THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Fall 2014

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

I.1. Let $f : \mathbb{R}^n_+ \to \mathbb{R}_+$ be a continuous, strictly increasing and strictly quasi-concave production function. Fix $q > 0$, and let $c : \mathbb{R}^n_{++} \to \mathbb{R}_{++}$ satisfy

$$c(w) = \min\{wx : f(x) \geq q\}$$

for each $w \in \mathbb{R}^n_{++}$. Is c necessarily differentiable? If your answer is "yes," give a proof. If your answer is "no," give a counterexample.

I.2. Consider a consumer with preferences $\succeq$ on $\mathbb{R}^L_+$. The expenditure function, $e : \mathbb{R}^L_{++} \times \mathbb{R}^L_+ \to \mathbb{R}_+$ is defined by

$$e(p, x) = \inf\{px' : x' \succeq x\}.$$

a) State assumptions on $\succeq$ that are sufficient to imply that for each $p \in \mathbb{R}^L_{++}$, $e(p, \cdot)$ is a utility function for $\succeq$. Be as general as you can. Do **not** give a proof.

For the next two parts of this question, assume in addition to the assumptions you stated in part (a) that the consumer has a continuously differentiable demand function $d : \mathbb{R}^{L+1}_{++} \to \mathbb{R}^L_+$. Let $(p^o, m^o) \in \mathbb{R}^{L+1}_{++}$ and $x^o = d(p^o, m^o)$.

b) Define the Slutsky matrix $S(p^o, m^o)$ for the demand function d at (p^o, m^o).

c) Explain why $S(p^o, m^o) = D^2_p e(p^o, x^o)$. Your explanation should be clear and detailed but need not be a complete proof.

I.3. Consider a pure exchange economy in which the preference relation $\succeq_i$, for each consumer i, can be represented by a utility function $u_i : \mathbb{R}^L_+ \to \mathbb{R}$. Let A denote the set of feasible allocations, that is,

$$A = \{(x_i)_i \in \mathbb{R}^{IL}_+ : \Sigma_i x_i = \bar{w}\}.$$

a) Let $(x^*_i)_i$ be a Pareto efficient allocation. Using additional assumptions, if necessary, show that there exists $\alpha \in \mathbb{R}^I_+$, $\alpha \neq 0$ such that

$$(x^*_i)_i \text{ maximizes } \Sigma_i \alpha_i u_i(x_i) \text{ subject to } (x_i)_i \in A.$$

Be sure to state clearly any additional assumptions you use, and be as general as you can.

b) Suppose that an allocation $(x^*_i)_i$ maximizes u_1 on the set

$$\{(x_i)_i \in A : u_i(x_i) \geq u_i(x^*_i) \text{ for all } i \geq 2\}.$$

Using additional assumptions, if necessary, show that $(x^*_i)_i$ is Pareto efficient. Be sure to state clearly any additional assumptions you use, and be as general as you can.

Section II

1. This question has two separate parts.

   (a) Consider (weak) preferences $\succsim$ on $\Delta(Z)$, where Z is a finite set of consequences. Assume that $\succsim$ is complete, transitive and reflexive. Define strict preference $\succ$ and indifference $\sim$ in the usual way. Suppose that $\succsim$ satisfies

      i. (*Continuity*) : for all $x, y, z \in \Delta Z$, $\{\alpha : \alpha x + (1-\alpha)z \succsim y\}$ and $\{\beta : y \succsim \beta x + (1-\beta)z\}$ are closed subsets of $[0,1]$; and

      ii. (*Herstein-Milnor independence*): for all x, y, z in $\Delta(Z)$, $x \sim y$ implies $\frac{1}{2}x + \frac{1}{2}z \sim \frac{1}{2}y + \frac{1}{2}z$.

      Show that for all x, y, z in $\Delta(Z)$ and all $\lambda \in (0,1]$, $x \succ y$ implies $\lambda x + (1-\lambda)z \succ \lambda y + (1-\lambda)z$.

   (b) The local risk aversion function associated with a Bernoulli utility function u is defined by $r(x) = -u''(x)/u'(x)$. Show that any two utility functions with the same local risk aversion function must have the same ranking over lotteries.

2. An object worth \$$V$ is being sold to one of two buyers. Each buyer i submits a sealed-bid b_i and the person bidding higher wins the object (ties are resolved using coin toss). How much each pays to the seller is specified below.

   (a) First, suppose that the object is sold using an all-pay auction in which both buyers pay the amount they bid, regardless of who wins, and so the total payment received by the seller is $b_1 + b_2$.

      i. Argue that the all-pay auction has no pure strategy equilibrium.

      ii. Find a symmetric mixed strategy equilibrium of the all-pay auction in which both buyers bid according to a continuous and strictly increasing distribution function F defined over an interval $[x, y]$. Thus for all $z \in [x, y]$, $F(z)$ is the probability that a bid no greater than z is submitted. What is each buyer's payoff in such an equilibrium?

   (b) Next, suppose that the object is sold using a second-price all-pay auction in which if $b_1 > b_2$, then bidder 1 wins the object but both buyers pay b_2 to the seller and so the total payment received by the seller is $2b_2$.

      i. Find a symmetric mixed strategy equilibrium of the second-price all-pay auction.

      ii. How do the payoffs of the buyers and the revenue of the seller compare to those in part (a)?

3. Suppose two firms A and B are searching for buried treasure of value 1 in one of two "sites" S_1 and S_2. (Say they are doing research on some problem using one of two different approaches.) Assume that one and only one site contains the treasure. The probability that the treasure is located in site S_i is p_i with $p_1 + p_2 = 1$.

   The firms choose which site to go to, independently and simultaneously. (Say each builds an observable lab to pursue one of the approaches.) Once they have chosen sites, they choose, again simultaneously and independently, effort levels, π_A and π_B, both in $[0, 1]$. *Conditional* on the treasure being at the chosen site, the probability that A (resp. B) will succeed in finding it is just π_A (resp. π_B). The cost of effort π is $c(\pi)$ where $c(\cdot)$ is a strictly increasing and strictly convex function satisfying $c'(0) = 0$ and $c'(x) \to \infty$ as $x \to 1$.

   If only A is at site S_i, then A's payoff is $p_i \pi_A - c(\pi_A)$. If both A and B are at the same site S_i, then A's payoff is $p_i \pi_A (1 - \pi_B) + \frac{1}{2} p_i \pi_A \pi_B - c(\pi_A)$ (assuming that if both discover the treasure, it is equally shared).

   (a) In a symmetric Nash equilibrium, do firms exert more effort if they are at the same site or if they are at different sites?

   (b) Suppose $p_i = \frac{1}{2}$ and $c(\pi) = 2\pi^2$ (this does not satisfy the limiting condition mentioned above). Assuming effort levels are chosen optimally in the second stage, which sites should A and B choose (in equilibrium)?