\} & \text{if } m_1 \leq 1 - 2b_2 \text{ and } m_2 \geq m_1 + 2b_2 \\ 1 - b_2 & \text{if } m_1 > 1 - 2b_2 \end{cases}$$

Proof that the strategies form a PBE

Decision Maker Clearly, the decision maker is optimizing given his beliefs.

Expert 2 Now consider, expert 2. We will argue that given the strategy of the decision maker, for all θ and m_1 , expert 2 cannot do better than to send message $\mu_2(\theta, m_1)$. The following terminology is useful. Say that expert 2 “agrees” with expert 1 if $m_2 < m_1 + 2b_2$ and that expert 2 “disagrees” with expert 1 if $m_2 \geq m_1 + 2b_2$.

1. First, suppose $\theta \leq 1 - 2b_2$.
 - (a) Suppose expert 1’s message m_1 is such that $\theta \leq m_1 \leq 1 - 2b_2$. In that case, if expert 2 agrees, he induces action m_1 and if he disagrees, he can induce actions in the set $[m_1 + 2b_2, 1]$. Since $\theta \leq m_1$ none of the actions that expert 2 can induce by disagreeing are preferred to m_1 .

- (b) Suppose expert 1's message m_1 is such that $m_1 < \theta$. By agreeing, expert 2 can induce action m_1 ; whereas by disagreeing he can induce any action $y \in [m_1 + 2b_2, 1]$. There are two cases to consider.
- i. If $\theta + b_2 \in [m_1 + 2b_2, 1]$, then clearly expert 2 can do no better than to follow $\mu_2(\theta, m_1)$ and induce his bliss action.
 - ii. If $\theta + b_2 < m_1 + 2b_2$ and the best inducible action by disagreeing is $\mu_2(\theta, m_1) = m_1 + 2b_2$. Further, this is preferred by expert 2 to agreeing with expert 1 since $m_1 < \theta$ and hence $m_1 + 2b_2$ is closer to expert 2's ideal action $\theta + b_2$ than is m_1 .
2. Next, suppose $\theta > 1 - 2b_2$.
- (a) If $m_1 > 1 - 2b_2$, expert 2's message is irrelevant; hence he can do no better than to follow μ_2 .
 - (b) Suppose $m_1 \leq 1 - 2b_2$. In this case, by agreeing expert 2 can induce m_1 whereas by disagreeing expert 2 can induce $y \in [m_1 + 2b_2, 1]$.
 - i. If $\theta + b_2 \in [m_1 + 2b_2, 1]$, then clearly expert 2 can do no better than to induce his bliss action.
 - ii. If $\theta + b_2 < m_1 + 2b_2$, then expert 2 can do no better by disagreeing than to induce $y = m_1 + 2b_2$ and this is preferred to agreeing with expert 1 for reasons identical to the Case 1.b.ii above.
 - iii. If $\theta + b_2 > 1$, then expert 2 can do no better than to follow μ_2 and thereby induce action $y = 1$.

We have thus argued that in any state θ and following any message m_1 , expert 2 cannot do better than to follow μ_2 .

Expert 1 Finally, consider expert 1. Suppose expert 2 and the decision maker follow the strategies μ_2 and y , respectively.

1. First, suppose $\theta \leq 1 - 2b_2$. By sending message $\mu_1(\theta) = \theta$ the action θ will result.
 - (a) If expert 1 sends a message m_1 such that $\theta < m_1 \leq 1 - 2b_2$, then expert 2 will also send the message m_1 and the resulting action will also be m_1 which is worse for expert 1 than action θ . If he sends a message $m_1 > 1 - 2b_2$ then the action $1 - b_2$ will be induced, and this is also worse for expert 1 than action θ .
 - (b) If expert 1 sends a message $m_1 < \theta$, then expert 2 will send the message $\max\{m_1 + 2b_2, \theta + b_2\}$ and the induced action will be $\max\{m_1 + 2b_2, \theta + b_2\}$ which is higher than θ and hence, from expert 1's perspective worse than θ .

2. Next, suppose $\theta > 1 - 2b_2$. By sending message $\mu_1(\theta)$ the action $1 - b_2$ will result.
- (a) If expert 1 sends a message m_1 such that $m_1 > 1 - 2b_2$, then again the action $1 - b_2$ will result.
 - (b) If expert 1 sends a message m_1 such that $m_1 \leq 1 - 2b_2$, then expert 2 will send the message $\mu_2(\theta, m_1) = \max\{m_1 + 2b_2, \theta + b_2\}$ and the resulting action will be $\min\{1, \max\{m_1 + 2b_2, \theta + b_2\}\} \geq 1 - b_2$. But since $b_1 \leq -b_2$, in any state $\theta > 1 - 2b_2$, expert 1 prefers action $1 - b_2$ to any higher action.

Thus, we have shown that the strategies specified above constitute a PBE.

Proof of Proposition 3 To prove Proposition 3, suppose that at least one of the experts is a moderate so that $\min\{|b_1|, |b_2|\} < \frac{1}{2}$.

If $|b_1| \geq |b_2|$ so that $b_1 \leq -b_2$ then the semi-revealing equilibrium constructed above is more informative than the most informative equilibrium obtained if the more loyal expert 2 is consulted alone.

If $|b_1| \leq |b_2|$ so that $b_1 \geq -b_2$ then consider a situation in which expert 2 offers advice first, and then expert 1 having heard expert 2's message offers his advice. A semi-revealing equilibrium similar to the one constructed above exists in which in all states $\theta \geq -2b_1$ the state is revealed and in all states $\theta < -2b_1$ there is pooling, resulting in action $-b_1$. By the same argument as above, this equilibrium is more informative than the most informative equilibrium obtained if the more loyal expert 1 is consulted alone.