Who Cares More? Allocation with Diverse Preference Intensities

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Idea

Common **allocation problems**:

Public housing

Medical Appointments

School choice

Restaurant lines

Often:

Same **Ordinal** Pref

Different **Intensity**

No transfers

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Our Question

What is **optimal way** for a social planner to allocate?

Best **incentive compatible** mechanism?

Main Insights

With **observable** preferences intensities:

- Generally: give best items to those who want them most
- But: sometimes **involves a lottery**

  Chance of very desirable and not desirable

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With **Unobservable** preference intensities:

- Optimal incentive-compatible mechanism: full separation
- Always involves lotteries
- **May coincide with First-Best** (with lotteries)
- May involve **artificial disposal** of services

Plan for today

- Lit (brief)
- Framework
- First-best: observable intensities
- Second-best: unobservable intensities
- N types
- Market alternative
- Variants

Literature

- Matching with incomplete information
  - Decentralized (Static and Dynamic)

    Liu *et al* 14, Agranov *et al.* 20, Ferdowsian Niederle Yariv 20
  - Centralized (Static and Dynamic)

    Fernandez Rudov Yariv 20, Leshno 19
- Different preference intensities

  Abdulkadiroglu Che Yasuda 11, 15

- Timing as a screening device

  Dimakopoulos Heller 18, Ely Szydlowski 17, Leshno 17
- Screening time-inconsistent agents

  Della Vigna Malmendier 06, Eliaz Spiegler 06
- Screening risk-averse agents (with prices)

  Rothschild Stiglitz 76, Maskin Riley 84, ...
- Disposal can help selection:

  Alatas *et al* 06, Austen-Smith Banks 00

framework

A Simple Model of Service Allocation

- Continuum of goods: $[0, T]$
    - Public housing provided at different times
    - Schools varying in quality
    - Doctor appointments varying in physician's expertise or dates
- $\diamond$ denotes the outside option
- Supply $f(t)$
    - Continuous density $f(t)$, CDF F
    - Support $[0, T]$

Agents

- 2 types: P and I, masses $\mu_P, \mu_I > 0$
- Each consumes single indivisible good
- Same **ordinal** preferences: $u_k(\cdot)$ **decreasing** on $[0, T]$
- Difference **Cardinal** ones:
$$\frac{u_P''(t)}{u_P'(t)} > \frac{u_I''(t)}{u_I'(t)} \quad \text{for all } t \in [0, T]$$
  Ranked in terms of curvature:
  - I **care more** about getting high quality
  - P more risk-averse than I
- u can have any shape, as long as monotone and ranked
- $u_k(0) = 1$, $u_k(T) \geqslant u_k(\diamond) = 0$

Examples

1. Heterogenous goods ranked identically
   - Colleges and U.S. News and World Report ranking
   - CRRA or CARA utilities with different parameters ranked

2. Identical goods with different **delivery date**
   - Many examples

     Public housing, Medical Appointments, Restaurants

   - Normalize good "value" at 1
   - **Patient** (P) discount rate r_P : $u_P(t) = e^{-r_P t}$

     Impatient (I) discount rate r_I: $u_I(t) = e^{-r_I t}$

     $0 < r_P < r_I$

   - **Lead example for today**

Lotteries, Allocations, Welfare

- **Allocation** $q = (q_P, q_I)$, where q_k is density on $[0, T] \cup \{\diamond\}$
- **Feasibility**: $\mu_P q_P(t) + \mu_I q_I(t) \leqslant f(t)$
- **Assume Sufficient supply** (today): $\mu_P + \mu_I \leqslant F(T)$
- **Expected payoff**: $V_k(q_k) = \int_0^T u_k(t) q_k(t) dt \quad k \in \{P, I\}$
- **Welfare**: $W(q) = \mu_P V_P(q_P) + \mu_I V_I(q_I)$
  - Here: equal weights on all types
  - In paper: arbitrary weight allowing under-weighting P-agents

A Note on Storage

- Some applications (e.g., public housing) allow storage
- If Q is CDF of lottery :

  Feasibility with storage: $\mu_P Q_P(t) + \mu_I Q_I(t) \leqslant F(t)$
- Results is the same: storage never used

first-best

The First-Best Solution

- First, suppose utilities/types are observable
- Relevant for some applications
  - Urgency of appointment seekers
  - BMI of individuals waiting for food
- If timing allocation:

Do you give goods to impatient first?

Computing the First Best

- Benefit of allocating to P relative to I at time t :

$$g(t) = e^{-r_P t} - e^{-r_I t}$$

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  - Give to P when $g(t)$ is **high**
- $g(0) = 0 \Rightarrow$ give to I initially

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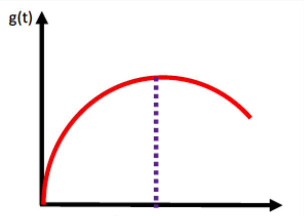
- $g(0) = 0 \Rightarrow$ give to I initially
- But $g(t)$ is **single-peaked**



Computing the First Best

- Let $\bar{T}$ be minimal time to service everyone:

$$\bar{T} = \inf\{t \mid F(t) \geqslant \mu_P + \mu_I\}$$

- Let $\bar{t}$ be minimal time to service only I:

$$\bar{t} = \inf\{t \mid F(t) \geqslant \mu_I\}$$

When to Service all I-agents First?

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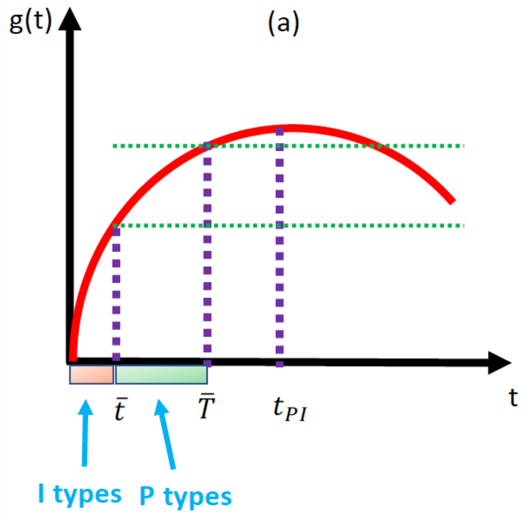
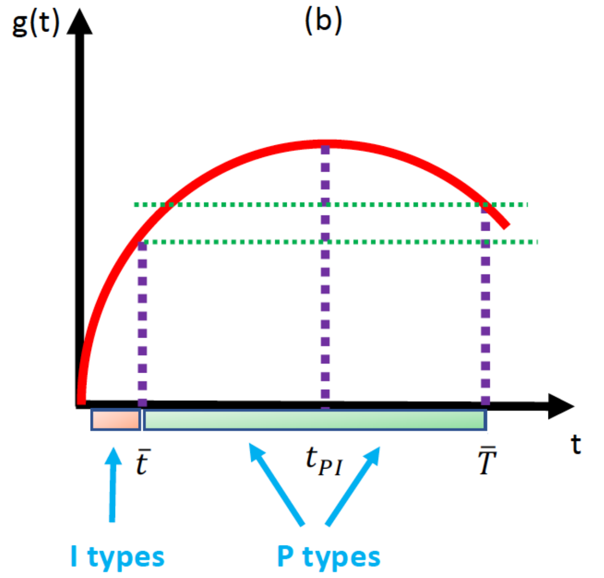
- When costs are low up to $\bar{t}$: $g(\bar{t}) \leqslant g(\bar{T})$



When to Service all I-type Agents First?



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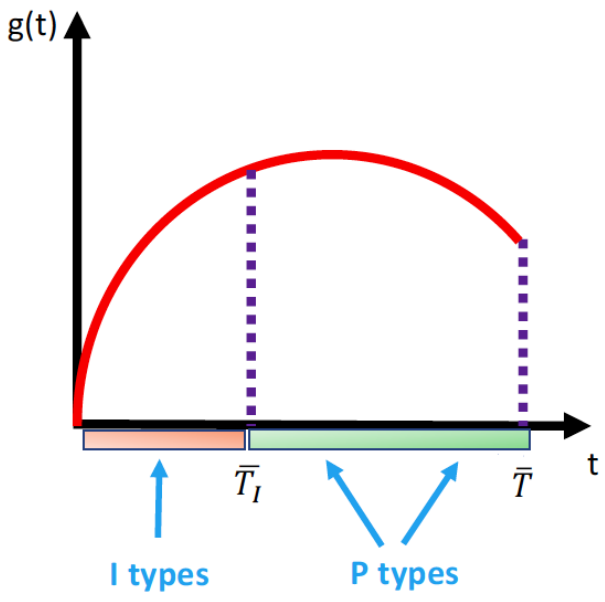

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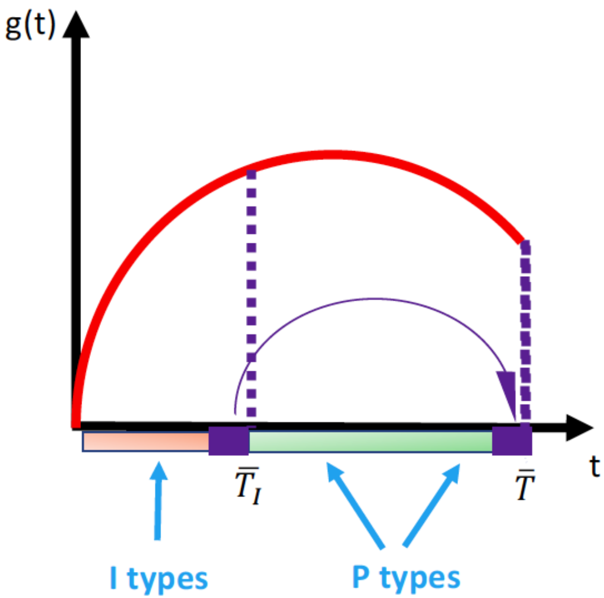
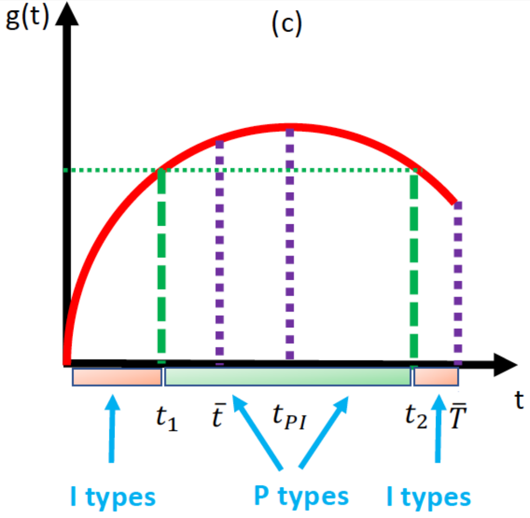


When to Serve I-agents with a "Lottery"?



Lotteries in the First-best Allocation

Proposition

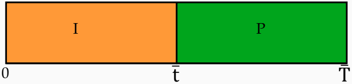
First best exists and is unique. Moreover,

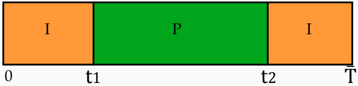
- *when* $g(\bar{t}) \leqslant g(\bar{T})$, *First-Best gives*
  - $[0, \bar{t})$ *to I;*
  - $(\bar{t}, \bar{T}]$ *to P;*

- *when* $g(\bar{t}) > g(\bar{T})$, *First-Best gives*
  - $[0, t_1) \cup (t_2, \bar{T}]$ *to I*
  - $[t_1, t_2]$ *to P*

  where t_1 *and* t_2 *are unique and identified by*

$$g(t_1) = g(t_2) \quad \text{and} \quad F(t_2) - F(t_1) = \mu_P.$$





Intuition: Risk Attitudes

- Expected Discounted Ut. $\Rightarrow$ **risk seeking over time lotteries**
- Compare $t = 2$ for sure vs. $t = 1$ or $t = 3$ with equal chances

$$\begin{aligned} \beta^2 u(x) &< \frac{1}{2}\beta^1 u(x) + \frac{1}{2}\beta^3 u(x) \\ \beta^2 &< \frac{1}{2}\beta^1 + \frac{1}{2}\beta^3 \end{aligned}$$

- β^t is **convex** $\rightarrow$ **risk seeking**

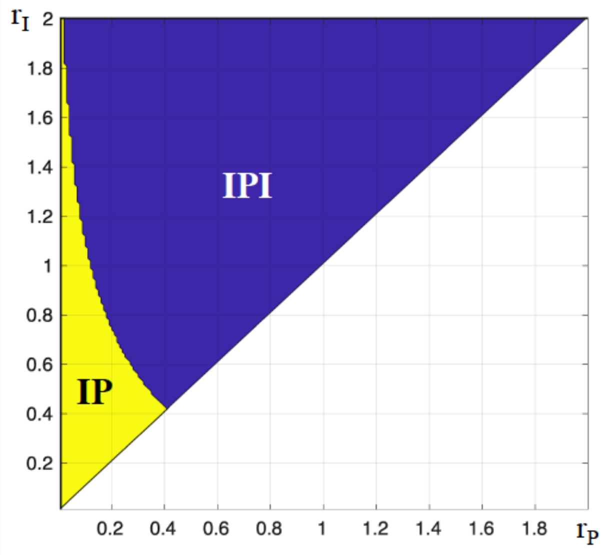
  [Dejarnette Dillenberger Gottlieb Ortoleva 2020]

- More discounting $\Rightarrow$ more risk seeking

- *I* strictly more risk seeking than *P*
- *I* benefit from lottery that places high probability on early

The First-best Allocation

$\mu_I = \mu_P = 1/2$, uniform supply



incentive-compatible mechanism

Incentive-Compatible Mechanism

- What if intensity is unobserved?
- Relevant for many settings:
    - Family circumstances of public-housing customers
    - Urgency in need of attention in scheduling settings
    - Restaurants..
- One obvious mechanism: give randomly
- **Can I do better**?

Mechanism Designer Problem (Formal Statement)

$$\max_{q(t)\geqslant 0}\left[\sum_{k=P,I}\mu_k\int_0^\infty u_k(t)q_k(t)dt\right]$$

such that

$$IC_{kj} \quad : \quad \int_0^\infty u_k(t)q_k(t)dt \geqslant \int_0^\infty u_k(t)q_j(t)dt \quad \forall k,j=P,I$$

$$\text{Feasibility} \quad : \quad \sum_{k=P,I}\mu_k q_k(t)\leqslant f(t) \quad \forall x\in[0,\infty)$$

Can it be First Best?

- When I are serviced before P:
  - P want to imitate I
  - Cannot be incentive compatible

  $\Longrightarrow \text{SB} \neq \text{FB}$

Can it be First Best?

- When I are serviced before P:
  - P want to imitate I
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    $\Longrightarrow$ SB $\neq$ FB

- When I-agents receive a lottery?
  - Not obvious any more
  - Could it be that FB is incentive compatible?
  - Could it be that SB = FB?

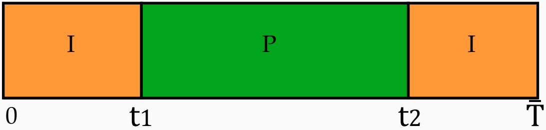
Second-best = First-best

Proposition

For a positive measure of discount factors, the first-best allocation is incentive compatible.

Intuition

- Suppose FB has lottery: type IPI
  - otherwise no hope



- I served in $[0, t_1) \cup (t_2, \bar{T}]$, P served in $[t_1, t_2]$

- I really care about early service

  $\Rightarrow$ more willing to take risk, prefer lottery

- P doesn't mind waiting

  $\Rightarrow$ less willing to take risks, prefer $[t_1, t_2]$

Second-best Allocation more Generally

- If all type-k served before all type-m
  $\Rightarrow$ type-m want to imitate type-k

- Therefore, we cannot have 'dominance'

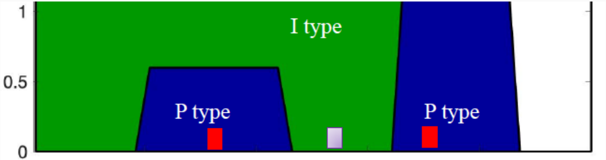
- **Need** lotteries

- What can we say?
- Let's proceed in steps

- [Note: sloppy formal statements in slides]

Step 1: No "Inverted Spreads"

Definition: **Inverted Spread** if "some I served between some P," or some P not served at all



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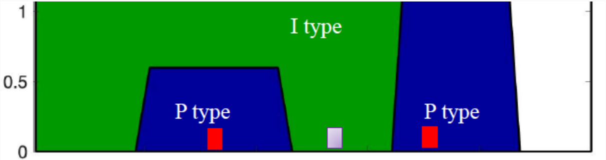
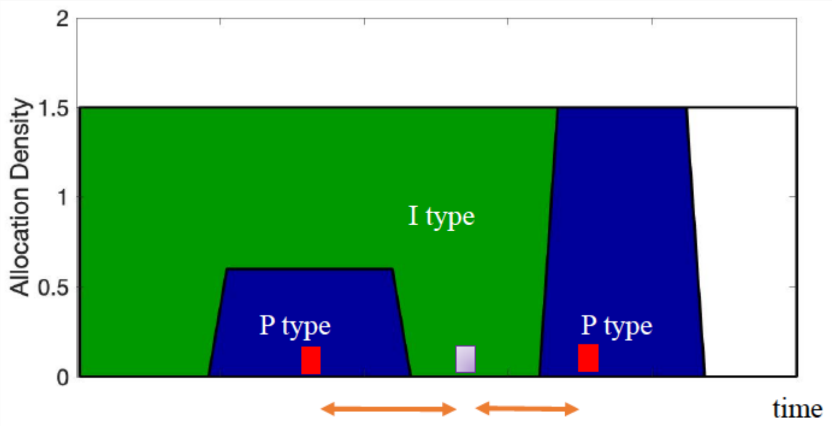


Lemma

No Solution of the MD problem exhibits Inverted spread.

Intuition of No Inverted Spreads



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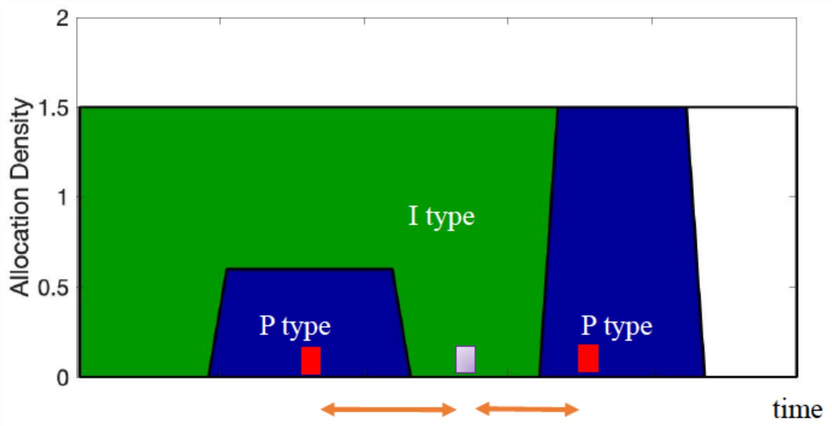

- P indifferent between δ_t and $\lambda\delta_{t'} + (1-\lambda)\delta_{t''}$
  $\Rightarrow$ I strictly prefers lottery
- Trade increases welfare, preserves incentive constraints