

Trade, Firm-Delocation, and Optimal Climate Policy

Farid Farrokhi
Purdue University

Ahmad Lashkaripour
Indiana University

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Abstract

To what extent can trade policy help reduce carbon emissions? To answer this question, we introduce transboundary carbon externality into a multi-country, multi-industry quantitative trade model. Our framework accommodates a rich set of policy considerations, including firm delocation in response to policy, multilateral carbon leakage, and returns to scale in production and abatement. We derive simple analytic formulas for optimal carbon, production, and trade taxes within this framework. With the aid of these formulas, we quantify the extent to which trade policy can reduce CO₂ emissions under two widely-discussed scenarios. First, we show that *carbon border adjustments* adopted non-cooperatively by all governments can reduce global CO₂ emissions by only 3% of the reduction attainable under globally optimum carbon taxes. Second, we find that [Nordhaus's \(2015\) climate club](#) proposal can prompt global climate cooperation in which global CO₂ emissions reduce by 82%. This successful outcome, however, hinges on both the US and EU committing to the climate club as core members, using their collective trade penalties to enforce global climate cooperation.

1 Introduction

Climate change is accelerating at a worrying pace. Yet governments have failed in their many attempts to forge an agreement that can effectively combat climate change. Major climate agreements, like the 1997 Kyoto Protocol and the 2015 Paris climate accord, have failed to deliver a meaningful reduction in carbon emissions. This failure is often attributed to the free-riding problem: Countries have an incentive to free ride on the rest of the world's reduction in carbon emissions without undertaking proportionate abatement themselves.

The failure of exiting climate agreements has urged experts to devise alternative remedies that are immune to the free-riding problem. Two proposal, in particular, have gained public traction:

Proposal #1: Climate-conscious governments use carbon border adjustments in their trade taxes (as a second-best policy) to reduce carbon emissions in the rest of the world.

Proposal #2: Climate-conscious governments form a “climate club,” and incentivize climate cooperation by non-members via trade penalties (Nordhaus 2015).

These proposals are not new but measuring their efficacy at carbon reduction has proven challenging to this date. To uncover the full potential of *Proposals 1* and *2*, we must determine optimal carbon border adjustments or optimal trade penalties under the climate club model. This task has proven elusive: Existing analyses of *Proposals 1* and *2* typically resort to arbitrarily chosen—i.e., sub-optimal—border carbon tariffs or trade penalties. As such, they only uncover the partial efficacy of border adjustments or the climate club. At the same time, existing analyses do not account for scale economies in production/abatement or firm delocation in response to policy. Recent evidence from micro-level data, however, suggest that these previously overlooked margins are key to measuring emission reduction in response to policy (Shapiro and Walker 2018).

In this paper, we overcome these challenges to uncover the full potential of the aforementioned policy proposals. To this end, we introduce carbon externalities, abatement, and scale economies into a multi-country, multi-industry general equilibrium trade model. We then derive simple analytic formulas for unilaterally optimal taxes that incorporate carbon-reducing and terms-of-trade motives. We take these formulas to data on trade, production, and carbon emissions and evaluate the effectiveness of carbon border adjustments, and the climate club scheme.

Section 2 presents our theoretical model, which is a general equilibrium semi-parametric Krugman model with many countries and industries. We introduce international carbon externalities and abatement into the model à la Copeland and Taylor (2004). This framework is particularly attractive as it combines carbon externality, terms-of-trade, and misallocation-correcting rationales for policy intervention in tractable and transparent fashion. Our theoretical model exhibits several features that distinguish it from prior theories of climate policy in open economies. First, our framework accommodates firm delocation and multilateral carbon leakage in response to policy. These effects are often absent in prior theories that employ partial equilibrium two-by-two models. Second, firm-entry, in our framework, creates economies of scale in both production and abatement. As it turns out, these previously overlooked scale effects have important implications for the ability of trade policy to combat carbon emissions.

Sections 3 and 4 derive simple analytic formulas for optimal trade, production, and carbon taxes in our multi-country, multi-industry general equilibrium framework. Our formulas for optimal car-

bon border adjustments and domestic carbon taxes, in particular, present a notable advance over traditional theories. Most importantly, our formulas are amenable to quantitative analysis, whereas traditional theories of optimal carbon taxes in open economies are based on stylized models that are difficult to map to data.

Our derivation of optimal policy is grounded on a new envelope result that transforms our general equilibrium optimal policy problem into a simpler quasi-partial equilibrium problem. Our envelope result consists of two proposals: First, we show that general equilibrium wage effects are welfare-neutral when the government has access to a complete set of tax instruments. Second, we show that general equilibrium income effects are welfare-neutral at the optimum. As a result, optimal taxes can be derived as if Marshallian demand functions were income inelastic and the vector of national wages were invariant to policy.

Our analytic formulas indicate that the unilaterally optimal carbon tax equals the CPI-adjusted domestic disutility from carbon. This choice is sub-optimal from a global standpoint as it fails to internalize home's carbon externality on the rest of the world. Optimal domestic production subsidies are carbon-blind and solely restore marginal-cost-pricing. Optimal import tariffs and export subsidies are composed of a border adjustment that penalizes carbon-intensive imports and promotes clean exports in each industry. In both cases, the carbon border adjustment is smaller the higher the degree of scale economies in the targeted industry.

To put the above results in perspective, we also characterize optimal policy under global climate cooperation. In this case, trade taxes should be set to zero, as they are inefficient from a global standpoint. Globally optimal carbon taxes are, however, higher than the non-cooperative rate and equal to the CPI-adjusted global disutility from home's carbon emissions. In other words, the globally optimal carbon tax in each country internalizes not only that country's carbon externality on domestic consumers but also consumers all over the world.

Sections 5 and 6 employ our analytic tax formulas together with required sufficient statistics to uncover the efficacy of carbon border adjustments and the climate club proposal at reducing carbon emissions. As noted earlier, this task can be computationally infeasible without the aid of our optimal tax formulas. Indeed, traditional analyses have often relied on arbitrarily-chosen—rather than optimal—trade and carbon taxes. This approach, while fruitful, cannot uncover the full potential of either border carbon tariffs or the climate club.

With the aid of our theory, the effectiveness of *Proposals 1* and *2* can be computed as a function of the following sufficient statistics: First, observable shares that can be constructed from national and

environmental accounts data. Second, the governments' perceived disutility from climate changed, which can be inferred from applied energy/carbon taxes. Third, structural elasticities including the industry-level trade, scale, and emission elasticities—all of which can be estimated via existing techniques in the literature. We construct these sufficient statistics by merging trade, production, emission, and tax data from multiple sources. Our compiled database includes 19 broadly-defined traded and non-traded industries as well as 13 major countries, the European Union, and an aggregate of the rest of the world.

Our quantitative analysis indicates that carbon border adjustments have limited efficacy—even when they are set optimally. If all countries adopt their optimal carbon border adjustments, global CO₂ emissions will decline by a mere 2.3%. This is a modest reduction, which amounts to only 3% of the CO₂ reduction attainable under the globally-first-best carbon taxes. This inefficacy of border adjustments is driven by two factors. First, border adjustments discriminate by country of origin. As such, their ability to regulate firm-level abatement in foreign locations is limited. Second, half of CO₂ emissions are generated by the non-traded industries. Hence, border carbon tariffs are unable to target a large portion of transboundary CO₂ emissions.

We find that the climate club model—with optimal trade penalties—can be remarkably effective at reducing CO₂ emissions.¹ The climate club's success, however, depends crucially on the makeup of its core members. If the EU alone initiates a climate club, by committing to globally optimum carbon taxes and penalizing non-members with trade penalties, no other country will find it optimal to join the climate club.² However, if the climate club is initiated by the US and EU as core members, all other countries will join the club in succession. As a result, countries achieve global climate cooperation in which global CO₂ emission will decline by 82%. The intuition is that the EU, alone, does not possess sufficient market power to maintain a climate club with bilateral trade penalties. The US and EU,

¹Characterizing the Nash equilibria of the climate club game is computationally challenging for two reasons. First, without appropriate analytical formulas, the computation of optimal trade penalties is not a workable approach with standard optimization techniques. As mentioned above, we overcome this barrier using our analytic formulas that characterize the efficient terms-of-trade penalty on non-cooperative countries. Second, with N countries and m core members, there are 2^{N-m} combinations of players' strategies. In the absence of a systematic approach, every of these combinations must be examined to pin down the set of Nash equilibria. We address this computational hurdle by solving the game based on the iterative elimination of dominated strategies. This approach exploits a key property of the climate club game, in that the net gain of joining the club rises in the size of the club.

²We specify the climate club game by letting the EU and possibly the US be the core members of the club, while other countries play strategically. Core members commit to the rules of membership: they impose unilaterally optimal trade taxes against non-members, while setting zero trade taxes against each other and adopting cooperative emission taxes that correct for their global emission externality. A non-member country can retaliate by adopting its non-cooperative trade taxes against members while keeping its other taxes at the status quo—i.e., applied emission taxes domestically and applied tariffs against other non-members. By joining the club a country evaluates the trade-off between a production loss that it incurs by adopting a larger emission tax against benefits of escaping trade penalties imposed by club members.

however, possess enough collective market power to establish and maintain a global climate club.³

Related Literature

Our work is related to several strands of literature. We integrate efforts to characterize optimal policies in modern trade theories with the literature on trade and environment, in a manner that can be connected to data for quantitative analyses.

First, we contribute to the theoretical literature on optimal trade and emission taxes in open economy. A central insight from this literature is that optimal unilateral tariffs include a tax on transboundary emission (e.g., [Markusen \(1975\)](#); [Copeland \(1996\)](#)). For all its merits, this body of literature has generally relied on partial equilibrium or two-country models that abstract from product differentiation, endogenous abatement, or firm-delocation. As a result, the results from this literature have been rarely used to guide the general equilibrium quantitative analysis of trade and environmental policy. We complement this literature by characterizing optimal policy in a multi-country general equilibrium trade model that accommodates several previously-overlooked features of the global economy and is straightforward to calibrate to data.

More closely to our paper is the work of [Kortum and Weisbach \(2020\)](#) who characterize the optimal trade, production, and carbon taxes in a setting that combines elements from [Markusen \(1975\)](#) and [Dornbusch et al. \(1977\)](#). Our paper complements [Kortum and Weisbach \(2020\)](#) in three ways. First, in terms of modeling emissions, they explicitly specify markets for energy whereas our model is built on [Copeland and Taylor \(2004\)](#) in which energy markets are implicitly defined. Second, in terms of methodology, rather than the primal approach, we adopt the dual approach, which we refine and customize for a class of GE trade models. Third, our theory is designed to be taken to data on multiple countries and industries for quantitative exercises.

Second, our analysis is related to an emerging body of quantitative work that analyzes the efficacy of trade policy at tackling environmental emission (e.g., [Babiker \(2005\)](#), [Elliott et al. \(2010\)](#), [Nordhaus \(2015\)](#), [Böhringer et al. \(2016\)](#)). Despite their rich structure, existing analyses have mostly quantified the efficacy of easy-to-implement but sub-optimal trade policy initiatives. This approach

³To be more specific, we find that the club-of-all-nations is a Nash equilibrium, no matter who core members are. This means that every country gains from staying in the club-of-all-nations relative to withdrawing unilaterally. These gains are larger for smaller countries, such as Canada, while smaller for larger countries whose perceived disutility from carbon emissions is not particularly high, such as China. In addition, and importantly, we find that for the club-of-all-nations to be the unique Nash equilibrium, it is not sufficient to include only the EU as the core member. However, if the core members consist of the EU and US, then the club-of-all-nations becomes the unique outcome. These results tell us that trade taxes can be remarkably effective at enforcing a climate cooperation, provided that the US joins the EU to form an initial climate club.

allows researchers to circumvent the computational difficulties associated with optimal policy analysis. However, it does not uncover the full potential of trade taxes at tackling environmental pollution. In comparison, we derive analytic formulas for optimal policy, which help us bypass computational difficulties, making us able to uncover the full potential of trade taxes at tackling environmental emission.

Third, our intermediate envelope result speaks to an emerging literature that studies optimal policy in modern quantitative trade models (Costinot et al. (2015, 2016), Lashkaripour and Lugovskyy (2016), Bartelme et al. (2019), Beshkar and Lashkaripour (2020)). These studies have bridged a longstanding divide between classic partial equilibrium trade policy models and modern general equilibrium trade theories. This divide is partly driven by classic trade policy models assuming away general equilibrium wage and income effects. Our envelope result is a step forward in filling this divide. Specifically, it shows that the simplifying assumptions in dealing with wage and income effects can be relaxed without sacrificing richness of analysis.

Finally, we contribute to the ongoing revival and enhancement of quantitative trade theories. Over the past two decades, quantitative trade models have been enriched to account for firm-selection, scale economies, input-output linkages, multinational production, and more (Costinot and Rodríguez-Clare (2014)). But less attention has been paid to embedding environmental externalities into the state-of-the-art quantitative trade models (Cherniwchan et al. (2017)). Our conceptual framework and optimal policy formulas can help bridge this gap. They can enable future analyses of trade liberalization to formally account for environmental costs and benefits.

This paper is organized as follows: In Section 2 we present our theoretical framework. In Section 3 we present our intermediate envelope result which we use to derive simple formulas for optimal unilateral policy. In Section 4 we discuss international cooperative and non-cooperative Nash outcomes. In Section 5 we map our theory with our optimal policy formulas to data, which we use in Section 6 to quantify the efficacy of trade policy at reducing global carbon emissions.

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2 Theoretical Setup

The global economy consists of multiple countries indexed by $i, j, n \in \mathbb{C}$ and multiple industries indexed by $k, g \in \mathbb{K}$. Each country i is endowed by \bar{L}_i units of workers who are perfectly mobile across industries but immobile across countries.

2.1 Demand

We denote by subscript ji, k the composite variety that corresponds to origin country j , destination country i , and industry k . The representative consumer in country i maximizes a non-parametric utility function $U_i(\mathbf{Q}_i)$ by choosing the vector of quantities, $\mathbf{Q}_i = \{Q_{ji,k}\}_{j \in \mathbb{C}, k \in \mathbb{K}}$ subject to the budget constraint,

$$Y_i = \sum_j \sum_k \tilde{P}_{ji,k} Q_{ji,k}, \quad (1)$$

where Y_i denotes national income, and $\tilde{P}_{ji,k}$ denotes the consumer price index of composite variety ji, k . The tilde notation on price distinguishes between after-tax consumer prices ($\tilde{P}_{ji,k}$) and before-tax producer prices ($P_{ji,k}$). Let $\tilde{\mathbf{P}}_i = \{\tilde{P}_{ji,k}\}$ denote the entire vector of after-tax consumer prices in country i . The consumer's problem implies an indirect utility function, $V_i(Y_i, \tilde{\mathbf{P}}_i)$, and a Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, for each variety ji, k . The demand function is characterized by a set of demand elasticities. First, the elasticity of demand for (ji, k) with respect to the price of

variety (ni, g) is,

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,g}},$$

Second, the elasticity of demand for ji, k with respect to income is:

$$\eta_{ji,k} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln Y_i}.$$

While we impose no parametric restriction on the demand function, we require it to satisfy standard conditions of Marshallian demand. We denote the own price elasticity of demand by $\varepsilon_{ji,k} \equiv \varepsilon_{ji,k}^{(ji,k)} \leq 1$. Lastly, since an individual consumer is one in many, he does not internalize the impact of his consumption on emissions.

2.2 Supply

Firms and Market Structure. Production in each *origin* j -industry k takes place by monopolistically competitive firms indexed by $\omega \in \Omega_{j,k}$. A large pool of ex-ante identical firms can pay an entry cost $w_j \bar{f}_{j,k}$ to supply their differentiated variety to various destinations. w_j denotes the labor wage rate in origin j and $\bar{f}_{j,k}$ is the labor requirement for entry.

Upon entry, a firm's production technology aggregates a Cobb-Douglas combination of labor and a bundle of emission-intensive input. Specifically, production quantity of variety ji, k by firm ω is given by $\bar{d}_{ji,k} q_{ji,k}(\omega) = \left(m_{ji,k}(\omega)\right)^{\alpha_{j,k}} \left(\bar{\varphi}_{j,k} l_{ji,k}(\omega)\right)^{1-\alpha_{j,k}}$ where $\bar{d}_{ji,k} q_{ji,k}(\omega)$, $m_{ji,k}(\omega)$, and $l_{ji,k}(\omega)$ are firm-level output, emission-intensive input, and labor input. Production is subject to iceberg trade costs, $\bar{d}_{ji,k} \geq 1$ with $\bar{d}_{jj,k} = 1$, and $\bar{\varphi}_{j,k}$ is labor productivity in origin country j , industry k .

The firm's problem, following [Copeland and Taylor \(2004\)](#), can be equivalently expressed as a choice of production and abatement: Every firm ω devotes a fraction $a_{j,k}(\omega) \in [0, 1]$ of its labor input to abatement activities, and the rest to production. The choice of $a_{j,k}(\omega)$ is regulated by an origin-industry specific emission tax, $\tau_{j,k}$. Firms (from the same origin) can be treated as symmetric: They all choose a common abatement level $a_{j,k} \equiv a_{j,k}(\omega)$, deliver quantity $q_{jik} \equiv q_{ji,k}(\omega)$ to destination i (for which they require to produce $\bar{d}_{ji,k} q_{jik}$), and use $m_{ji,k} \equiv m_{ji,k}(\omega)$ emission-intensive input. The use of emission-intensive input per unit of output, $m_{ji,k} / (\bar{d}_{ji,k} q_{jik})$, equals $(1 - a_{j,k})^{1/\alpha_{j,k}-1}$, where $\alpha_{j,k} > 0$ is the "emission elasticity" which can vary across producer countries $j \in \mathbb{C}$ and industries $k \in \mathbb{K}$.

The use of emission-intensive input, $m_{ji,k}$, generates two types of emissions: CO2 emissions, $z_{ji,k}$, that create a *global* externality; and, a bundle of local pollutants, $z_{ji,k}^0$, that create a *local* externality. We

let one unit of emission-intensive input generate one unit of CO2 emissions, and $\bar{\zeta}_{j,k}$ units of local emissions. Hence, $z_{ji,k} = m_{ji,k}$, and $z_{ji,k}^0 = \bar{\zeta}_{j,k} z_{ji,k}$.⁴

Given the choice of abatement $a_{j,k}$, the marginal cost of production equals $c_{ji,k} = \bar{d}_{ji,k}(1 - \alpha_{j,k})^{-1}(1 - a_{j,k})^{-1}(w_j / \bar{\varphi}_{j,k})$. A higher level of abatement implies that firms produce less emissions and pay less in emission taxes, but face a higher marginal cost of production.

Industry-Level Aggregates. The composite output of ji,k , $Q_{ji,k}$, aggregates over firm-level quantities $q_{ji,k}(\omega)$,

$$Q_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} q_{ji,k}(\omega)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega \right)^{\frac{\gamma_k}{\gamma_k - 1}}$$

with $\gamma_k > 1$ denoting the elasticity of substitution across firm-level varieties from the same origin. Facing with substitution elasticity γ_k , firms charge a constant markup over their marginal cost, which implies the following producer price index for composite variety ji,k :

$$P_{ji,k} = M_{j,k}^{\frac{1}{1 - \gamma_k}} \frac{\gamma_k}{\gamma_k - 1} \frac{\bar{d}_{ji,k} w_j}{\bar{\varphi}_{j,k} (1 - \alpha_{j,k}) (1 - a_{j,k})}. \quad (\text{Price})$$

In the above expression, $M_{j,k} \equiv |\Omega_{j,k}|$ denotes the mass of firms. It is pinned down by the free entry condition, that requires entry costs, $M_{j,k} w_j \bar{f}_{j,k}$, be equal to gross profits across all destinations, $\sum_i \frac{1}{\gamma_k} P_{ji,k} Q_{ji,k}$. Putting these together with $P_{ji,k} = \bar{d}_{ji,k} P_{jj,k}$ and $Q_{j,k} = \sum_i \bar{d}_{ji,k} Q_{ji,k}$, yields the following expression for the mass of firms:

$$M_{j,k} = \frac{P_{jj,k} Q_{j,k}}{\gamma_k \bar{f}_{j,k} w_j} \quad (\text{Entry})$$

Using the Equations (Price) and (Entry), we can express industry-level variables as functions of abatement and output in each origin-industry:

$$P_{ji,k}(w_j, a_{j,k}; Q_{j,k}) = \bar{d}_{ji,k} \bar{p}_{jj,k} w_j (1 - a_{j,k})^{\frac{1}{\gamma_k} - 1} Q_{j,k}^{-\frac{1}{\gamma_k}} \quad (2)$$

$$Z_{j,k}(a_{j,k}; Q_{j,k}) = \bar{z}_{j,k} (1 - a_{j,k})^{\frac{1}{\alpha_{j,k}} + \frac{1}{\gamma_k} - 1} Q_{j,k}^{1 - \frac{1}{\gamma_k}} \quad (3)$$

⁴We let $\bar{\zeta}_{i,k}$ vary across industry-country pairs. This means that, for example, one tonne of CO2 emissions in agriculture vs chemical manufacturing in China is associated with different emissions of local pollutants such as carbon monoxide, and this relationship also differs from one country to the other.

$$M_{j,k}(a_{j,k}; Q_{j,k}) = \bar{m}_{j,k}(1 - a_{j,k})^{-1 + \frac{1}{\gamma_k}} Q_{j,k}^{1 - \frac{1}{\gamma_k}} \quad (4)$$

In the above expressions, $\bar{p}_{j,k}$, $\bar{z}_{j,k}$, $\bar{m}_{j,k}$ are exogenous shifters of price, CO2 emission, and mass of firms.⁵ The industry-level local emissions is given by $Z_{j,k}^0 = \bar{z}_{j,k} Z_{j,k}$. Notice, internal economies of scale operate through the endogenous mass of firms, which is given by Equation (4). The resulting scale effects impact both industry-level price $P_{j,k}$ and emission $Z_{j,k}$, as reflected by the term $(Q_{j,k}/(1 - a_{j,k}))^{-1/\gamma_k}$ in Equations (2) and (3). This formulation indicates that scale economies are operative through both production and abatement to a common extent, and are governed by γ_k .

2.3 Policy Instruments, Price Wedges & Emissions

The government in country i has access to a full set of tax instruments necessary to replicate the (unilaterally) first-best outcome. These tax instruments include:⁶

1. An import tax, $t_{ji,k}$, applied to each imported variety ji, k ($t_{ii,k} = 0$ by design)
2. An export subsidy, $x_{ij,k}$, applied to each exported variety ij, k ($x_{ii,k} = 0$ by design)
3. A production subsidy, $s_{i,k}$, applied to all outputs in *origin i -industry k* irrespective of the location of final sales.
4. An emission tax, $\tau_{i,k}$, applied to all outputs in *origin i -industry k* irrespective of the location of final sales.

The first three tax instruments create a wedge between the after-tax, consumer price and before-tax, producer price of a given variety. Specifically, after-tax consumer prices are related to before-tax producer prices according to:⁷

$$\tilde{P}_{ji,k} = \frac{(1 + t_{ji,k})}{(1 + s_{i,k})(1 + x_{ij,k})} P_{ji,k}$$

⁵Specifically, $\bar{p}_{j,k} \equiv (\gamma_k \bar{f}_{j,k})^{1/\gamma_k} \left(\frac{\gamma_k}{\gamma_k - 1} \frac{1}{\bar{\varphi}_{j,k}(1 - \alpha_{j,k})} \right)^{(\gamma_k - 1)/\gamma_k}$, $\bar{z}_{j,k} \equiv (\gamma_k \bar{f}_{j,k} / \bar{p}_{j,k})^{1/(\gamma_k - 1)}$, $\bar{m}_{j,k} \equiv \bar{p}_{j,k} / (\gamma_k \bar{f}_{j,k})$.

⁶Adding consumption and abatement taxes does not bring any new potential in policy since the entire effect from these two taxes can be replicated by an appropriate choice of the current instruments.

⁷An alternative way of representing this relationship is $\tilde{P}_{ji,k} = (1 + t_{ji,k})(1 + s_{i,k}^a)(1 + x_{ij,k}^a)P_{ji,k}$. Since the policy tools related to production and exports are typically understood as *subsidies*, we have replaced $(1 + s_{i,k}^a) = 1/(1 + s_{i,k})$ and $(1 + x_{ij,k}^a) = 1/(1 + x_{ij,k})$.

In the case that only home country i sets taxes, the following one-to-one mapping holds between the set of instruments $\{t_{ji,k}, x_{ij,k}, s_{i,k}\}_{j,k}$ and the set of after-tax prices $\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}\}_{j \neq i,k}$,

$$(1 + t_{ji,k}) = \frac{\tilde{P}_{ji,k}}{P_{ji,k}}, \quad (1 + x_{ij,k}) = \frac{P_{ij,k} P_{ii,k}}{\tilde{P}_{ij,k} \tilde{P}_{ii,k}}, \quad (1 + s_{i,k}) = \frac{P_{ii,k}}{\tilde{P}_{ii,k}} \quad (5)$$

Hence, the government can replicate any choice of trade and production tax-cum-subsidies with the right choice of consumer/producer price wedges.

The emission tax, $\tau_{j,k}$, is a tax imposed on the emission-intensive input used in country j -industry k . The government in country j can incentivize higher levels of abatement in country-industry jk by imposing a higher emission tax, $\tau_{j,k}$. Using the relation between abatement and emission, the choice of abatement by cost-minimizing firms in origin j is given by:⁸

$$(1 - a_{j,k}) = \left(\frac{\alpha_{j,k}}{1 - \alpha_{j,k}} \right)^{\alpha_{j,k}} \left(\frac{w_j / \bar{\varphi}_{j,k}}{\tau_{j,k}} \right)^{\alpha_{j,k}}. \quad (6)$$

The above equation indicates that firms' choice of abatement is a function of the wage to emission tax ratio, with the extent of the relationship controlled by the emission elasticity $\alpha_{j,k}$.

2.4 General Equilibrium

Revenues. Total income in country i , which pins down total expenditure per Equation (1), is the sum of wage payments, $w_i \bar{L}_i$, lump-sum tax revenues, T_i :

$$Y_i = w_i \bar{L}_i + T_i \quad (7)$$

⁸Specifically, the unit cost of ji,k is given by $c_{ji,k} = \bar{d}_{ji,k} \alpha_{j,k}^{-\alpha_{j,k}} (1 - \alpha_{j,k})^{-(1-\alpha_{j,k})} \tau_{j,k}^{\alpha_{j,k}} (w_j / \bar{\varphi}_{j,k})^{1-\alpha_{j,k}}$, emission by $z_{ji,k} = \alpha_{j,k} c_{ji,k} \bar{d}_{ji,k} q_{ji,k} / \tau_{j,k}$, and labor by $l_{ji,k} = (1 - \alpha_{j,k}) c_{ji,k} \bar{d}_{ji,k} q_{ji,k} / w_j$. Replacing these in the relation between abatement and emission, $(1 - a_{j,k}) = \left(z_{ji,k} / \bar{\varphi}_{j,k} l_{ji,k} \right)^{\alpha_{j,k}}$, delivers equation (6). In addition, note that our framework nests a model with exogenous emission intensities (no abatement) if $\alpha_{j,k} \rightarrow 0$. In this case, $\bar{d}_{ji,k} q_{ji,k}(\omega) = \varphi_{j,k} l_{ji,k}(\omega)$, and $a_{j,k} = 0$. Also, for completeness, (for $0 < \alpha_{j,k} < 1$) we specify that $a_{j,k} = 0$ if $\tau_{j,k} \leq \tau_{j,k}^{\min} \equiv \frac{\alpha_{j,k}}{1 - \alpha_{j,k}} (w_j / \bar{\varphi}_{j,k})$.

The tax revenue T_i is the sum of revenues collected from emission and import taxes and the revenues exhausted by production and export subsidies:

$$\begin{aligned}
T_i = & \underbrace{\sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} \left(\alpha_{j,k} \frac{\gamma_k - 1}{\gamma_k} P_{ij,k} Q_{ij,k} \right)}_{\text{emission taxes}} + \underbrace{\sum_{k \in \mathbb{K}} [(\tilde{P}_{ii,k} - P_{ii,k}) Q_{ii,k}]}_{\text{production subsidies}} \\
& + \underbrace{\sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}, j \neq i} [(\tilde{P}_{ji,k} - P_{ji,k}) Q_{ji,k}]}_{\text{imports taxes}} + \underbrace{\sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}, j \neq i} [(\tilde{P}_{ij,k} - P_{ij,k}) Q_{ij,k}]}_{\text{exports subsidies}}
\end{aligned} \tag{8}$$

Definition. For a given vector of taxes $\{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{j,k}$, *equilibrium* is a vector of wages $\{w_j\}$ such that before-tax prices $\{P_{ji,k}\}$ are given by (2), emission $\{Z_{j,k}\}$ by (3), mass of firms $\{M_{j,k}\}$ by (4), abatement $\{a_{j,k}\}$ by (6), demand quantities by $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$ in which after-tax prices $\tilde{\mathbf{P}}_i \equiv \{\tilde{P}_{ji,k}\}$ are given by (5) and national expenditure Y_i in equation (1) equals national income according to (7) with tax revenues given by (8), and labor markets clear:⁹

$$w_i \bar{L}_i - \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k} Q_{ij,k} = 0. \tag{9}$$

Expenditure/Revenue Shares and emission Intensity. To streamline the presentation of our theory, we define the following variables. The within-industry expenditure share of country i on variety ji, k is denoted by $\lambda_{ji,k}$ (trade share of country i on supplying country j within industry k), and the overall expenditure share of country i on industry k is denoted by $e_{i,k}$,

$$\lambda_{ji,k} \equiv \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\sum_{\hat{j} \in \mathbb{C}} \tilde{P}_{\hat{j},k} Q_{\hat{j},k}}; \quad e_{i,k} = \frac{\sum_{\hat{j} \in \mathbb{C}} \tilde{P}_{\hat{j},k} Q_{\hat{j},k}}{\sum_{\hat{j} \in \mathbb{C}} \sum_{\hat{k} \in \mathbb{K}} \tilde{P}_{\hat{j},\hat{k}} Q_{\hat{j},\hat{k}}} = \frac{\sum_{\hat{j} \in \mathbb{C}} \tilde{P}_{\hat{j},k} Q_{\hat{j},k}}{Y_i} \tag{10}$$

Country i 's overall share of expenditure on variety ji, k is denoted by $e_{ji,k}$,

$$e_{ji,k} \equiv \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\sum_{\hat{j} \in \mathbb{C}} \sum_{\hat{k} \in \mathbb{K}} \tilde{P}_{\hat{j},\hat{k}} Q_{\hat{j},\hat{k}}} = \frac{\tilde{P}_{ji,k} Q_{ji,k}}{Y_i} = \lambda_{ji,k} e_{i,k} \tag{11}$$

⁹The labor market clearing condition (LMC) is equivalent to the balance trade condition (BTC), $\sum_{k \in \mathbb{K}} \sum_{j \neq i \in \mathbb{C}} (P_{ji,k} Q_{ji,k} - \tilde{P}_{ij,k} Q_{ij,k}) = 0$, where exports and imports of every country i are measured in values outside the border of i (that are, exports are after-tax, but imports are before-tax). In our policy analysis, we sometimes use (BTC) instead of (LMC).

The within-industry share of origin j 's revenues from sales of variety ji, k is denoted by $r_{ji,k}$, and the share of industry k in origin j 's total revenues is denoted by $\rho_{j,k}$:

$$r_{ji,k} \equiv \frac{P_{ji,k} Q_{ji,k}}{\sum_{\hat{i} \in \mathbb{C}} P_{j\hat{i},k} Q_{j\hat{i},k}}; \quad \rho_{j,k} = \frac{\sum_{\hat{k} \in \mathbb{K}} P_{j\hat{i},\hat{k}} Q_{j\hat{i},\hat{k}}}{\sum_{\hat{j} \in \mathbb{C}} \sum_{\hat{k} \in \mathbb{K}} P_{j\hat{i},\hat{k}} Q_{j\hat{i},\hat{k}}}. \quad (12)$$

Lastly, we use $v_{j,k}$ to denote the CO2 emission intensity per unit value of output in *origin j -industry k* :

$$v_{j,k} \equiv \frac{Z_{j,k}}{P_{jj,k} Q_{j,k}} = \frac{\gamma_k - 1}{\gamma_k} \frac{\alpha_{j,k}}{\tau_{j,k}}. \quad (13)$$

2.5 Governments' Objective Function

In this section, we present the objective function that governments aim to maximize. To this end, we first specify the environmental cost of economic activities perceived by governments. We express country i 's disutility from local and global emissions as $\phi_i^0 \sum_k Z_{i,k}^0 + \phi_i \sum_n \sum_k Z_{n,k}$, where ϕ_i^0 and ϕ_i reflect the disutility per unit of local, non-CO2 and global, CO2 emissions. Using $Z_{j,k}^0 = \bar{\zeta}_{j,k} Z_{j,k}$ and defining $\phi_{ik} \equiv \phi_i^0 \bar{\zeta}_{i,k}$, we can then express country i 's *disutility from emissions*, $\Delta_i(\mathbf{Z})$, as:

$$\Delta_i(\mathbf{Z}) = \sum_n \sum_k (\delta_{ni,k} Z_{n,k}), \quad \text{where} \quad \delta_{ni,k} = \begin{cases} \phi_i & n \neq i \\ \phi_{i,k} + \phi_i & n = i \end{cases} \quad (14)$$

This specification allows $\phi_{i,k}$ to vary across industries insofar as a unit of CO2 emission across industries within country i comes with different units of local emissions. While our specification makes room for this connection, it allows us to track merely CO2 emissions, where $\mathbf{Z} = [Z_{n,k}]_{n \in \mathbb{C}, k \in \mathbb{K}}$ is the long vector of CO2 emissions from worldwide country-industry pairs. We interpret country i 's disutility from emissions, $\Delta_i(\mathbf{Z})$, as the present value cost of CO2 emissions *perceived* by the government of country i . We note that governments' perceptions of the cost of emissions might differ from academic estimates of the expected damage.

Let \mathbb{I}_i stack the instruments of policy for the government in country i , $\mathbb{I}_i \equiv \{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{j,k}$. The objective function of the government in country i is given by:

$$W_i = V_i(Y_i(\mathbb{I}_i, \mathbf{w}), \tilde{\mathbf{P}}_i) - \Delta_i(\mathbf{Z}) \quad (15)$$

The first term in this objective function reproduces indirect utility from consumption, taking into account that income Y_i depends on the vector of wages $\mathbf{w} = \{w_i\}$ as well as policy instruments \mathbb{I}_i . In

addition, we will use $\tilde{\phi}_i \equiv \tilde{P}_i \phi_i$, $\tilde{\phi}_{i,k} \equiv \tilde{P}_i \phi_{i,k}$, and $\tilde{\delta}_{ni,k} \equiv \tilde{P}_i \delta_{ni,k} = \tilde{\phi}_{i,k} \mathbf{1}(n = i) + \tilde{\phi}_i$ as the *CPI-adjusted* welfare cost per unit use of emission-intensive input, where $\tilde{P}_i \equiv (\partial V_i(\cdot) / \partial Y_i)^{-1}$ is the consumer price index in country i . With the government's objective function at hand, we can now define the optimal unilateral policy.

Definition. The *Optimal Unilateral Policy* for country i is achieved by choosing policy instruments, \mathbb{I}_i , that maximize country i 's welfare, W_i (Equation 15), subject to equilibrium conditions (1)-(9).

To elucidate the above definition, Appendix A.1 presents a minimal set of equations that describe the unilateral policy problem.

3 Optimal Unilateral Policy

In this section, we characterize a country's optimal unilateral tax schedule. The unilaterally optimal policy corrects three types of distortion from the standpoint of a non-cooperative government who acts in their self interest:

1. [*Emission Externalities*] The externality imposed on domestic consumers from local and trans-boundary emissions.
2. [*Markup Distortions*] The misallocation caused by cross-industry markup heterogeneity.
3. [*Terms-of-Trade*] The unexploited unilateral gains from exercising national-level export and import market power.

We currently have a limited understanding of how these distinct policy channels interact. To shed light on their interaction, we analytically characterize the optimal unilateral policy schedule for each country. This is a challenging task, which explains why previous characterizations of optimal trade and emission taxes have typically restricted attention to two-country or partial equilibrium setups, with all or some of these simplifying assumptions: perfect competition, fixed location of firms, fixed set of products, exogenous emission intensities, and constant-returns-to-scale production technologies.

Before presenting our optimal policy formulas, we discuss our methodological approach. Our goal here is to demonstrate that we have a systematic way of characterizing optimal policy with applications beyond this particular work. We present this point via an intermediate envelope result,

which forms the basis of our subsequent optimal policy formulas. Throughout the paper, if not reported in the main text, our derivations and proofs are presented in the appendix.

We proceed in following order: We first present our intermediate envelop result in Section 3.1. Then, we derive optimal unilateral policy formulas in Section 3.2. In Subsections 3.3 and 3.4 we discuss special cases of formulas and highlight the key trade-offs that underlie them. In Subsection 3.5, we examine second-best scenarios in which a government is afforded fewer tax instruments than is necessary to attain the (unilateral) first-best outcome.

3.1 Intermediate Envelope Result

In this section, we present an intermediate envelope result that greatly facilitates our optimal policy analysis. In summary, this result allows us to convert our general equilibrium optimization problem into a simpler problem characterized by a set of partial equilibrium derivatives. We establish this result in three steps.

Step 1: Reformulate the optimal policy problem in terms of consumer prices and abatement

The government in i can choose consumer prices $\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}\}_{j \neq i,k}$ to replicate any set of trade and production tax-cum-subsidies $\{t_{ji,k}, x_{ij,k}, s_{i,k}\}_{j,k}$ according to Equation (5), and can choose abatement levels $\{a_{i,k}\}_k$ to replicate any set of emission taxes $\{\tau_{i,k}\}$ according to Equation (6). Shifting the focus from the vector of taxes $\mathbb{I}_i \equiv \{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{j,k}$ to their target variables $\mathbb{P}_i \equiv \{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}, a_{i,k}\}_{j \neq i,k}$ is useful, as it emphasizes the economic variable each tax instrument directly targets. As a point of reference, we provide a formal definition of \mathbb{P}_i .

Definition 1. $\mathbb{P}_i \equiv \{\tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}, \mathbf{a}_i\}$ denotes the vector of policy instruments for country i in the reformulated optimal policy problem, where $\tilde{\mathbf{P}}_{ji} = \{\tilde{P}_{ji,k}\}_{j \neq i,k'}$, $\tilde{\mathbf{P}}_{ij} = \{\tilde{P}_{ij,k}\}_{j \neq i,k'}$, $\tilde{\mathbf{P}}_{ii} = \{\tilde{P}_{ii,k}\}_k$, and $\mathbf{a}_i = \{a_{i,k}\}_k$.

We can simplify our optimal policy problem by re-casting it as a problem of choosing consumer prices and abatement levels instead of tax instruments. The following lemma establishes that after solving such a problem, the optimal taxes can be recovered using Equations (5) and (13).

Lemma 1. *Given optimal prices and abatement levels, $\mathbb{P}_i^* = \{\tilde{P}_{ji,k}^*, \tilde{P}_{ij,k}^*, \tilde{P}_{ii,k}^*, a_{i,k}^*\}_{j \neq i,k}$, optimal taxes/subsidies*

$\mathbb{I}_i^* = \{t_{ji,k}^*, x_{ij,k}^*, s_{i,k}^*, \tau_{i,k}^*\}_{j,k}$ can be recovered according to the following one-to-one mapping:

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}}, \quad 1 + x_{ij,k}^* = \frac{P_{ij,k} P_{ii,k}}{\tilde{P}_{ij,k}^* \tilde{P}_{ii,k}^*}, \quad 1 + s_{i,k}^* = \frac{P_{ii,k}}{\tilde{P}_{ii,k}^*}, \quad \tau_{i,k}^* = \frac{\gamma_k - 1}{\gamma_k} \frac{\alpha_{i,k}}{v_{i,k}(a_{i,k}^*)}$$

By appealing to the above lemma, we hereafter formulate the optimal policy problem as a choice of \mathbb{P}_i which maximizes the objective function W_i subject to equilibrium constraints. The next two steps establish neutrality results that greatly simplify this reformulated problem.

Step 2: Conditional welfare-neutrality of wage effects

The choice of \mathbb{P}_i affects the vector of wages whose subsequent effect on welfare complicates the analysis. We show that conditional on holding policy \mathbb{P}_i fixed, general equilibrium wage effects are welfare-neutral. To make this point, we formulate all variable outcomes as a function of \mathbb{P}_i and the wage vector \mathbf{w} . As detailed in Appendix A.2, this formulation results from a system that solves all equilibrium relationships with the exception of the labor market clearing condition. It characterizes welfare in country i as $W_i(\mathbb{P}_i; \mathbf{w})$, prices as $P_{ij,k}(\mathbb{P}_i; \mathbf{w})$, quantities as $Q_{ij,k}(\mathbb{P}_i; \mathbf{w})$, etc. Note, though, that all $(\mathbb{P}_i; \mathbf{w})$ pairs are not feasible. Given \mathbb{P}_i , a feasible vector of wages must satisfy the labor market clearing condition in each country.

Definition 2. A policy-wage pair, $(\mathbb{P}_i; \mathbf{w})$ is feasible iff the vector of wages $\mathbf{w} \equiv \{w_n\}_{n \in \mathbb{C}}$ satisfy the labor-market clearing conditions, given the policy vector \mathbb{P}_i . Namely,

$$(\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_i^w \iff \sum_{j,k} \left[\left(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k} \right) P_{nj,k}(\mathbb{P}_i; \mathbf{w}) Q_{nj,k}(\mathbb{P}_i; \mathbf{w}) \right] = w_n \bar{L}_n, \quad \text{for all } n \in \mathbb{C}. \quad (16)$$

Using this definition, we express the government's problem (P1) as:

$$\max_{\mathbb{P}_i} W_i(\mathbb{P}_i; \mathbf{w}), \quad \text{subject to } (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_i^w \quad (\text{P1})$$

where $W_i(\mathbb{P}_i; \mathbf{w}) = V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) - \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; \mathbf{w})$. The inner product $\delta_i \cdot \mathbf{Z}(\mathbb{P}_i; \mathbf{w}) = \sum_{j,k} \delta_{ij} Z_{j,k}(\mathbb{P}_i; \mathbf{w})$ summarizes the disutility from global emission to country i . The necessary condition for the optimality of each policy instrument $\mathcal{P} \in \mathbb{P}_i$ is then given by:

$$\frac{dW_i(\mathbb{P}_i; \mathbf{w})}{d \ln \mathcal{P}} = \frac{\partial V_i(\cdot)}{\partial \ln \mathcal{P}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; \mathbf{w})}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i; \mathbf{w})}{\partial \mathbf{w}} \frac{d \mathbf{w}}{d \ln \mathcal{P}}}_{\text{wage effects}} = 0,$$

Recall that $V_i(\cdot) \equiv V_i(Y_i, \tilde{\mathbf{P}}_i)$ denotes the indirect utility function from consumption, and $\frac{\partial V_i(\cdot)}{\partial \ln \mathcal{P}}$ is nonzero only if \mathcal{P} is one of prices faced by home consumers, i.e., $\mathcal{P} \in \tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\}$. In the above FOC, the first three terms correspond to the effects of policy $\mathcal{P} \in \mathbb{P}_i$ on welfare holding $\mathbf{w} = \{w_n\}_{n \in \mathcal{C}}$ fixed.¹⁰ The last term accounts for the general equilibrium wage effects. By choice of numeraire, we normalize wage in one of the foreign countries, say n , to unity. That implies $\frac{dw_n}{d \ln \mathcal{P}} = 0$. We show in Appendix A.4 that

$$r_{ji} \times \lambda_{li,k} \approx 0 \quad \text{if } (j \neq i) \wedge (\ell \neq i) \implies \frac{d\mathbf{w}_{-\{i,n\}}}{d \ln \mathcal{P}} \approx 0.$$

Throughout this section we maintain the assumption that $r_{ji} \times \lambda_{li,k} \approx 0$ if j and $\ell \neq i$. Later, when mapping out theory to data, we show that this assumption is strikingly consistent with actual data. Regardless, the most important wage effect is the one corresponding to own's wage, w_i . Accounting for the change in w_i (relative to \mathbf{w}_{-i}) has proven a major obstacle when solving problems like (P1). The next lemma allows us to overcome this obstacle. It states that for any $(\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_i^w$, if the government has access to all policy instruments, country i 's own wage effects are also welfare-neutral.

Lemma 2. *Within the feasible policy-wage set $(\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_i^w$, conditional on a choice of policy vector \mathbb{P}_i , welfare in country i is invariant to wage w_i :*

$$\left(\frac{\partial W_i(\mathbb{P}_i; \mathbf{w})}{\partial w_i} \right)_{\mathbf{w}_{-i}} = 0, \quad \forall (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_i^w.$$

To take stock, the above result indicates that home's wage has no effect on home's welfare, provided that the labor market clearing condition holds and the government has access to all policy instruments. This result would hold even if the government did *not* choose the policy vector optimally. To provide intuition, note that as long as policy \mathbb{P}_i is fixed, w_i affects welfare, W_i , only through its effect on income Y_i . Lemma 2 can, thus, be established by showing that $\partial Y_i / \partial w_i = 0$. To show that $\partial Y_i / \partial w_i = 0$, note that an increase in w_i has two opposing but equal-sized effects on income Y_i , as long as the policy vector, \mathbb{P}_i , is held fixed. On one hand, an increase in wage w_i raises income Y_i directly through wage incomes $w_i \bar{L}_i$. On the other hand, it decreases income indirectly through raising origin i 's producer prices, which amounts to lower tax revenues. This latter effect arises because the

¹⁰On our notation: (1) For any vector \mathbf{y} , $\mathbf{y}_{-n} \equiv \mathbf{y} / \{y_n\}$. (2) In cases where there might be ambiguity, we include endogenous variables that we hold fixed in the subscript of a derivative. For function $G(\mathbf{x}; \mathbf{y})$ with \mathbf{x} as the policy vector, and \mathbf{y} as the vector of endogenous variables, $\left(\frac{\partial G(\mathbf{x}; \mathbf{y})}{\partial x_m} \right)_{\mathbf{y}}$ denotes the derivative of G wrt x_m , holding fixed \mathbf{y} and \mathbf{x}_{-m} , and $\left(\frac{\partial G(\mathbf{x}; \mathbf{y})}{\partial y_n} \right)_{\mathbf{y}_{-n}}$ denotes the derivative of G wrt y_n , holding fixed \mathbf{y}_{-n} and \mathbf{x} .

after-tax price of home-made varieties, $\{\tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{ii}\}$, are held fixed as part of \mathbb{P}_i . Importantly, these two opposing effects sum up to zero, because the tax revenue effect is proportional to country i 's total sales, and total (net) sales equal wage incomes in equilibrium.

Lemma 2 greatly facilitates our analysis, since it allows us to identify the optimal policy by treating the wage vector \mathbf{w} as fixed. Once we fix \mathbf{w} , income in the rest of the world, $\mathbf{Y}_{-i} = \mathbf{w}_{-i} \odot \bar{\mathbf{L}}_{-i}$, is also fixed by construction. The next step shows that income in country i , Y_i , can also be treated as fixed, since domestic income effects are welfare neutral at the optimum.

Step 3: Conditional welfare-neutrality of income effects at the optimum

Following Step 2, we treat wages as invariant to policy. This means that, for a given vector of policy \mathbb{P}_i , we can hold wages fixed at their values that satisfy market clearing conditions, $\mathbf{w} = \bar{\mathbf{w}}$. This intermediate result also implies that we can hold income in foreign countries fixed, $Y_n = \bar{Y}_n$ for $n \neq i$. With these considerations, we re-formulate all equilibrium variables as a function of the policy vector \mathbb{P}_i and income Y_i . As detailed in Appendix A.3, this formulation derives from solving a system that imposes all equilibrium relationships except the budget constraint, $Y_i = \bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i)$. Notice, tax revenues T_i depend on income Y_i because home's demand schedule, which dictates these revenues, depends on income. This brings us to define feasible pairs of policy-income as follows.

Definition 3. A policy-income pair, $(\mathbb{P}_i; Y_i)$ is feasible iff income Y_i equals total wages plus tax revenues, given policy \mathbb{P}_i . Namely,

$$(\mathbb{P}_i; Y_i) \in \mathbb{F}_i^Y \iff Y_i = \bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i). \quad (17)$$

We continue with an observation that further facilitates our analysis. Restricting the system to the feasible policy-income pairs, we observe that income Y_i affects welfare *exclusively* through demand quantities. Behind this observation is that income affects producer prices, emissions, and taxes only through income effects in demand, meaning that we can express these variables as $P_{ni,g} = P_{ni,g}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, $Z_{n,g} = Z_{n,g}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, $T_i = T_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, where $\mathbf{Q}_i \equiv \{Q_{ni,g}, Q_{in,g}\}_{n \in \mathbb{C}, g \in \mathbb{K}}$ is the vector of country i 's output and consumption quantities. The equilibrium value for consumption quantities are given by $Q_{ni,g} = \mathcal{D}_{nig}(Y_i, \tilde{\mathbf{P}}_i)$. Export quantities are $Q_{in,g} = \mathcal{D}_{ing}(\tilde{Y}_j = \bar{w}_j \bar{L}_j, \tilde{\mathbf{P}}_{in}, \tilde{\mathbf{P}}_{-in}(\bar{\mathbf{w}}_{-i}))$.

The optimal policy problem of country i can now be expressed as:

$$\max_{\mathbb{P}_i} W_i(\mathbb{P}_i; \mathbf{Q}_i(\mathbb{P}_i, Y_i)) \quad \text{subject to } (\mathbb{P}_i; Y_i) \in \mathbb{F}_i^Y \quad (\text{P2})$$

where:

$$W_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i)) = V_i(\underbrace{\bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))}_{Y_i}, \tilde{\mathbf{P}}_i) - \delta_i \cdot \mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$$

Capitalizing on the reformulation (P2), we can now explain the welfare neutrality of income effects. The first order condition w.r.t. to policy instrument $\mathcal{P} \in \mathbb{P}_i$ is given by $\left[\frac{\partial V_i}{\partial Y_i} \frac{\partial T_i(\cdot)}{\partial \mathcal{P}} + \frac{\partial V_i}{\partial \mathcal{P}} - \frac{\partial \delta_i \cdot \mathbf{Z}(\cdot)}{\partial \mathcal{P}} \right] = 0$. We expand the components of this equation using the following derivatives,

$$\begin{cases} \frac{\partial T_i(\cdot)}{\partial \mathcal{P}} = \left(\frac{\partial T_i}{\partial \mathcal{P}} \right)_{\mathbf{Q}_i} + \frac{\partial T_i}{\partial \mathbf{Q}_i} \cdot \left[\left(\frac{\partial \mathbf{Q}_i}{\partial \mathcal{P}} \right)_{Y_i} + \frac{\partial \mathbf{Q}_i}{\partial Y_i} \frac{dY_i}{d\mathcal{P}} \right] \\ \frac{\partial \delta_i \cdot \mathbf{Z}(\cdot)}{\partial \mathcal{P}} = \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathcal{P}} \right)_{\mathbf{Q}_i} + \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \cdot \left[\left(\frac{\partial \mathbf{Q}_i}{\partial \mathcal{P}} \right)_{Y_i} + \frac{\partial \mathbf{Q}_i}{\partial Y_i} \frac{dY_i}{d\mathcal{P}} \right] \end{cases} ,$$

where $\frac{dY_i}{d\mathcal{P}}$ can be calculated by applying the Implicit Function Theorem to Equation 17 to ensure feasibility. To elaborate on the above expressions, tax revenues $T_i(\cdot)$ and emission disutility $\delta_i \cdot \mathbf{Z}(\cdot)$ react to policy \mathcal{P} directly holding quantities, and indirectly through changes in demand quantities. Note that, once we hold quantities fixed, we are also holding income fixed. Putting these points together, and recalling that $\tilde{P}_i \equiv \left(\frac{\partial V_i(\cdot)}{\partial Y_i} \right)^{-1}$, the FOC collapses to:

$$\underbrace{\tilde{P}_i \frac{\partial V_i(\cdot)}{\partial \mathcal{P}} + \left(\frac{\partial T_i}{\partial \mathcal{P}} \right)_{\mathbf{Q}_i} - \tilde{P}_i \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathcal{P}} \right)_{\mathbf{Q}_i} + \left[\frac{\partial T_i}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \right] \cdot \left(\frac{\partial \mathbf{Q}_i}{\partial \mathcal{P}} \right)_{Y_i}}_{\left(\frac{\partial W_i}{\partial \mathcal{P}} \right)_{Y_i}} + \underbrace{\left[\frac{\partial T_i}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \right] \cdot \frac{\partial \mathbf{Q}_i}{\partial Y_i} \frac{dY_i}{d\mathcal{P}}}_{\frac{\partial W_i}{\partial Y_i}} = 0 \quad (18)$$

In equation (18), the first four terms represent the direct welfare effect of policy instrument \mathcal{P} holding income fixed, $\left(\frac{\partial W_i}{\partial \mathcal{P}} \right)_{Y_i}$, and the last term represents the indirect general equilibrium effect of policy \mathcal{P} on welfare through changes in income (hence, the term ‘‘income effects’’). We will rely on this system of FOCs to solve for the optimal policy schedule, but we pause that analysis for the moment to illustrate the conditions for the neutrality of income effects.

Suppose \mathcal{P} is one of the consumer prices in home $\tilde{P}_{j,k} \in \tilde{\mathbf{P}}_i$, be it either a domestic ($j = i$) or an

imported ($j \neq i$) variety. In this case,

$$\left(\frac{\partial T_i}{\partial \tilde{P}_{ji,k}} \right)_{\mathbf{Q}_i} = Q_{ji,k}, \quad \tilde{P}_i \left(\frac{\partial V_i}{\partial \tilde{P}_{ji,k}} \right) = -Q_{ji,k}, \quad \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \tilde{P}_{ji,k}} \right)_{\mathbf{Q}_i} = 0 \implies \tilde{P}_i \frac{\partial V_i(\cdot)}{\partial \tilde{P}_{ji,k}} + \left(\frac{\partial T_i}{\partial \tilde{P}_{ji,k}} \right)_{\mathbf{Q}_i} - \tilde{P}_i \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \tilde{P}_{ji,k}} \right)_{\mathbf{Q}_i} = 0,$$

where the first equality reflects the direct effect of consumer price $\tilde{P}_{ji,k}$ on tax revenues holding the demand schedule fixed; the second equality follows from Roy's identity; and the third equality holds because emission is fully determined by abatement levels and quantities. From setting $\tilde{P}_i \frac{\partial V_i(\cdot)}{\partial \tilde{P}} + \left(\frac{\partial T_i}{\partial \tilde{P}} \right)_{\mathbf{Q}_i} - \tilde{P}_i \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \tilde{P}} \right)_{\mathbf{Q}_i} = 0$ in equation (18), it follows that

$$\left[\frac{\partial T_i}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \right] \cdot \left(\frac{\partial \mathbf{Q}_i}{\partial \mathcal{P}} \right)_{Y_i} + \left[\frac{\partial T_i}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \right] \cdot \frac{\partial \mathbf{Q}_i}{\partial Y_i} \frac{dY_i}{d\mathcal{P}} = 0$$

Noting that $\frac{\partial \mathbf{Q}_{in}}{\partial \tilde{P}_{ji,k}} = \frac{\partial \mathbf{Q}_{in}}{\partial \tilde{P}_{ii,k}} = \frac{\partial \mathbf{Q}_{in}}{\partial Y_i} = 0$ if $n \neq i$, we can conclude that a trivial solution in case of $\mathcal{P} = \tilde{P}_{ji,k}$ or $\tilde{P}_{ii,k} \in \tilde{\mathbf{P}}_i \subset \mathbb{P}_i$ is achieved where

$$\sum_{n=1}^N \sum_{k=1}^K \left[\frac{\partial T_i}{\partial Q_{ni,k}} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial Q_{ni,k}} \right] = 0,$$

which means that the income effect is neutral at the optimum. We show in the next section that this trivial solution is also the unique solution to the system of FOCs. Our above discussion shows that the optimal choice with respect to $\tilde{P}_{ii,k} \in \tilde{\mathbf{P}}_i$ and $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$ entails that income effects are welfare neutral: $\frac{\partial W_i}{\partial Y_i} = 0$. We summarize this conclusion in the following lemma.

Lemma 3. *Within the feasible policy-income set, $(\mathbb{P}_i; Y_i) \in \mathbb{F}_i^Y$, if $\tilde{\mathbf{P}}_i \subset \mathbb{P}_i$ is chosen optimally, then income effects are welfare-neutral, $\frac{\partial W_i}{\partial Y_i} = 0$.*

Putting the Three Steps Together

We outline the results from Lemmas 1,2,3 in the following proposition.

Proposition 1. [Intermediate Envelope Result] *Country i 's optimal policy, \mathbb{P}_i^* , is the solution to a system of equations that asserts optimality w.r.t. all $\mathcal{P}_i \in \mathbb{P}_i$, holding fixed wages and income,*

$$\tilde{P}_i \frac{\partial V_i(\cdot)}{\partial \ln \mathcal{P}} + \left(\frac{\partial T_i}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}, \mathbf{Q}_i} - \tilde{P}_i \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}, \mathbf{Q}_i} + \left[\left(\frac{\partial T_i}{\partial \mathbf{Q}_i} \right)_{\mathbf{w}} - \tilde{P}_i \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathbf{Q}_i} \right)_{\mathbf{w}} \right] \cdot \left(\frac{\partial \mathbf{Q}_i}{\partial \mathcal{P}} \right)_{Y_i} = 0 \quad (\star).$$

We refer to Proposition 1 as an intermediate envelop result, because it reduces our general equilibrium optimal policy problem into one in which wage and income effects can be ignored. In other

words, we can derive the optimal policy schedule while treating \mathbf{w} as constant and ignoring Y_i 's impact on country i 's demand schedule. Below, we discuss several aspects of this intermediate envelope result.

As noted in the build up to Lemma 3, the first three terms in Equation (\star) collapse to zero when $\mathcal{P} = \tilde{P}_{ji,k}$ or $\tilde{P}_{ii,k} \in \mathbb{P}_i$. Relatedly, $\frac{\partial V_i(\cdot)}{\partial \ln(1-a_{i,k})} = \frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ij,k}} = 0$ since neither $a_{i,k}$ or $\tilde{P}_{ij,k}$ explicitly enter the indirect utility function. Furthermore, $\left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}, \mathbf{Q}_i} = 0$ since $\tilde{P}_{ij,k}$ affects emission only through its effect on output quantities, \mathbf{Q}_i ; and $\left(\frac{\partial \mathbf{Q}_i}{\partial \ln(1-a_{i,k})}\right)_{Y_i} = 0$ since holding prices (which are in \mathbb{P}_i) and income fixed, abatement has not effect on the demand schedule. Accounting for these equal-to-zero terms, \mathbb{P}_i^* solves the following system according to Proposition 1:

$$\begin{cases} \left(\frac{\partial T_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}, \mathbf{Q}_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}, \mathbf{Q}_i} = 0 & [a_{i,k}] \\ \tilde{P}_{ij,k} \mathcal{Q}_{ij,k} + \sum_{n \in \mathbb{C}} \sum_{k \in \mathbb{K}} \left[\left(\frac{\partial T_i}{\partial \mathcal{Q}_{nj,k}} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathcal{Q}_{nj,k}}\right) \frac{\partial \mathcal{D}_{nj,k}(\cdot)}{\partial \ln \tilde{P}_{nj,k}} \right] = 0 & [\tilde{P}_{ij,k}] \\ \sum_{n \in \mathbb{C}} \sum_{k \in \mathbb{K}} \left[\frac{\partial T_i}{\partial \mathcal{Q}_{ni,k}} - \tilde{P}_i \frac{\partial \delta_i \cdot \mathbf{Z}}{\partial \mathcal{Q}_{ni,k}} \right] = 0 & [\tilde{P}_{ji,k}, \tilde{P}_{ii,k}] \end{cases} \quad (19)$$

Discussion. Before solving the above system, a few details about Proposition 1 are in order. Above all, Proposition 1 holds when country i 's government has access to all *price-related* policy instruments. As for wage effects, if the government is prohibited from setting any of the instruments that correspond to the after-tax prices of varieties originating from home (namely, $\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ij}\}$), then Lemma 2 fails. The intuition is the following: The government in country i can improve its terms-of-trade by inflating its wage, w_i , relative to \mathbf{w}_{-i} . The gains from inflating w_i can be perfectly mimicked with an appropriate adjustment in production and export subsidies, $\{s_{i,k}, x_{ij,k}\}_{j \neq i,k}$. This adjustment in our reformulated problem corresponds to an appropriate choice vis-à-vis price vectors, $\tilde{\mathbf{P}}_{ii}$ and $\tilde{\mathbf{P}}_{ij}$. The argument is that a proper adjustment in production and export subsidies can achieve any level of national sales; and, provided that labor markets clear, national sales pin down home's wage, w_i . This argument holds even if the choice with respect to $\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ij}\}$ is not optimal, but it fails if the government is prohibited from manipulating any element of these price vectors. In that case, wage effects become non-neutral and should be properly tracked when solving the optimal policy problem

A similar argument applies to Lemma 3 which states that if the government can set all price variables associated with the local consumption market optimally, then income effects are redundant. Because any gains from raising Y_i are already internalized by the vector of consumer prices in home. But if the government is prohibited from manipulating any element in $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}$, the argument no longer holds. Also notice, the welfare-neutrality of income effects explain why income elasticities

of demand play no role in the optimal policy schedule that follows.

Finally, note that the ability to set prices in foreign markets, \tilde{P}_{ij} , is only relevant to Lemma 2 but irrelevant to 3. So even if the government cannot set \tilde{P}_{ij} , we can still invoke Lemma 3 to simplify the optimal policy problem. In addition, if abatement \mathbf{a}_i is set sub-optimally, Lemmas 2 and 3 continue to hold. Hence, Proposition 1 applies to second-best scenarios where governments cannot tax emission but can manipulate the entire vector of after-tax prices, $\{\tilde{P}_{ii}, \tilde{P}_{ji}, \tilde{P}_{ij}\}$, associated with their economy.

3.2 Characterizing the Optimal Tax Schedule

Proposition 1 describes the system of F.O.C.s that characterize the optimal policy schedule. Since we assume a non-parametric demand function, we present this system using the own- and cross-price demand elasticities defined in Section 2.1. The following lemma summarizes this step, with detailed derivations provided in Appendixes B.2 and B.4.

Lemma 4. *Country i 's optimal policy, $\mathbb{P}_i^* = \{\tilde{P}_{ij}^*, \tilde{P}_{ji}^*, \tilde{P}_{ii}^*, \mathbf{a}_i^*\}$, solves the following system of F.O.C.s:*

$$\begin{aligned}
[a_{i,k}] \quad & \tilde{\delta}_{ii,k} v_{i,k}(a_{i,k}^*) - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} = 0; \\
[\tilde{P}_{ni,k} = \tilde{P}_{ji,k}, \tilde{P}_{ii,k}] \quad & \sum_{n \neq i} \sum_g \left[\left(\frac{\tilde{P}_{ji,g}^*}{P_{ji,g}} - \left(1 + \omega_{ji,g} + \tilde{\delta}_{ji,g} v_{j,g} \frac{\gamma_g - 1}{\gamma_g} \right) \right) e_{ji,g} \varepsilon_{ji,g}^{(ni,k)} \right] \\
& + \sum_g \left[\left(\frac{\tilde{P}_{ii,g}^*}{P_{ii,g}} - \left(1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g} + \tilde{\delta}_{ii,g} v_{i,g} \right) \frac{\gamma_g - 1}{\gamma_g} \right) e_{ii,g} \varepsilon_{ii,g}^{(ji,k)} \right] = 0 \\
[\tilde{P}_{ij,k}] \quad & 1 - \sum_{\ell \neq i} \sum_g \left[\left(\omega_{\ell i,g} + \tilde{\delta}_{\ell i,g} v_{\ell,g} \frac{\gamma_g - 1}{\gamma_g} \right) \frac{e_{\ell j,g}}{e_{ij,k}} \varepsilon_{\ell j,g}^{(ij,k)} \right] \\
& + \sum_g \left[\left(1 - \left(1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g} + \tilde{\delta}_{ii,g} v_{i,g} \right) \frac{\gamma_g - 1}{\gamma_g} \frac{P_{ij,g}^*}{\tilde{P}_{ij,g}} \right) \frac{e_{ij,g}}{e_{ij,k}} \varepsilon_{ij,g}^{(ij,k)} \right] = 0
\end{aligned}$$

where $\omega_{ji,k}$ denotes the inverse of good ji, k 's "general equilibrium" export supply elasticity.¹¹

The optimality condition w.r.t. $a_{i,k}$ equalizes the marginal utility loss that stems from raising the marginal cost of production and the marginal utility gains associated with lower emissions. Com-

¹¹As elaborated in Appendix XX, $\omega_{ji,k}$ summarizes how a contraction in good ji, k 's export supply affects the entire vector of producer prices associated with country i 's economy. The same appendix provides an exact characterization of $\omega_{ji,k}$ as well as a first-order approximation that appears as follows:

$$\omega_{ji,k} \approx \frac{-\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1 - \frac{\mu_k}{1+\mu_k} \sum_{\ell \neq i} r_{j\ell,k} \varepsilon_{j\ell,k}} \left[1 - \frac{\mu_k}{1+\mu_k} \frac{w_i L_i}{w_j L_j} \sum_{n \neq i} \rho_{i,k} r_{in,k} \varepsilon_{in,k}^{(jn,k)} \right].$$

binning the F.O.C w.r.t $a_{i,k}$ with Equation (13) that relates emission intensity to emission tax, yields the following formula for the optimal emission tax:

$$\tau_{i,k}^* = \tilde{\delta}_{ii,k} = \tilde{\phi}_i + \tilde{\phi}_{i,k}, \quad (20)$$

where recall that $\tilde{\phi}_i \equiv \tilde{P}_i \phi_i$, $\tilde{\phi}_{i,k} \equiv \tilde{P}_i \phi_{i,k}$, and $\tilde{\delta}_{ii,k} \equiv \tilde{P}_i \delta_{ii,k}$. The F.O.C.s w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k}$ are interdependent, and contain price ratios in the form of $\frac{\tilde{P}_{ji,k}^*}{P_{ji,g}}$ that do not show up in the F.O.C.s w.r.t. $\tilde{P}_{ij,k}$. Setting $\tau_{i,k}^* = \tilde{\delta}_{ii,k}$, these F.O.C.s amount to NK equations and NK unknowns, which are summarized by the following matrix equation:

$$\begin{bmatrix} e_{1i,1} \varepsilon_{1i,1}^{(1i,1)} & \cdots & e_{Ni,1} \varepsilon_{Ni,1}^{(1i,1)} & \cdots & e_{1i,K} \varepsilon_{1i,K}^{(1i,1)} & \cdots & e_{Ni,K} \varepsilon_{Ni,K}^{(1i,1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ e_{1i,1} \varepsilon_{1i,1}^{(Ni,K)} & \cdots & e_{Ni,1} \varepsilon_{Ni,1}^{(Ni,K)} & \cdots & e_{1i,K} \varepsilon_{1i,K}^{(Ni,K)} & \cdots & e_{Ni,K} \varepsilon_{Ni,K}^{(Ni,K)} \end{bmatrix} \begin{bmatrix} \frac{\tilde{P}_{1i,k}^*}{P_{1i,1}} - \left(1 + \omega_{1i,k} + \tilde{\delta}_{1i,k} v_{1,k} \frac{\gamma_k - 1}{\gamma_k}\right) \\ \vdots \\ \frac{\tilde{P}_{ii,k}^*}{P_{ii,k}} - \frac{\gamma_k - 1}{\gamma_k} \\ \vdots \\ \frac{\tilde{P}_{Ni,k}^*}{P_{Ni,k}} - \left(1 + \omega_{Ni,k} + \tilde{\delta}_{Ni,k} v_{N,k} \frac{\gamma_k - 1}{\gamma_k}\right) \end{bmatrix} = \mathbf{0}. \quad (21)$$

The first matrix is $NK \times NK$ and the second is $NK \times 1$. Importantly, the above equation identifies the optimal tariff, $1 + t_{ji,k}^* = \tilde{P}_{ji,k}^* / P_{ji,k}$, and production subsidy, $1 + s_{i,k}^* = P_{ii,k} / \tilde{P}_{ii,k}^*$ independently from the choice of export subsidies, $1 + x_{ij,k}^* = P_{ij,k} / \tilde{P}_{ij,k}^*$. To solve the above matrix equation, we invoke on another intermediate result that ensures the invertibility of the system.

Lemma 5. *The square matrix, $\Xi = \left[e_{ji,k} \varepsilon_{ji,k}^{(ni,g)} \right]_{ng,jk}$, is non-singular, with $|\det(\Xi)| > \prod_{n,k} e_{ni,k} > 0$.*

Given Lemma 5, the unique solution to Equation 21 is the trivial solution, which indicates that:

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}} = 1 + \omega_{ji,k} + \tilde{\delta}_{ji,k} v_{j,k} \frac{\gamma_k - 1}{\gamma_k}; \quad 1 + s_{i,k}^* = \frac{P_{ii,k}}{\tilde{P}_{ii,k}^*} = \frac{\gamma_k}{\gamma_k - 1}.$$

where it is understood that the CPI-adjusted disutility from transboundary emission, $\tilde{\delta}_{ji,k}$, is blind to which country-industry generates it, and so, $\tilde{\delta}_{ji,k} = \tilde{\phi}_i$. Lastly, we can plug the already-derived values of $\{\tau_{i,k}^*, t_{ji,k}^*, s_{i,k}^*\}_{j \neq i,k}$ (or equivalently, $\{a_{i,k}^*, \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}}, \frac{\tilde{P}_{ii,k}^*}{P_{ii,k}}\}_{j \neq i,k}$) into the first-order conditions w.r.t. $\{\tilde{P}_{ij,k}\}_{j \neq i,k}$. This final step, which solves for $x_{ij,k}^*$, is outlined in Appendix B.4. The following theorem summarizes the optimal policy schedule in its final form.

Theorem 1. *The optimal unilateral tax schedule for country i is given by*

$$\begin{aligned}
[\text{import tax}] \quad 1 + t_{ji,k}^* &= 1 + \omega_{ji,k} + \tilde{\phi}_i v_{j,k} \frac{\gamma_k - 1}{\gamma_k} & \forall j, k \\
[\text{export subsidy}] \quad 1 + x_{ij,k}^* &= \left(1 + \frac{1}{\varepsilon_{ij,k}}\right) \chi_{ij,k} & \forall j, k \\
[\text{domestic subsidy}] \quad 1 + s_{i,k}^* &= \frac{\gamma_k}{\gamma_k - 1} & \forall k \\
[\text{emission tax}] \quad \tau_{i,k}^* &= \tilde{\phi}_i + \tilde{\phi}_{i,k} & \forall k
\end{aligned} \tag{22}$$

where $\chi_{ij,k}$ is an export subsidy intended at lowering the emission of product varieties competing with ij, k , and

$$\text{is given by } \chi_{ij} = \left[\frac{\tilde{e}_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{\tilde{e}_{ij,k} \varepsilon_{ij,k}} \right]_{k,g}^{-1} \left[\frac{\sum_{n \neq i} t_{ni,g}^* \tilde{e}_{ni,g} \varepsilon_{ni,g}^{(ij,k)}}{\sum_{\hat{n} \neq i, \hat{g}} \tilde{e}_{\hat{n},\hat{g}} \varepsilon_{\hat{n},\hat{g}}^{(ij,k)}} \right]_{k,g} \mathbf{1}_K.$$

To put in words, the optimal unilateral policy for country i includes (i) a uniform Pigouvian tax on CO2 emission, $\tilde{\phi}_i$, adjusted for its implied local externality, $\tilde{\phi}_{i,k}$, (ii) an industry-specific Pigouvian production subsidy, $s_{i,k}^*$, that eliminates the cross-industry markup heterogeneity, (iii) import taxes, $t_{ji,k}^*$ that penalize high-emission imports and also take advantage of unexploited import market power, and (iv) export subsidies, $x_{ij,k}^*$, that promote low-emission exports and also take advantage of unexploited export market power. The fact that production subsidies are *emission-blind* is a manifestation of the *Targeting Principle*. By this principle, emission taxes are the optimal instrument to tackle the externality of local emission on domestic consumers, because they correct the externality at its source.

Optimal trade taxes are designed to both improve the terms-of-trade (ToT) and correct transboundary emissions. So, a decomposition of these taxes is in order. First, consider the import tax on variety ji, k . The optimal rate, as implied by Proposition 1, can be decomposed as follows:

$$1 + t_{ji,k}^* = \underbrace{1 + \omega_{ji,k}}_{\text{ToT driven}} + \underbrace{\tilde{\phi}_i v_{j,k} \frac{\gamma_k - 1}{\gamma_k}}_{\text{CO2 reducing}}. \tag{23}$$

The ToT-driven component is motivated by country i 's collective import market power vis-à-vis partner j . It corresponds to an optimal mark-down on the producer price of goods imported from country j . This mark-down equals the inverse of the *export supply elasticity*, $\omega_{ji,k} < 0$. The CO2-reducing term is intended to tackle the transboundary CO2 emission externality of goods imported from origin j . Notice that, in a non-cooperative setting, country j does not internalize the transboundary CO2 externality that it generates for country $i \neq j$. Hence, country i 's optimal tariff on j incorporates a markup

that is proportional to the carbon-intensity of imported goods from j , $v_{j,k}$ —hence, the term *border carbon adjustment*.

Likewise, export subsidies are designed to both improve the terms-of-trade (ToT) and correct transboundary CO2 emissions. The export subsidy on good ij,k , therefore, exhibit two distinct components:

$$1 + x_{ij,k}^* = \underbrace{\left(1 + \frac{1}{\varepsilon_{ij,k}}\right)}_{\text{ToT driven}} \times \underbrace{\chi_{ij,k}}_{\text{CO2 reducing}}. \quad (24)$$

The ToT-driven term equals an optimal markup, if country i were pricing its composite export good as a single representative monopolist. The CO2-reducing term subsidizes exports of varieties that compete with carbon-intensive (high- v) foreign varieties in market j . Note that this term internalizes the import tax charged by country i on various trading partners. The intuition can be put as follows. If export subsidies prompt more exports from home country i to destination j , then a third country n would reallocate its exports away from j and possibly back to home country i . This gives home country i some leverage to curb CO2 emissions in every third-country n . We provide a more detailed discussion when we consider CES-Cobb-Douglas preferences.

3.3 Optimal Policy Formulas in Special Cases

Special Case: Ricardian Model. In the limit where $\gamma_k \rightarrow \infty$ and $f_k^e \rightarrow 0$, firms can be viewed as perfectly competitive and our framework reduces to a Ricardian trade model. The Ricardian special case of our framework is isomorphic to the multi-industry [Eaton and Kortum \(2002\)](#) model. The optimal tax formulas in the Ricardian case can be attained by plugging the following values into Theorem 1:

$$\frac{\gamma_k}{\gamma_k - 1} \rightarrow 1; \quad \omega_{ji,k} \rightarrow 0 \quad (\text{Ricardian Model})$$

Note that, in principle, Theorem 1 applies equally to a model with a continuum of industries. As a result, in the limit where $\varepsilon_{ij,k} \rightarrow \infty$, our optimal tax formulas characterize the optimal policy in the [Dornbusch et al. \(1977\)](#) model studied by [Costinot et al. \(2015\)](#).

Special Case: Cobb-Douglas-CES preferences. To gain further intuition about the optimal policy schedule, consider the special case where preferences have a Cobb-Douglas-CES formulation,

$$U_i(\mathbf{Q}_i) = \prod_k \left(\sum_j b_{ji,k}^{1/\sigma_k} Q_{ji,k}^{\frac{\sigma_k-1}{\sigma_k}} \right)^{e_{i,k} \frac{\sigma_k}{\sigma_k-1}}, \quad (25)$$

where $e_{i,k}$ is the expenditure share of country i on industry k , and σ_k is the (Armington) elasticity of substitution between origin countries. The Marshallian demand elasticities in this special case are given by the following formulas:

$$\varepsilon_{ji,k} \equiv \varepsilon_{ji,k}^{(ji,k)} = -1 - (\sigma_k - 1)(1 - \lambda_{ji,k}), \quad \varepsilon_{ni,k}^{(ji,k)} = (\sigma_k - 1)\lambda_{ji,k} \quad (n \neq j); \quad \varepsilon_{ni,g}^{(ji,k)} = 0 \quad (g \neq k).$$

Plugging the above values into Theorem 1, yields the following formula for optimal trade taxes,

$$1 + t_{ji,k}^* = 1 + \tilde{\phi}_i v_{j,k}^* \frac{\gamma_k - 1}{\gamma_k} - \frac{r_{ji,k}^*}{\gamma_k - \sum_{n \neq i} r_{jn,k}^* \left[1 + \varepsilon_k(1 - \lambda_{jn,k}^*) \right]}$$

$$1 + x_{ij,k}^* = \left[1 + \frac{\gamma_k - 1}{\gamma_k} \sum_{n \neq i} \tilde{\phi}_i v_{n,k}^* \frac{\lambda_{nj,k}^*}{1 - \lambda_{ij,k}^*} \right] \left(1 + \frac{1}{(\sigma_k - 1)(1 - \lambda_{ij,k}^*)} \right)^{-1}. \quad (26)$$

where the term inside the brackets in the second line corresponds to $\chi_{ij,k}$ in Theorem 1. Absent emission externalities (i.e., set $v_{n,k} = 0$ for all n, k), the above formulas collapse to the familiar optimal trade tax formulas in multi-industry quantitative trade models (see [Lashkaripour and Lugovskyy \(2016\)](#)).

3.4 The Trade-Off Between Emission Correction, Scale Economies, and the ToT

The formulas under Equation (26) uncover some primitive trade-offs facing border adjustment carbon taxes. The first trade off is reflected in the emission-correcting term in the optimal import tax formula. This term is the product of (1) $\tilde{\phi}_i v_{j,k}$, which taxes carbon-intensive imports, and (2) $\frac{\gamma_k-1}{\gamma_k}$, which operates as a tax deflator for industries with high returns-to-scale in emission (low- γ_k). As such, the effectiveness of import taxes at reducing transboundary CO2 emissions is dictated by $\text{Cov}_k(v_{j,k}, \gamma_k)$. In the case where $\text{Cov}_k(v_{j,k}, \gamma_k) < 0$, import taxes are an ineffective emission-reducing instrument because the high-emission industries that have to be penalized are also the high-returns-to-scale industries whose production should not be contracted. Alternatively, if $\text{Cov}_k(v_{j,k}, \gamma_k) > 0$ import taxes become quite effective as they hit two birds with one stone.

A similar trade-off faces export subsidies: the optimal export subsidy includes an emission-correcting term (in brackets) that promotes country i 's clean exports against its high-emission competition in market j .¹² This term, though, is smaller the higher the degree of scale economies in an industry, i.e., the lower $(\gamma_k - 1)/\gamma_k$ in the second line of Equation (26). The intuition is the same: promoting one's exports against carbon-intensive competition leads to an additional increase in the competition's carbon-intensity through scale effects. So, the optimal level of promotion is accordingly weaker.¹³

The second trade-off occurs between the *ToT-driven* and *emission-correcting* terms. With regards to import taxes, if $\text{Cov}_k(v_{j,k}, \gamma_k) < 0$ the ToT-driven component asks for a lower import tariff on carbon-intensive industries. With regards to export subsidies, if $\text{Cov}_k(v_{j,k}, \sigma_k) > 0$ the ToT-driven component asks for a higher export subsidy (or a lower export tax) on carbon-intensive industries. Hence, in both case, the optimal policy net of climate objectives may exhibit a climate or environmental bias. The direction or magnitude of these trade-offs is ultimately an empirical issue, which we will come back to in Section 5 when our model is mapped to data.¹⁴

3.5 Discussion: Optimal Policy in Second-Best Scenarios

Theorem 1 concerns a unilaterally first-best scenario in which the government has access to a complete set of policy instruments. In many scenarios, governments may face limitations in using the policy instruments necessary to achieve the first best. In addition to prevalent political economy issues, second-best scenarios may arise from agreements on environment or trade that tie the policy-maker's hands with regards to certain policy tools. In the section, we derive three sets of results that shed light on these second-best scenarios.

¹²Specifically, suppose that good ij, k competes with high-emission (high- $\tilde{\delta}_{ni}v_{n,k}$) varieties in market j . In that case, country i 's government will apply a relatively high export subsidy (or a lower export tax) to good ij, k to increase its sales in market j against high-emission rivals there. Recall from Theorem 1, that emission-correcting term is governed by the emission externality of rival varieties ($\{\tilde{\delta}_{ni}v_{n,k}\}_{n \neq i}$) and the degree of cross-substitutability between ij, k and these rival varieties ($\epsilon_{nj,g}^{(ij,k)}$). The latter effect in this special case is factored out in the term that depends on σ_k .

¹³To dig deeper, the magnitude of the emission-correcting term depends on the interaction between three terms. First, the lower γ_k , the larger the scope for scale economies in abatement. Hence, penalizing foreign varieties with export subsidies is less effective. Second, the smaller the perceived disutility from foreign emissions (lower $\delta_{ni}v_{ni}$ for $n \neq i$), the larger the incentive to use export policy to correct these emissions. Third, the greater the market share of high-emission international varieties in market j (higher $\lambda_{nj,k}$), the greater the incentive to promote exports of clean, locally-produced varieties to that market.

¹⁴We can connect this result to a recent literature that documents an environmental bias in *applied* tariffs. Shapiro (2020) attributes this bias to upstream, carbon-intensive industries being more organized than downstream industries. As such, they can demand a greater degree of tariff protection from governments. Our theory instead indicates that such biases can arise purely from terms-of-trade considerations.

First, we examine optimal taxes in a second-best scenario where a country takes emission taxes as given. This case is relevant for a country under a commitment to international agreements on climate, which prevents it from setting emission taxes according to unilateral objectives. In that case, the government alters its production subsidies to:

$$1 + s_{i,k}^{**} = \frac{\gamma_k}{\gamma_k - 1} [1 + \tilde{\delta}_{ii,k}(v_{i,k} - v_{i,k}^*)]^{-1}, \quad (27)$$

where v_k^* is the emission intensity attainable under the first-best unilateral policy. Consider the case where emission intensity, $v_{i,k}$, is smaller than $v_{i,k}^*$ because country i is abiding with an international climate agreement. In that case, $(1 + s_{i,k}^{**})$ includes an extra subsidy that promotes domestic production. This extra subsidy acts against the climate goals in international agreements. Hence, it is important for international agreements on climate to couple emission taxes with restrictions on production subsidies.¹⁵

Second, we consider the second-best case in which countries do not have access to export policies, resembling the ban on export subsidies under the WTO. As detailed in Appendix C, optimal production subsidies and emission taxes remain *ToT-blind* in this case. More specifically, the formula for production subsidies and emission taxes as well as the emission-correcting term in import taxes remain unchanged. The only alteration is a uniform multiplier applied to the ToT-improving component in the optimal import tax formula under Equation (23). Intuitively, by the Lerner symmetry, import taxes are strictly more effective than emission taxes or production subsidies at mimicking export subsidies. As such, when import taxes are applicable, there is no rationale for altering emission or production taxes to compensate for the absence of export subsidies.

Lastly, there is widespread skepticism that environmental policies are occasionally used as protection in disguise. The argument is that when governments are banned from exercising trade or industrial policies, they may turn to carbon taxes as a second best trade-restricting instrument. To make the case, we suppose that all tax instruments aside from carbon taxes are banned. We show in Appendix C, for a simplified version of our model, that in this case, optimal carbon taxes will be no longer uniform. Instead, it is optimal for a country to apply carbon taxes above the first-best rate in industries where the trade elasticity, $(\sigma_k - 1)$, is low. Doing so, enables a country to contract exports

¹⁵On a related note, our model collapses to one with exogenous emission intensity à la Markusen (1975) as the emission elasticity approaches zero, i.e., $\alpha_{i,k} \rightarrow 0$ (See footnote (8)). Here, emission taxes can be dropped from the model as firms do not undertake abatement. In this case, the optimal production subsidy includes the markup-correcting term $\frac{\gamma_k}{\gamma_k - 1}$ plus an extra term that taxes high-emission (high- v) industries. Namely, $1 + s_{i,k}^{**} = \frac{\gamma_k}{\gamma_k - 1} (1 + \tilde{\delta}_{ii,k}v_{i,k})^{-1}$. This formula can be also derived by setting $v_{i,k}^* = 0$ in equation (27).

in high-market-power industries as an indirect means to extract markups from the rest of the world.

4 Non-cooperative Nash Equilibrium *vs.* Global Climate Cooperation

In this section, we discuss policy outcomes when many countries simultaneously set their policies. We first discuss the case of global climate cooperation in Subsection 4.1. Such a case yields the globally optimum outcome via deep international cooperation. Then, in Subsection 4.2, we characterize the non-cooperative Nash equilibrium where non-cooperative countries who act in their self interest simultaneously apply their optimal unilateral policy. Equipped with these theoretical results, Section 6 quantifies the consequences of global cooperation versus non-cooperation on climate issues.

4.1 Global Climate Cooperation

The globally optimum outcome is attainable when all countries coordinate their emission taxes, while internalizing their emission externality on the rest of the world. Such a scenario is akin to a deep multilateral agreement on trade and climate. Below, we formally define this scenario, which we label global climate cooperation.

Definition. *Global Climate Cooperation* corresponds to an equilibrium wherein all governments set their policy instruments cooperatively in order to maximize global welfare, $\sum_i W_i$, subject to equilibrium conditions (1)-(9).

Under global climate cooperation all countries apply zero trade taxes, as these taxes create inefficient distortions from a global perspective. Globally optimal production subsidies solely correct markup distortions, by restoring marginal cost pricing in each industry. Globally optimal emission taxes are of Pigouvian nature, correcting each origin's local and transboundary emission externality. In formal terms, the aforementioned policy schedule can be expressed as follows for each country i :

$$\mathbf{x}_i^* = \mathbf{t}_i^* = \mathbf{0}; \quad 1 + s_{i,k}^* = \frac{\gamma_k}{\gamma_k - 1}; \quad \tau_{i,k}^* = \tilde{\phi}_{i,k} + \underbrace{\sum_{j \in \mathbb{C}} \tilde{\phi}_j}_{\tilde{\phi}^W} \quad (28)$$

Globally optimal emission taxes in country-industry (i, k) consists of two components: (i) one that corrects for externalities from local pollution, $\tilde{\phi}_{i,k}$, and (ii) one that corrects for the global externality of CO2 emissions, $\tilde{\phi}^W = \sum_{j \in \mathbb{C}} \tilde{\phi}_j$. To put this formula in perspective, consider the required raise in

emission taxes of country i when it goes from the unilaterally optimal to the globally optimal emission taxes. Assuming away from the equilibrium changes to the consumer price index \tilde{P}_i between these two policy schedules, the required raise in the emission tax equals:

$$\tau_{i,k}^{\text{Global}} - \tau_{i,k}^{\text{Unilateral}} \approx \tilde{\phi}^W - \tilde{\phi}_i \quad (29)$$

The free-riding problem that impedes global cooperation is manifested by Equation (29). In a non-cooperative setting, a country does not have the incentive to correct for the CO2 externality it generates for residents of other countries. As a consequence, any compensation scheme (such as transfers between countries to incentivize cooperation) or penalty device (such as using trade taxes to punish non-cooperation) may enforce a global climate cooperation only when it implies benefits that exceeds the costs, $(\tilde{\phi}^W - \tilde{\phi}_i)$, for every country i .

4.2 Non-Cooperative Nash Equilibrium

As in the previous case, we start with a formal definition of the non-cooperative Nash equilibrium.

Definition. The *Non-Cooperative Nash Equilibrium* corresponds to a case where non-cooperative countries simultaneously choose their optimal unilateral policy taking policy choices in the rest of the world as given.¹⁶

In the Nash equilibrium, the unilaterally optimal emission tax and production subsidy formulas are still characterized by Theorem 1. However, the trade share, $\lambda_{nj,k}$, and carbon intensities, $v_{j,k}$, in these formulas now depend on policy choices in the rest of the world. Specifically, Consider country i 's optimal export subsidies and import taxes. They depend on transboundary carbon intensities, $\{v_{j,k}\}_{j \neq i}$, which are regulated by optimal emission taxes adopted by other countries ($j \neq i$). Using equation (13) and given that $\tau_{j,k}^* = \tilde{\delta}_{jj,k}$ for all $j \in \mathbb{C}$,

$$v_{j,k}^* = \alpha_{j,k} \frac{\gamma_k - 1}{\gamma_k} \tilde{\delta}_{jj,k}^{-1}$$

Supposing preferences are Cobb-Douglas-CES and each country is sufficiently small relative to the rest of the world, we can plug the above expression into Equation (26) to arrive at the following optimal trade tax schedule.

¹⁶This situation is akin to a one-shot non-cooperative Nash game.

Proposition 2. *The non-cooperative Nash equilibrium is characterized by each country applying the following tax schedule:*

$$\begin{aligned}
[\textit{import tax}] \quad 1 + t_{ji,k}^* &= 1 + \left(\frac{\gamma_k - 1}{\gamma_k} \right)^2 \frac{\alpha_{j,k} \tilde{\delta}_{ji,k}}{\tilde{\delta}_{jj,k}} \\
[\textit{export subsidy}] \quad 1 + x_{ij,k}^* &= \left[1 + \sum_{n \neq i} \left(\left(\frac{\gamma_k - 1}{\gamma_k} \right)^2 \frac{\alpha_{n,k} \tilde{\delta}_{ni,k}}{\tilde{\delta}_{nn,k}} \lambda_{nj,k} \right) \right] \left(\frac{\sigma_k}{\sigma_k - 1} \right)^{-1} \\
[\textit{domestic subsidy}] \quad 1 + s_{i,k}^* &= \frac{\gamma_k}{\gamma_k - 1} \\
[\textit{carbon tax}] \quad \tau_{i,k}^* &= \tilde{\delta}_{ii,k} = \tilde{\phi}_{i,k} + \tilde{\phi}_i
\end{aligned}$$

The optimal emission taxes and production subsidies remain the same (as in Equation 22), even though all countries simultaneously apply taxes and subsidies. That is because, when governments have access to a complete set of trade tax instruments, their unilaterally optimal choice with respect to emission taxes and production subsidies is independent of economic variables in the rest of the world—see Theorem 1.

As in the unilateral case, optimal trade taxes in country i correct transboundary CO2 externalities. But the extent of these externalities, here, depends on cross-national differences in the perceived cost of CO2 emissions. Consider two polar cases. First, suppose that the perceived disutility from CO2 emissions by country i is negligible compared to that by country j — e.g. because country i is very small relative to j , or it discounts the future effectively at a much higher rate—, then $\tilde{\delta}_{ji,k}/\tilde{\delta}_{jj,k} \approx 0$, and border carbon adjustment of i over j becomes negligible. Second, suppose, $\tilde{\phi}_{i,k} = 0$, and $\tilde{\delta}_{ji,k} = \tilde{\phi}_i$, meaning that the perceived disutility from emissions is exclusively from CO2 emissions. In this case, $\tilde{\delta}_{ji,k}/\tilde{\delta}_{jj,k} = \tilde{\phi}_i/\tilde{\phi}_j$. If country i 's government cares more about CO2 emissions than its counterpart in country j , it will impose a higher border carbon adjustment. If however countries i and j have the same perceived cost of carbon emission, i.e., $\tilde{\phi}_i = \tilde{\phi}_j$, the optimal import tax of country i imposed on country j equals $1 + t_{ji,k}^* = 1 + \alpha_{j,k} ((\gamma_k - 1)/\gamma_k)^2$. In any of these events, from country i 's perspective, country j 's domestic emission tax on good ji, k is sub-optimal as it does not internalize the transboundary cost of CO2 emissions. So, it is optimal for country i to tax imports originating from *high- $\alpha_k \times$ high- γ_k* industries in country j to partially correct the transboundary CO2 externality associated with these imports.¹⁷

¹⁷A similar logic explains why the square of the inverse markup, $\left(\frac{\gamma_k - 1}{\gamma_k} \right)^2$, appears in formulas specified under Proposition 2. According to equation (3), carbon intensity per unit of production, $Z_{n,k}/Q_{n,k}$ is proportional to $(Q_{n,k}/(1 - a_{n,k}))^{-1/\gamma_k}$. That is, carbon intensity is affected by scale economies in both production and abatement, gov-

5 Mapping Theory to Data

In this section, we describe how our equilibrium relationships, including our optimal policy formulas, map to data. Our objective is to use this mapping to quantify the full effectiveness of trade policy at reducing global CO2 emissions. For quantification purposes, we consider the Cobb-Douglas-CES case of our model. To simplify the presentation of our quantitative approach, we define $\rho_{n,k} \equiv L_{n,k}/\bar{L}_n$ as the share of national labor employed in industry k . In addition, we suppose that domestic emission taxes, $\{\tau_{i,k}\}$, that we observe in the baseline data are unilaterally optimal. In this regard, our exercises are not designed to examine the impact of a higher perceived disutility from emission. Rather, our exercises are meant to shed light on the effects of (i) incorporating border carbon adjustments in trade taxes, and (ii) exercising cooperative emission taxes as opposed to acting non-cooperatively.

5.1 Non-cooperative Nash Equilibrium

Using our optimal tax formulas, we fully characterize the change in equilibrium values when moving from the factual (baseline) equilibrium to the counterfactual *non-cooperative* outcome. Our information set consists of baseline values for key economic variables $\mathcal{B}_v \equiv \{\lambda_{ni,k}, r_{ni,k}, \rho_{i,k}, \tilde{\delta}_{ni,k}, e_{n,k}, w_n \bar{L}_n, Y_n\}_{ni,k}$, applied tax/subsidy rates $\mathcal{B}_t \equiv (s_{n,k}, x_{in,k}, t_{ni,k})_{ni,k}$ and structural elasticities $\mathcal{B}_e = \{\sigma_k, \gamma_k, \alpha_{n,k}\}_{n,k}$. Using the exact hat-algebra notation, for any generic variable z , we denote its counterfactual value in the non-cooperative equilibrium as z^* , and its change as $\hat{z} \equiv z^*/z$. Invoking this notation and given full information on $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_e\}$, we can determine the entire vector of optimal tax/subsidy rates, $\mathcal{R}_t \equiv \{s_{n,k}^*, \tau_{n,k}^*, x_{in,k}^*, t_{ni,k}^*\}_{ni,k}$, using our optimal tax formulas and the change in key economic variables, $\mathcal{R}_v \equiv \{\hat{w}_n, \hat{Y}_n, \hat{P}_n, \hat{\rho}_{n,k}\}_{ni,k}$, using the equilibrium conditions. Then, given $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_e\}$ and $\mathcal{R} \equiv \{\mathcal{R}_v, \mathcal{R}_t\}$, we can characterize the counterfactual level of welfare, carbon emission, and other key economic outcomes for each country.

For a clearer exposition, we write baseline variables and structural elasticities $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_e\}$ in blue, the independent unknown variables $\mathcal{R} \equiv \{\mathcal{R}_v, \mathcal{R}_t\}$ in red, and the intermediate unknown variables in black. Following Section 3.2, the optimal tax/subsidy formulas in the Cobb-Douglas-CES

erned by a common parameter γ_k . In the formula for optimal import taxes $t_{ji,k}^*$, the first $(\gamma_k - 1)/\gamma_k$ reflects the importing country i 's desire to dampen the CO2-reducing tariff given scale economies in "production". The second $(\gamma_k - 1)/\gamma_k$ is due to the origin country j 's emission taxes interacting with scale economies in "abatement".

case are given by:

$$\left\{ \begin{array}{l} 1 + t_{ni,k}^* = 1 + \tilde{\delta}_{ni,k} v_{n,k} \hat{v}_{n,k} \hat{P}_i^{\frac{\gamma_k-1}{\gamma_k}} - \frac{r_{ni,k} \hat{r}_{ni,k}}{\gamma_k - \sum_{\ell \neq i} r_{n\ell,k} \hat{r}_{n\ell,k} (1 + (\sigma_k - 1)(1 - \lambda_{n\ell,k} \hat{\lambda}_{n\ell,k}))} \quad \text{a) optimal imp tax } (ni, k) \\ 1 + x_{in,k}^* = \left[1 + \frac{\gamma_k-1}{\gamma_k} \sum_{\ell \neq i} \tilde{\delta}_{\ell i,k} \hat{P}_i v_{\ell,k} \hat{v}_{\ell,k} \frac{\lambda_{\ell n,k} \hat{\lambda}_{\ell n,k}}{1 - \lambda_{in,k} \hat{\lambda}_{in,k}} \right] \left(1 + \frac{1}{(\sigma_k - 1)(1 - \lambda_{in,k} \hat{\lambda}_{in,k})} \right)^{-1} \quad \text{b) optimal exp tax } (in, k) \\ \widehat{1 + t_{ni,k}} = \frac{1 + t_{ni,k}^*}{1 + t_{ni,k}}; \quad \widehat{1 + x_{in,k}} = \frac{1 + x_{in,k}^*}{1 + x_{in,k}}; \quad \widehat{1 + s_{n,k}} = \frac{\gamma_k / (\gamma_k - 1)}{1 + s_{n,k}}; \quad \hat{\tau}_{n,k} = \hat{P}_n \quad \text{c) in changes} \end{array} \right. \quad (30)$$

The change in the variety-level producer prices and the corresponding change in the CES and Cobb-Douglas consumer price indexes can be described as:

$$\left\{ \begin{array}{l} \hat{P}_{ni,k} = \hat{w}_n (\hat{\rho}_{n,k})^{\frac{1}{1-\gamma_k}} (1 - a_{n,k})^{-1} \quad \text{a) producer price } (ni, k) \\ \hat{P}_{ni,k} = \frac{(1 + t_{ni,k})}{(1 + x_{ni,k})(1 + s_{n,k})} \hat{P}_{ni,k} \quad \text{b) consumer price } (ni, k) \\ \hat{P}_{i,k} = \left[\sum_{n=1}^N \lambda_{ni,k} (\hat{P}_{ni,k})^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}} \quad \text{c) consumer price } (i, k) \\ \hat{P}_i = \prod_k (\hat{P}_{i,k})^{e_{i,k}} \quad \text{d) consumer price } (i) \end{array} \right. \quad (31)$$

Note that $\hat{P}_{ni,k}$, in the above equation, encompasses changes in wage, \hat{w}_n , scale, $\hat{\rho}_{n,k}$, and abatement, $(1 - a_{n,k})$, which are each governed by equilibrium conditions. Given the change in consumer prices, the change in within-industry expenditure and revenue shares can be expressed as

$$\left\{ \begin{array}{l} \hat{\lambda}_{ni,k} = \left(\frac{\hat{P}_{ni,k}}{\hat{P}_{i,k}} \right)^{1-\sigma_k} \quad \text{a) within-ind exp share } (ni, k) \\ \hat{r}_{ni,k} = \frac{(1 + t_{ni,k})^{-1} (1 + x_{ni,k}) \hat{\lambda}_{ni,k} \hat{Y}_i}{\sum_{\ell} r_{j\ell,k} (1 + t_{n\ell,k})^{-1} (1 + x_{n\ell,k}) \hat{\lambda}_{n\ell,k} \hat{Y}_\ell} \quad \text{b) within-ind rev share } (ni, k) \end{array} \right. \quad (32)$$

The change in industry-level output, carbon emission, carbon intensity, and abatement are given by:

$$\left\{ \begin{array}{l} \hat{Q}_{n,k} = (\hat{\rho}_{n,k})^{1 + \frac{1}{\gamma_k - 1}} (1 - a_{n,k}) \quad \text{a) output quantity of country-industry } (n, k) \\ \hat{Z}_{n,k} = (1 - a_{n,k})^{\frac{1}{\alpha_{n,k}} + \frac{1}{\gamma_k} - 1} \hat{Q}_{n,k}^{1 - \frac{1}{\gamma_k}} \quad \text{b) emission from country-industry } (n, k) \\ \widehat{1 - a_{n,k}} = (\hat{w}_n / \hat{\tau}_{n,k})^{\alpha_{n,k}} \quad \text{c) abatement in country-industry } (n, k) \\ \hat{v}_{n,k} = 1 / \hat{\tau}_{n,k} \quad \text{d) emission intensity of country-industry } (n, k) \end{array} \right. \quad (33)$$

The expression for $\hat{Q}_{n,k}$ in the above equation derives from the equilibrium price equation, the free entry condition, and the definition of $\rho_{n,k} \equiv L_{n,k} / \bar{L}_n$.¹⁸ The expressions for $\hat{Z}_{n,k}$, $\widehat{1 - a_{n,k}}$, and $\hat{v}_{n,k}$

¹⁸Labor market clearing for country-industry (n, k) requires that: $w_n \rho_{n,k} \bar{L}_n = (1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k}) P_{nn,k} Q_{n,k}$. Replacing for $P_{nn,k}$ from Equation 2, we have: $w_n \rho_{n,k} \bar{L}_n = \left(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k} \right) \bar{p}_{nn,k} w_n \left(\frac{Q_{n,k}}{1 - a_{n,k}} \right)^{1 - 1/\gamma_k}$. Applying hat algebra delivers Equation (33-

respectively derive from applying the hat-algebra notation to Equations 3, 6, and 13. The change in wages and industry-level labor shares are governed by the labor market clearing condition expressed in changes:

$$\begin{cases} \hat{w}_n \hat{\rho}_{n,k} \rho_{n,k} \bar{w}_n \bar{L}_n = \sum_j \left[\frac{(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k})(1 + s_{n,k}^*)(1 + x_{nj,k}^*)}{(1 + t_{nj,k}^*)} \hat{\lambda}_{nj,k} \lambda_{nj,k} e_{j,k} \hat{Y}_j Y_j \right] & \text{a) LMC } (n, k) \\ \sum_k \hat{\rho}_{n,k} \rho_{n,k} = 1 & \text{b) sum of shares=1 } (n) \end{cases} \quad (34)$$

The first line in the above equation ensures that the industry-level wage bill equals total sales net of taxes/subsidies. The second line ensures that the industry-level labor shares add up to one in the counterfactual equilibrium (i.e., $\sum_k \rho_{n,k}^* = 1$). Finally, the change in national income \hat{Y}_n is governed by the representative consumer's budget constraint (BC) in country i , expressed in changes:

$$\begin{aligned} \hat{Y}_n Y_n = & \hat{w}_n \bar{w}_n \bar{L}_n + \sum_k \sum_j \left[\frac{(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k})(1 + s_{n,k}^*)(1 + x_{nj,k}^*)}{(1 + t_{nj,k}^*)} \hat{\lambda}_{nj,k} \lambda_{nj,k} e_{j,k} \hat{Y}_j Y_j \right] \\ & + \sum_k \sum_j \left[\frac{[1 - (1 + s_{n,k}^*)(1 + x_{nj,k}^*)]}{(1 + t_{nj,k}^*)} \hat{\lambda}_{nj,k} \lambda_{nj,k} e_{j,k} \hat{Y}_j Y_j + \frac{t_{jn,k}^*}{1 + t_{jn,k}^*} \hat{\lambda}_{jn,k} \lambda_{jn,k} e_{n,k} \hat{Y}_n Y_n \right]. \quad \text{BC } (n) \end{aligned} \quad (35)$$

The above equation ensures that total income equals the wage bill plus tax revenues. The first and second sums respectively denote the carbon and non-carbon tax revenues expressed in changes. Recall that $\mathcal{B}_v \equiv \{\lambda_{ni,k}, r_{ni,k}, \rho_{i,k}, \tilde{\delta}_{ni,k}, e_{n,k}, \bar{w}_n \bar{L}_n, Y_n\}_{ni,k}$, $\mathcal{B}_t \equiv \{s_{n,k}, x_{in,k}, t_{ni,k}\}_{ni,k}$, and $\mathcal{B}_e = \{\sigma_k, \gamma_k, \alpha_{n,k}\}_{n,k}$ consist of either observable values or estimable parameters. Also, note that $\hat{\lambda}_{ni,k}$, $\hat{r}_{ni,k}$, $\hat{Z}_{n,k}$, $\hat{P}_{ni,k}$, $\hat{P}_{i,k}$, $1 - \widehat{a_{n,k}}$, and $\hat{v}_{i,k}$ are automatically determined given information on $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_e\}$, $\mathcal{R}_v \equiv \{\hat{w}_n, \hat{Y}_n, \hat{P}_n, \hat{\rho}_{n,k}\}_{n,k}$ and $\mathcal{R}_t \equiv \{x_{in,k}^*, t_{ni,k}^*\}_{ni,k}$.¹⁹ As such, Equations 30-35 constitute a system of $2N(N-1)K + NK + 3N$ independent equations and unknowns. The independent unknowns are the elements of $\mathcal{R} \equiv \{\mathcal{R}_v, \mathcal{R}_t\}$, which consist of $N(N-1)K$ optimal import tax rates $t_{ji,k}^*$, $N(N-1)K$ optimal export subsidy rates $x_{ji,k}^*$, NK changes in industry-level labor shares $\hat{\rho}_{i,k}$, and $3N$ changes in national wage rates, \hat{w}_i , income levels, \hat{Y}_i and consumer price indexes, \hat{P}_i . Solving the system characterized by Equations 30-35 fully characterizes the change in all equilibrium values, when moving from the observed baseline to the counterfactual non-cooperative equilibrium. The following proposition summarizes this point.

a).

¹⁹Since $s_{i,k}^* = (\gamma_k - 1)/\gamma_k$ and $\tau_i^* = \tilde{\delta}_{ni} \hat{P}_i$ we can exclude these tax rates from \mathcal{B}_t . That is because they are implicitly-determined with information on the remaining elements of \mathcal{B} and \mathcal{R} .

Proposition 3. Solving the system of Equations 30-35, determines $\mathcal{R}_v \equiv \{\hat{w}_n, \hat{Y}_n, \hat{P}_n, \hat{\rho}_{n,k}\}_{ni,k}$ and $\mathcal{R}_t \equiv \{x_{in,k}^*, t_{ni,k}^*\}_{ni,k}$ as a function of observables and structural elasticities, $\mathcal{B}_v \equiv \{\lambda_{ni,k}, r_{ni,k}, \rho_{i,k}, \tilde{\delta}_{ni,k}, e_{n,k}, \omega_n \bar{L}_n, Y_n\}_{ni,k}$, $\mathcal{B}_t \equiv \{s_{n,k}, x_{in,k}, t_{ni,k}\}_{ni,k}$, and $\mathcal{B}_e = \{\sigma_k, \gamma_k, \alpha_{n,k}\}_{n,k}$. Given information on $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_e\}$ and $\mathcal{R} \equiv \{\mathcal{R}_v, \mathcal{R}_t\}$, the effect of non-cooperative taxes/subsidies on welfare and carbon emissions can be calculated as

$$\hat{W}_i = \underbrace{\frac{Y_i}{Y_i - \sum_{n,k} \tilde{\delta}_{ni,k} Z_{n,k}} \left(\frac{\hat{Y}_i}{\hat{P}_i} \right)}_{\text{change to real consumption}} - \underbrace{\sum_{n,k} \frac{\tilde{\delta}_{ni,k} Z_{n,k}}{Y_i - \sum_{n,k} \tilde{\delta}_{ni,k} Z_{n,k}} \hat{Z}_{n,k}}_{\text{change to disutility from emissions}}. \quad (36)$$

To put Proposition 3 in perspective, the welfare effect specified by \hat{W}_i can be used to determine the effectiveness of trade policy as an enforcement tool in climate negotiations. \hat{W}_i determines what nations lose or gain from *not* joining a global climate agreement and opting for non-cooperative policy choices. To make sense of this number, we have to contrast it with how much countries gain or lose from joining a global climate agreement.

5.2 Global Climate Cooperation

Applying a similar logic, we can quantify the gains from global climate cooperation using the cooperative tax schedule presented under Equation 28. Cooperative carbon taxes are given by $\tau_{i,k}^* = \sum_{j \in \mathcal{C}} \tilde{\delta}_{ij,k} = \tilde{\phi}_{i,k} + \sum_{j \in \mathcal{C}} \tilde{\phi}_j$. Maintaining our conservative assumption that applied domestic carbon taxes are consistent with the unilaterally optimal rate, the change in carbon taxes when transitioning from the factual to the counterfactual cooperative equilibrium is, thus, given by

$$\hat{\tau}_{i,k} \equiv \frac{\tau_{i,k}^*}{\tau_{i,k}} = \sum_{j \in \mathcal{C}} \left(\frac{\tilde{\delta}_{ij,k} \hat{P}_j}{\tilde{\delta}_{ii,k}} \right) = \hat{P}_i + \sum_{j \neq i} \left(\frac{\tilde{\phi}_j}{\tilde{\phi}_{i,k} + \tilde{\phi}_i} \hat{P}_j \right).$$

Given that $1 + s_{j,k}^* = \gamma_k / (\gamma_k - 1)$ and $t_{ji,k} = x_{ji,k} = 0$, the change in non-carbon tax instruments can be expressed as:

$$\widehat{1 + s_{j,k}} = \frac{\gamma_k / (\gamma_k - 1)}{1 + s_{i,k}}; \quad \widehat{1 + x_{ji,k}} = \frac{1}{1 + x_{ji,k}}; \quad \widehat{1 + t_{ji,k}} = \frac{1}{1 + t_{ji,k}}.$$

Solving the above equations along-side Equations 31-35 determines $\hat{\tau}_{i,k}, \hat{v}_{i,k}, \widehat{1 - a_{i,k}}, \hat{\lambda}_{ji,k}, \hat{\rho}_{i,k}, \hat{r}_{ji,k}, \hat{w}_i, \hat{Y}_i$ and \hat{P}_i as a function of observables $\mathcal{B}_v \equiv \{\lambda_{ni,k}, r_{ni,k}, \rho_{i,k}, \tilde{\delta}_{ni,k}, e_{n,k}, \omega_n \bar{L}_n, Y_n\}_{ni,k}$, $\mathcal{B}_t \equiv \{s_{n,k}, x_{in,k}, t_{ni,k}\}_{ni,k}$ and estimable parameters, $\mathcal{B}_e = \{\sigma_k, \gamma_k, \alpha_{n,k}\}_k$. With knowledge of these variables, we can immedi-

ately calculate the change in real income $\hat{V}_i = \hat{Y}_i / \hat{P}_i$ and carbon emissions $\hat{Z}_{n,k} = \hat{v}_{n,k} \hat{p}_{n,k} \hat{w}_n$. Plugging these values into Equation 36 determines the change in (carbon adjusted) welfare when transitioning from the status quo to the cooperative climate & trade policy equilibrium.

5.3 Data Sources

Trade, Production, and Emissions. Data on international emission and expenditure levels are taken from the 2009 WIOD database on Input-Output Tables and Environmental Accounts (Timmer et al. (2012)).²⁰ The WIOD reports the full matrix of international expenditures across 41 major countries and 35 ISIC-level industries. Since the European Union (EU) acts as one tax-imposing authority, we aggregate all EU members into one tax-imposing region. To merge the WIOD data with our other datasets, we aggregate our sample into 19 industries, the details of which are listed in Table 1. After applying these aggregations, we are left with 15 economic regions ($N = 15$) and 19 industries ($K = 19$)—covering tradeables and nontradeables—, resulting in a $15 \times 15 \times 19$ matrix of expenditure levels, $\tilde{P}_{j,i,k} Q_{j,i,k}$, per origin j –destination i –industry k . Table 1 and 4 report respectively the list of industries and countries together with some of their relevant characteristics.

The WIOD Environmental Accounts report emissions of several air pollutants by origin country and industry. First, we use these data to calculate CO2 equivalent (CO2e) emissions based on global warming potential (GWP-100) from IPCC (2014) report. (Throughout the paper, we use CO2 as a shorthand way of CO2e.) The WGP-100 measures how much emissions of one tonne of a gas will be absorbed in the atmosphere in a period of 100 years relative to the emissions of one tonne of CO2. Using emission data of CO2 (carbon dioxide), CH4 (methane), and N2O (nitrous oxide), we calculate CO2e as $Z = Z_{\text{CO}_2} + 28 \times Z_{\text{CH}_4} + 265 \times Z_{\text{N}_2\text{O}}$ for every pair of origin country and industry. According to the Environmental Protection Agency, emissions of CO2, CH4, and N2O account for 97% of greenhouse gas emissions worldwide. Accordingly, we construct carbon intensity of *origin i–industry k* as:

$$v_{i,k} = \frac{Z_{i,k}}{P_{ii,k} Q_{i,k}} = \frac{\text{Emission}_{i,k}}{(\text{Gross Output})_{i,k}},$$

where the numerator is measured in tonnes of CO2e, and the gross output is measured in US dollars.

As for air emissions that contribute to local pollution, the WIOD Environmental Accounts reports under the category of acidification the air emissions of nitrogen oxides (NOx), sulfur oxides (SOx),

²⁰Our baseline year is 2009 as the most recent year with available information on trade & production, emission, and environmentally related taxes. Specifically, 2009 is the last year reported in WIOD Environmental Account and the first year with a large coverage in environmentally-related tax data.

and carbon monoxide (CO), for every pair of origin country and industry. We consider the aggregate of these local emissions as $Z_{i,k}^0$, and define $\bar{\zeta}_{i,k} \equiv Z_{i,k}^0 / Z_{i,k}$, as the rate at which use of emission-intensive inputs in country i -industry k generates local emissions per tonne of CO₂e emission.

Applied Taxes on Trade and Emissions. We compile data on applied tariffs in year 2009 from the United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAINS). The 2009 version of UNCTAD-TRAINS covers 31 two-digit (in ISIC rev. 3) sectors, which are aggregated up into our 19 aggregate ISIC industries for which we have compiled international expenditure and emissions data. For each industry, the UNCTAD-TRAINS reports multiple industry-level measures for applied tariffs. As is standard in the quantitative trade literature, we use the “simple tariff line average” of the “effectively applied tariff” (AHS). The UNCTAD-TRAINS reports *origin–destination–industry-specific* tariffs rates for all 41 countries in the WIOD sample, except for instances where the destination country is a member of the European Union (EU). In such cases we assign applied tariff measures based on the fact that EU tariffs are reported in the UNCTAD-TRAINS, intra-EU trade is subject to zero tariffs, and all EU members impose a common external tariff on non-members. Finally, we assume that applied export and domestic subsidies are negligible in each country, i.e., $s_{i,k} \approx x_{ij,k} \approx 0$.

We make use of the data from Environmental Taxes by Economic Activity from EUROSTAT as well as Environmentally-related Taxes by OECD-PINE. The data from EUROSTAT report environmentally-related taxes at the level of country-industry, covering all European countries, based on NACE rev. 2 industries, which we map to our 19 ISIC industries. The data from OECD-PINE report environmentally-related tax data in every country as a percentage of that country’s GDP. These data are reported for four mutually exclusive categories of energy, pollution, resources, and transport (excluding fuels for transport). Among these, we take energy taxes for what we refer to in this paper as emission taxes, maintaining our interpretation that emission taxes are imposed on the use of emission-intensive inputs, which can be thought of as energy in the data. For more details on our data construction, see Appendix D.

5.4 Estimation/Calibration of Model Parameters

To evaluate policy outcomes, we need the following elasticity parameters per industry, trade elasticity, $\epsilon_k \equiv (\sigma_k - 1)$; emission elasticity, $\alpha_{n,k}$; and degree of firm-level market power, γ_k , which is tied to the markup, $\mu_k \equiv \gamma_k / (\gamma_k - 1)$. In addition, we require emission disutility parameters for every

country, $\tilde{\delta}_{ji,k} = \tilde{\phi}_{i,k}\mathbf{1}(j = i) + \tilde{\phi}_i$. We estimate trade elasticities following [Caliendo and Parro \(2015\)](#), estimate markups based on the production function approach, calibrate emission elasticities using environmentally-related tax data, and calibrate carbon disutility parameters using data on energy taxes.

Markups. We use firm-level COMPUSTAT data and closely follow [De Loecker et al. \(2020\)](#) and [Baqae and Farhi \(2020\)](#). To map COMPUSTAT data to industries as defined in WIOD, we map disaggregated NAICS-level industries from COMPUSTAT to the 19 aggregate 2-digit ISIC-level industries ($k = 1, \dots, K = 19$) as well as disaggregated 3-digit ISIC industries. For every industry-year, we first estimate the output elasticity with respect to variable input, based on Olley-Pakes procedure in which, in logs of real value, our dependent variable is sales, the variable input is COGS (Cost of Goods Sold), proxy variable is capital expenditure, state variable is gross capital stock, and following the usual practice in the literature, we control for a firm's share of sales within disaggregated industries (which are 3-digit ISIC in our data). The variable input is what a firm gets to choose in its cost minimization problem. The resulting estimated coefficient of log variable input gives us output elasticities $\theta_{k,t}$ for every industry-year k, t . In addition, for every firm ω in industry k at year t , variable input share is the ratio of variable input (COGS) to sales, $b_{\omega,k,t}$. Using the first order condition of the firm's cost minimization, markups are:

$$\mu_{\omega,k,t} = \frac{\theta_{kt}}{b_{\omega,k,t}}. \quad (37)$$

To obtain markups at the level of industries, we aggregate firm-level markups to the level of 19 industries, with the weight assigned to a firm being equal to within-industry firm's sales share in the three-year period of 2008, 2009, and 2010. We consider a three-year period to make our estimates not particularly sensitive to potential industry-level fluctuations in our baseline year of 2009. Our estimates of the output elasticity $\theta_{k,t}$ are on average 0.82, with 0.80 at 25th and 0.86 at 75th percentile. The variable input shares, $b_{\omega,k,t}$, which we observed in the data, are on average 0.65, with 0.45 at 25th and 0.79 at 75th percentile. The resulting firm-level markup estimates, $\mu_{\omega,k,t}$, are on average 1.58, with 1.07 at 25th and 1.84 at 75th percentile. We report our markup estimates by industries in [Table 1](#).

Emission Elasticity. We recover the emission elasticity from our data on emission taxes, emission intensities, and markups based on [Equation 13](#). Specifically, the emission elasticity is given by the

Table 1: Variables and Elasticities across Industries

	Industry	Emission Intensity (v)	Emission Elasticity (α)	Trade Elasticity (σ)	Markup ($\frac{\gamma}{\gamma-1}$)
1	Agriculture	100.00	0.040	3.16	1.46
2	Mining	40.72	0.022	2.89	1.53
3	Food	4.23	0.009	2.43	1.70
4	Textiles and Leather	4.23	0.009	1.90	2.11
5	Wood	5.33	0.010	4.60	1.28
6	Pulp and Paper	6.79	0.007	4.38	1.30
7	Coke and Petroleum	23.08	0.015	4.31	1.18
8	Chemicals	19.65	0.032	1.94	2.06
9	Rubber and Plastics	15.18	0.007	2.55	1.27
10	Non-Metallic Mineral	31.86	0.008	3.05	1.49
11	Metals	2.11	0.001	5.18	1.24
12	Machinery and Electronics	1.79	0.006	2.99	1.50
13	Transport Equipment	1.59	0.004	1.59	1.21
14	Manufacturing, Nec	10.05	0.009	1.59	1.91
15	Electricity, Gas and Water	206.45	0.019	8.14	1.12
16	Construction	2.09	0.009	8.14	1.10
17	Retail and Wholesale	2.61	0.013	8.14	1.14
18	Transportation	30.15	0.053	8.14	1.01
19	Other Services	4.11	0.006	2.68	1.60

Note: This table shows for every of the 19 industries the global average of emission intensity (tonnes of CO2 per dollar of output) normalized by that of agriculture, calibrated emission elasticity in the case of the EU, estimated trade elasticity, and estimated markups. All CO2 measures are CO2 equivalent.

markup \times emission tax per unit of emission \times emission intensity,

$$\alpha_{n,k} = \frac{\gamma_k}{\gamma_k - 1} \frac{\tau_{nk} Z_{n,k}}{P_{nn,k} Q_{n,k}} = \mu_{n,k} \tau_{n,k} v_{n,k}. \quad (38)$$

We recover the industry-specific emission elasticity $\{\alpha_{n,k}\}_k$ by evaluating the right-hand side of Equation 38. The numerator ($\tau_{nk} Z_{n,k}$) is total emission taxes in country n –industry k , and the denominator is gross output ($P_{nn,k} Q_{n,k}$). Together, with our markup estimates, we obtain $\alpha_{n,k}$ for every country n –industry k in our sample. The second column in Table 1 reports these calibrated values for the case of the EU. The emission elasticities for agriculture, mining, and electricity are quite high pointing to

the importance of non-manufacturing in the analysis.

Trade Elasticity. We estimate trade elasticities, $(\sigma_k - 1)$ by applying [Caliendo and Parro's \(2015\)](#) estimation technique to our 2009 data on trade values and applied tariffs.

Perceived Disutility from Emission. We recover countries' perceived CPI-adjusted disutility from emissions along two lines: (a) currently-applied energy taxes in a country rationalize its unilaterally optimal domestic emission tax; (b) the globally optimal CO2 tax equals world's disutility from CO2 emissions,

$$\begin{cases} T_i^E = \sum_k (\tilde{\phi}_{i,k} + \tilde{\phi}_i) Z_{i,k} & \text{(a)} \\ SCC = \sum_i \tilde{\phi}_i & \text{(b)} \end{cases} \quad (39)$$

Here, T_i^E is total energy tax collected in country i , and SCC refers to the Social Cost of Carbon. Estimating SCC is beyond the scope of our paper. For our benchmark exercises, we use the estimates from [Cai and Lontzek \(2019\)](#) which sets SCC for our base year of 2009 at \$68 per tonne of CO2.

To recover relative values of $\tilde{\phi}_i$ across countries, we consider two key aspects of countries' attitude toward climate policy. First, if every individual, no matter in which country, cared equally about global warming, then the disutility from CO2 emissions would be proportional to countries' sizes. To reflect the size effect, denoting the damage from climate change as a percentage of countries' real GDP requires $\frac{\phi_i}{\phi_j} \propto \frac{Y_i/\tilde{P}_i}{Y_j/\tilde{P}_j}$, which implies that $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{Y_i}{Y_j}$. Second, in the current state of the world, countries' per capita care about CO2 emissions are vastly different. It is outside the scope of our paper to explain why governments of different countries perceive the cost of climate change differently. We, however, make the observation that countries' care about CO2 emissions can be at least partly reflected in their current overall care toward the environmental cost of burning fossil fuels. Under this presumption, we let country i relative to j 's care about CO2 emissions be proportional to their current emission taxes per unit of CO2 emission, meaning that $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{(T_i^E/Z_i)}{(T_j^E/Z_j)}$. Putting together these two points, and defining $y_i \equiv Y_i/Y_W$ as country i 's share of world GDP, we specify country i 's perceived disutility from CO2 emissions as $\tilde{\phi}_i = \bar{h} y_i (T_i^E/Z_i)$. This specification leaves us with a single parameter, \bar{h} , which we calibrate using a given value of SCC and equation 39-(b). Given recovered values of $\{\tilde{\phi}_i\}$, we calibrate $\{\tilde{\phi}_{i,k}\}$ based on equation 39-(a). Details of this calibration is presented in Appendix D. Table 4 reports our calibrated $\tilde{\phi}_i$ for regions in our sample.²¹

²¹Notice, we recover CPI-adjusted disutility parameters, $\tilde{\phi}_{i,k} = \tilde{P}_i \phi_{i,k}$ and $\tilde{\phi}_i = \tilde{P}_i \phi_i$ rather than $\phi_{i,k}$ and ϕ_i . This is sufficient for our counterfactual equilibrium analyses, as shown in our quantitative strategy in Section 5.

6 Quantitative Exercises

In this section, we use our theory and estimates to evaluate policy proposals that combine emission taxes with trade policy to achieve climate objectives. Specifically, we consider two types of policy recommendations:

1. *Using Trade Policy Unilaterally to Influence Transboundary Emissions.* We examine the full potential of unilateral trade policy at reducing transboundary CO₂ emissions. First, we briefly examine the impact of carbon border adjustments when adopted optimally by the EU. Then, we study the effects on country's CO₂ emissions and welfare of non-cooperative taxes exercised by all governments, and compare the results to those attainable under global cooperation.
2. *Using Trade Policy as a Penalty Device to Incentivize a Global Climate Cooperation.* We explore the full effectiveness of trade taxes when they are used as an incentive structure to form and expand a "climate club" (Nordhaus 2015).

In what follows, we use the Cobb-Douglas-CES version of our model, at our estimated/calibrated parameters, along with our optimal policy formulas.

6.1 Proposal 1: Using Unilateral Policy for Climate Objectives

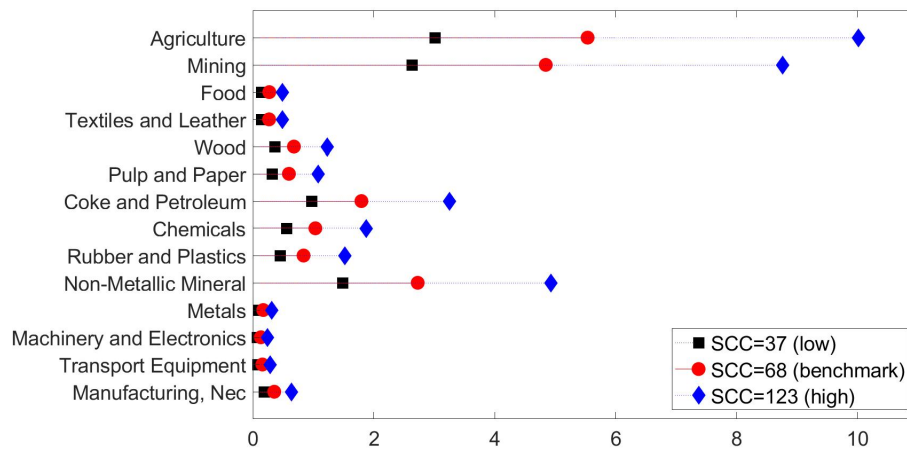
Impact of Carbon Border Adjustments We consider a counterfactual scenario wherein the EU adopts a unilaterally optimal policy while other countries remain passive. We benchmark the results from this exercise against a policy scenario in which the EU does not incorporate the border carbon adjustments in its policy schedule (i.e., by setting $\delta_{ni,k} = 0$ for $i = EU, n \neq EU$). We take the difference in the equilibrium outcomes between these two policy scenarios as the effect of the carbon-reducing component of the EU's unilaterally optimal trade policy.

We find that the optimal carbon border adjustments set by the EU are 0-3 percentage points across industries with an average of 1.1 p.p., and the optimal export subsidies are 0-4 percentage points with an average of 1.4 p.p. These carbon-reducing margins of trade taxes reflect the carbon bias of the optimal trade policy in the EU. Figure 1 shows the carbon bias of the EU's optimal tariffs across tradeable industries, for three different values of social cost of carbon.

This carbon bias of policy generates a further reallocation of the production from foreign countries into the EU region where the EU can directly tax CO₂ emissions, beyond the ToT-driven motives. As a result, the global CO₂ emission would fall by 0.21%. This is a very modest effect, particularly

taking into account that the EU is the largest region in terms of GDP and its care toward CO2 emissions is the highest in our sample. General equilibrium forces play an important role in neutralizing the EU's border carbon adjustments. Faced by these border adjustments, targeted countries reallocate their exports from the EU to other destinations, and also, their production partly shifts toward nontradeables. In addition, we notice that CO2 emissions rise slightly in the EU while they decrease elsewhere, leading to an overall reduction in global emissions. Our analysis also indicates that the EU's welfare net of the disutility from transboundary emissions increases by 0.25%.

Figure 1: Carbon Border Adjustments in the EU's Unilaterally Optimal Tariffs



Notes: This figure shows optimal carbon border adjustments in tariffs set by the EU. Each dot is an average across trade partners for an industry reported in percentage points. Social cost of carbon (SCC) for low and high values are chosen according to 10-th and 90-th percentile of the distribution of the climate damage in the DICE model for 2020 evaluated at the interest rate of 2.5%.

In this exercise, to single out the role of carbon-reducing components of trade taxes when exercised unilaterally, we let other countries be passive. We continue to examine the case where all countries set their taxes according to non-cooperative incentives.

6.1.1 Non-cooperative Nash Equilibrium

We examine the extent to which non-cooperative policies, when adopted simultaneously by all governments, can reduce global CO2 emissions. To put the outcome of this non-cooperative setting in perspective, we compare it to that of global cooperation. For this purpose, we compare the change in welfare and CO2 emissions when moving from the baseline equilibrium to (1) non-cooperative Nash

equilibrium in which all countries set their unilaterally optimal taxes (as detailed in Section 5.1), and (2) cooperative equilibrium in which countries set globally optimal taxes (as detailed in Section 5.2). We then contrast outcomes under these two counterfactual scenarios.

Table 2: Non-cooperative and Cooperative Outcomes

Country	Increasing Returns to Scale					
	Non-Cooperative			Global Cooperation		
	$\Delta\text{CO}_2\text{e}$	ΔV	ΔW	$\Delta\text{CO}_2\text{e}$	ΔV	ΔW
AUS	-7.5%	-1.7%	-1.6%	-82.2%	-1.0%	1.7%
EU	0.5%	-1.2%	-1.1%	-34.3%	-0.1%	6.7%
BRA	-6.8%	-1.3%	-1.2%	-91.0%	-1.0%	-0.1%
CAN	-15.6%	-4.1%	-4.0%	-80.1%	-1.0%	1.3%
CHN	2.1%	-1.0%	-0.9%	-94.5%	-1.0%	-0.5%
IDN	-4.1%	-1.7%	-1.7%	-93.1%	-1.2%	-0.5%
IND	-1.3%	-4.3%	-4.3%	-95.1%	-1.7%	-1.1%
JPN	3.3%	-0.9%	-0.7%	-64.2%	-0.5%	4.5%
KOR	-6.7%	-3.6%	-3.6%	-81.7%	-2.9%	-0.2%
MEX	0.9%	-0.7%	-0.7%	-89.2%	1.2%	2.4%
RUS	-9.1%	-5.2%	-5.2%	-91.2%	-0.8%	-0.3%
TUR	-7.0%	-3.5%	-3.4%	-72.2%	-1.2%	3.8%
TWN	44.9%	-2.1%	-2.1%	-91.8%	-1.2%	-0.3%
USA	0.3%	-1.7%	-1.7%	-83.5%	-0.5%	1.0%
RoW	-8.9%	-1.9%	-1.8%	-88.7%	-1.7%	-1.1%
Global	-2.3%	-1.6%	-1.6%	-82.4%	-0.7%	2.3%

Table 2 reports the percentage change in CO2 emissions and the corresponding welfare effects. We find that trade taxes, as a standalone device, are not notably effective at combating global CO2 emissions. Under the non-cooperative equilibrium, trade policies can lower global CO2 emissions by 2.3%. This number corresponds to only 2.8% of the total reductions in CO2 emissions possible under globally cooperative taxes (i.e., $2.3/82.4\%$).

Furthermore, the welfare consequences of non-cooperation are also relatively bleak. The average country loses 1.6% of its real GDP under the non-cooperative Nash equilibrium while gaining negligibly from emissions reductions. Under global cooperation, by comparison, the average country gains 2.3% in terms of emission-adjusted real GDP. At the global scale, this figure comprises a loss of 0.7% of real GDP at the benefit of gaining 3.0% of GDP-equivalent utility from emissions reductions. Turning to individual countries, percentage change to real consumption is smaller for every country under the non-cooperative outcome compared to global cooperation. It is also interesting that, in the

non-cooperative outcome, despite reductions in CO2 emissions at the global level, CO2 emissions increase in a few countries. For example, by reallocation of production to a large country like China, its CO2 emissions rise by 2.1%.

Increasing vs Constant Returns to Scale. To see more clearly the equilibrium implications of increasing returns, we repeat our exercise for the case of constant returns to scale (CRS). See Table 5 in the Appendix E. Before turning to compare CRS and IRS results, we note that the calibration of the emission elasticity requires that $\alpha_{n,k}^{CRS} = \alpha_{n,k}^{IRS} / (\text{markup}_k)$. This is a reflection of our sufficient statistic approach in which the micro-structure may remain inconsequential for sufficient elasticities.

In the non-cooperative Nash equilibrium, the reduction in global emissions is 2.7% under CRS that is somewhat larger than 2.3% under IRS. The comparable global emission reduction between CRS and IRS comes from notably different distribution of emission changes across countries. For example, under CRS emissions from the US fall by 1.1% while that rises by 0.3% under IRS. This is because increasing returns to scale imply a different allocation of resources between countries, and across industries within countries, leading to a different vector of production and emission across space and industries. The nature of these differences depend on the empirical correlation between markups and industry-level terms-of-trade changes under non-cooperative taxes.

6.2 The Effectiveness of Trade Taxes at Enforcing a Climate Club

Our previous findings indicated that non-cooperative taxes are remarkably less effective at reducing CO2 emissions than cooperative taxes. The implementation of cooperative climate policy is, however, complicated by the *free-riding* problem. Even countries with a high disutility from CO2 emissions have an incentive to free ride on the rest of the world's reductions in CO2 emissions without undertaking proportionate abatement measures themselves. Moreover, international misalignment in climate concerns may exacerbate the problem, as some governments may find the burden of cooperative CO2 taxes disproportionately too large in order for them to join an international climate agreement.

Seeking a solution to this problem, Nordhaus (2015) proposes that climate-conscience governments can form a *climate club*, and enforce climate cooperation by using trade taxes as a penalty device against non-member countries. Quantifying the full effectiveness of the climate club model, though, is a challenging task. It requires knowledge of optimal trade penalties, the computation of which is impractical with standard optimization techniques. Our optimal tax formulas, by design,

Table 3: Climate Club Game - Main Specification

	<i>Trade taxes set by</i>	
	<i>Members</i>	<i>Non-members</i>
<i>Against Members</i>	zero	unilaterally optimal
<i>Against Non-members</i>	unilaterally optimal	status quo (i.e., applied tariffs)

	<i>Emission taxes set by</i>	
	<i>Members</i>	<i>Non-members</i>
	globally optimal	status quo (i.e., unilaterally optimal)

characterize the extent to which trade taxes can optimally correct for transboundary CO₂ externalities while preserving ToT-driven objectives. In this sense, our optimal tax formulas determine the trade policy schedule that inflicts the greatest terms-of-trade penalty on non-cooperative trading partners. Our quantitative approach that builds on these formulas can, thus, uncover the full effectiveness of the climate club proposal.

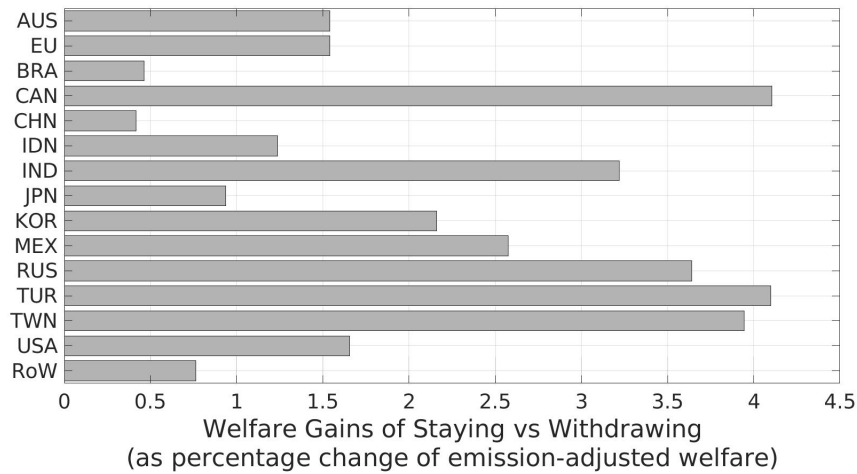
We first specify the structure of the game. We let the EU (and if needed, one or few other countries) be the core members of the club and all other countries play strategically. Table 3 shows the structure of the game. Players (countries) choose their strategies (whether or not to join the club) simultaneously, and the outcome is a Nash equilibrium in which no country has incentives to deviate:

- *Rules of Membership.* A member country must set zero trade taxes against other members while imposing unilaterally optimal trade taxes against non-members. A member must adopt a globally optimal emission tax that corrects for the global externality of its CO₂ emissions.
- *Non-members' Response.* A non-member country can retaliate against member countries by its best response, that is to adopt its unilaterally optimal trade taxes against them. Other than this, non-member countries remain in their status quo: They keep their existing applied tariffs against other non-members and maintain their existing domestic emission taxes.

Notice, we are here adopting a conservative approach in which the motivation of non-members to join the club is merely driven by members' penalty taxes. To see this clearly, suppose we were to allow non-member countries to set their unilaterally optimal trade taxes against other non-members. In that case, non-members would have an extra motive beyond climate-related concerns to join the club, as there would be a trade war between non-member countries and joining the club would be a way to escape from it. In contrast, our specification is meant to be an evaluation of trade taxes as a penalty device to incentivize cooperation.

The trade-off that a country faces in evaluating whether to join the club is that by joining the club the country incurs a production loss due to adopting larger emission taxes, but it can escape trade penalties imposed by club members. Intuitively, trade penalties must exceed the loss from larger emission taxes in order for the club to be formed and sustained. This also means that a larger club can exercise a higher collective market power to enforce cooperation.

Figure 2: Welfare Gains of Staying vs Leaving the Club-of-all-nations



Our first finding is that the club-of-all nations is a Nash equilibrium, no matter who the core members are. Figure 2 depicts for every country the welfare gains of staying in the club-of-all-nations relative to withdrawing unilaterally. Smaller countries such as Canada and Turkey has the largest net gains of staying in the club, while larger countries whose climate concerns are not particularly high, such as China or Brazil, have the lowest net gains.

We continue our analysis by examining whether the club-of-all-nations is the *unique* Nash equilibrium. This is a challenging task because with m core members, there will be 2^{N-m} combinations of countries' actions. We overcome this computational barrier by way of iterative elimination of dominated strategies. In the first round, we check for which countries it can be a dominant strategy to join the club. Given countries that join the club in the first round, we check for which countries joining the club will be a dominant strategy, and so on. A key property of the climate-club model makes this approach to work: that the net gains of joining the club rises in the size of the club.

We confirm that, in the case where the EU is the only core member, the club-of-only-EU is also a Nash equilibrium. This means that the net gains of joining the club-of-only-EU is negative for every country when all other countries stay out. To rule out this non-cooperative outcome, the set of core

members must be then larger than only the EU. We thus consider the case where both the EU and US are core members. This coalition creates sufficient incentives for main trade partners of the core members to join the club, incentivizing other countries at the margin to join in the next round, and so on. We find that with the EU and US as core members, the club-of-all-nations is the unique Nash equilibrium.

7 Conclusion

The realization that delays in taking action against global warming means only larger climate change damages, has raised public policy attention toward climate policy. In seeking to find a policy scheme that can help reduce global CO₂ emissions, many experts have advocated for the use of trade-related policy tools in addition to carbon pricing. We distinguish these policy proposals in their suggested use of trade taxes: One set of these proposals is based on using carbon border adjustments in unilateral trade taxes; and, the other is based on using trade taxes as a penalty device to enforce the formation of a climate club.

In this paper, we evaluated these policy proposals by characterizing optimal trade and carbon policies in a multi-country, multi-industry, general equilibrium trade model that features firm de-location, scale economies, and transboundary carbon externality. We derive fairly simple analytical formulas that characterize unilaterally (and, globally) optimal carbon and trade taxes. An important feature of our analysis is that it is designed to be taken to data for quantitative assessments of existing trade and carbon policy proposals—a task that has eluded the existing theoretical studies on optimal carbon and trade policy.

Our findings indicate that the unilateral use of trade and carbon policy—by way of optimal carbon border adjustments—can reduce global CO₂ emissions, only to a modest degree: around 3% of the emission reduction achievable under global cooperation. In contrast, using trade policy as a penalty device can lead to a more promising incentive structure to enforce an international climate club. We find that the climate club proposal is effective in forming a global cooperation, provided that both the EU and US commit to be core members of the club.

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Appendix

A Theoretical Preliminaries

A.1 Detailed Statement of the Optimal Unilateral Policy Problem

We derive optimal unilateral policy for the government in country i , which here we refer to as the home country. We denote by $\mathbb{P}_i \equiv \{\tilde{P}_{ii,k}, \tilde{P}_{ji,k}, \tilde{P}_{ij,k}, a_{i,k}\}_{j \neq i, j \in \mathbb{C}, k \in \mathbb{K}}$ the policy instruments in country i , by $\tilde{\mathbf{P}}_i \equiv \{\tilde{P}_{ji,k}\}_{j \in \mathbb{C}, k \in \mathbb{K}}$ the vector of consumer prices in country i , and by $\mathbf{w} \equiv \{w_j\}_{j \in \mathbb{C}}$ the vector of wages. The problem of the government in country i is:

$$\max_i V_i(Y_i, \tilde{\mathbf{P}}_i) - \sum_{n \in \mathbb{C}} \sum_{g \in \mathbb{K}} \delta_{ni} Z_{n,g}(a_{n,g}; Q_{n,g})$$

subject to the following equilibrium relationships, for all $i, j \in \mathbb{C}$, and $k \in \mathbb{K}$,

(Optimal Demand)	$Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$
(Producer Price)	$P_{ji,k}(w_j, a_{j,k}; Q_{j,k}) = \bar{d}_{ji,k} \bar{p}_{jj,k} w_j (1 - a_{j,k})^{\frac{1}{\gamma_k} - 1} Q_{j,k}^{-\frac{1}{\gamma_k}}$
(Pollution)	$Z_{j,k}(a_{j,k}; Q_{j,k}) \equiv \bar{z}_{j,k} (1 - a_{j,k})^{\frac{1}{\alpha_{j,k}} + \frac{1}{\gamma_k} - 1} Q_{j,k}^{1 - \frac{1}{\gamma_k}}$
(Income = Revenue)	$Y_i = w_i \bar{L}_i + \sum_{k, j \neq i} [(\tilde{P}_{ji,k} - P_{ji,k}) Q_{ji,k}] + \sum_{k, j} \left[\left(\tilde{P}_{ij,k} - (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k} \right) Q_{ij,k} \right]$
(Trade Deficit)	$B_i \equiv \sum_{j \neq i} \sum_k P_{ji,k} Q_{ji,k} - \sum_{j \neq i} \sum_k \tilde{P}_{ij,k} Q_{ij,k} = 0$

Here, we have written every variable as a function of (1) wages, (2) all or a subset of policy instruments, and (3) quantities. Equations for producer price and emission reproduce (2) and (3), in which $Q_{j,k} = \sum_i \bar{d}_{ji,k} Q_{ji,k}$. The equation for income reproduces (7) only in a more compact way by replacing for taxes from (8), and the trade deficit condition is equivalent to factor market clearing condition (See footnote 9). The demand function $\mathcal{D}_{ji,k}$ is characterized by the set demand elasticities defined in

Section 2.1. Throughout our proof, we assign the factor in one foreign country as the numeraire.

A.2 Expressing Equilibrium Outcomes as a Function of $(\mathbb{P}_i; \mathbf{w})$

Consider system (S^w) that incorporates all equilibrium conditions excluding the labor-market clearing condition. For all $n, j \in \mathbb{C}$, and $k \in \mathbb{K}$,

$$\begin{aligned}
(\text{Optimal Demand}) \quad Q_{nj,k}(\mathbb{P}_i; \mathbf{w}) &= \begin{cases} D_{ni,k}(\tilde{\mathbf{P}}_i, Y_i(\mathbb{P}_i; \mathbf{w})) & \text{if } j = i \\ D_{nj,k}(\tilde{\mathbf{P}}_{ij}, \{ \tilde{\mathbf{P}}_{nj}(\mathbb{P}_i; \mathbf{w}) \}_{n \neq i}, Y_j(\mathbb{P}_i; \mathbf{w})) & \text{if } j \neq i \end{cases} \\
(\text{Industry Output}) \quad Q_{n,k}(\mathbb{P}_i; \mathbf{w}) &= \sum_{j \in \mathbb{C}} \bar{d}_{nj,k} Q_{nj,k}(\mathbb{P}_i; \mathbf{w}) \\
(\text{Producer Price}) \quad P_{nj,k}(\mathbb{P}_i; \mathbf{w}) &= \bar{d}_{nj,k} \bar{p}_{nn,k} w_n (1 - a_{n,k})^{\frac{1}{\gamma_k} - 1} (Q_{n,k}(\mathbb{P}_i; \mathbf{w}))^{-\frac{1}{\gamma_k}} \\
(\text{Pollution}) \quad Z_{n,k}(\mathbb{P}_i; \mathbf{w}) &= \bar{z}_{n,k} (1 - a_{n,k})^{\frac{1}{\alpha_{n,k}} + \frac{1}{\gamma_k} - 1} (Q_{n,k}(\mathbb{P}_i; \mathbf{w}))^{-\frac{1}{\gamma_k}} \quad (S^w) \\
(\text{Tax Revenues}) \quad T_n(\mathbb{P}_i; \mathbf{w}) &= \begin{cases} \sum_{k, j \neq i} \left[\left(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{P}_i; \mathbf{w}) \right) Q_{ji,k}(\mathbb{P}_i; \mathbf{w}) \right] & \text{if } n = i \\ + \sum_{k, j} \left[\left(\tilde{P}_{ij,k} - \left(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k} \right) P_{ij,k}(\mathbb{P}_i; \mathbf{w}) \right) Q_{ij,k}(\mathbb{P}_i; \mathbf{w}) \right] & \\ 0 & \text{if } n \neq i \end{cases} \\
(\text{Income}) \quad Y_n(\mathbb{P}_i; \mathbf{w}) &= w_n \bar{L}_n + T_n(\mathbb{P}_i; \mathbf{w})
\end{aligned}$$

Here, $\tilde{\mathbf{P}}_i \subset \mathbb{P}_i$ is the vector of consumer prices in home country i , $\tilde{\mathbf{P}}_{ij} \subset \mathbb{P}_i$ is the vector of consumer prices in foreign country j of varieties produced in home, and $a_{i,k} \in \mathbb{P}_i$ is the abatement in home. All these are instruments of policy to be chosen by the home government. In contrast, every foreign country $n \neq i$ has some fixed abatement level $a_{n,k} = \bar{a}_{n,k}$ and no tax revenues $T_n = 0$. System (S^w) characterizes quantities, producer prices, emissions, tax revenues, and income in all economies as a function of $(\mathbb{P}_i, \mathbf{w})$. Correspondingly, welfare in country i can be formulated as,

$$W_i(\mathbb{P}_i; \mathbf{w}) = V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) - \sum_{n,k} \delta_{ni} Z_{i,k}(\mathbb{P}_i; \mathbf{w}).$$

By design, system (S^w) excludes the labor-market clearing condition, and it is understood that many wage vectors may satisfy (S^w) . For a given choice of policy, \mathbb{P}_i , a wage vector, \mathbf{w} , is in the feasible set \mathbb{F}_i^w if and only if it satisfies the labor-market clearing conditions:

$$\sum_{j,k} \left[\left(1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k} \right) P_{nj,k}(\mathbb{P}_i; \mathbf{w}) Q_{nj,k}(\mathbb{P}_i; \mathbf{w}) \right] = w_n \bar{L}_n, \quad \forall n.$$

A.3 Expressing Equilibrium Outcomes as a Function of $(\mathbb{P}_i; Y_i)$

Following Lemma 2, we treat wages as fixed. Consider system (S^Y) that incorporates all equilibrium conditions excluding the budget constraint. For all $n, j \in \mathbb{C}$, and $k \in \mathbb{K}$,

$$\begin{aligned}
(\text{Optimal Demand}) \quad Q_{nj,k}(\mathbb{P}_i; Y_i) &= \begin{cases} D_{ni,k}(\tilde{\mathbf{P}}_i, Y_i) & \text{if } j = i \\ D_{nj,k}(\tilde{\mathbf{P}}_{ij}, \{\tilde{\mathbf{P}}_{nj}(\mathbb{P}_i; Y_i)\}_{n \neq i}, \tilde{Y}_j) & \text{if } j \neq i \end{cases} \\
(\text{Industry Output}) \quad Q_{n,k}(\mathbb{P}_i; Y_i) &= \sum_{j \in \mathbb{C}} \bar{d}_{nj,k} Q_{nj,k}(\mathbb{P}_i; Y_i) \\
(\text{Producer Price}) \quad P_{nj,k}(\mathbb{P}_i; Y_i) &= \bar{d}_{nj,k} \bar{p}_{nn,k} \bar{w}_n (1 - a_{n,k})^{\frac{1}{\gamma_k} - 1} (Q_{n,k}(\mathbb{P}_i; Y_i))^{-\frac{1}{\gamma_k}} \\
(\text{Pollution}) \quad Z_{n,k}(\mathbb{P}_i; Y_i) &= \bar{z}_{n,k} (1 - a_{n,k})^{\frac{1}{\alpha_{n,k}} + \frac{1}{\gamma_k} - 1} (Q_{n,k}(\mathbb{P}_i; Y_i))^{1 - \frac{1}{\gamma_k}} \\
(\text{Taxes}) \quad T_n(\mathbb{P}_i; Y_i) &= \begin{cases} \sum_{k, j \neq i} [(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{P}_i; Y_i)) Q_{ji,k}(\mathbb{P}_i; Y_i)] & \text{if } n = i \\ + \sum_{k, j} \left[\left(\tilde{P}_{ij,k} - (1 - \alpha_{n,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\mathbb{P}_i; Y_i) \right) Q_{ij,k}(\mathbb{P}_i; Y_i) \right] & \\ 0 & \text{if } n \neq i \end{cases}
\end{aligned} \tag{S^Y}$$

System (S^Y) characterizes quantities, producer prices, emissions, and tax revenues in all economies as a function of (\mathbb{P}_i, Y_i) . Correspondingly, welfare in country i can be formulated as,

$$W_i(\mathbb{P}_i; Y_i) = V_i(\bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i), \tilde{\mathbf{P}}_i) - \sum_{n,k} \delta_{ni} Z_{i,k}(\mathbb{P}_i; Y_i).$$

A policy-income pair is feasible, denoted by $(\mathbb{P}_i, Y_i) \in \mathbb{F}_i^Y$, if and only if $Y_i = \bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i)$.

A.4 Characterizing Equilibrium Wage Effects

Suppose we formulate all equilibrium variables as a function of \mathbb{P}_i and \mathbf{w} (described in Appendix A.2). The feasible vector of wages, \mathbf{w} , solves the following system of labor market clearing conditions:

$$\begin{cases} f_1(\mathbb{P}_i; \mathbf{w}) \equiv w_1 L_1 - \sum_{j \in \mathbb{C}} \sum_{k \in \mathbb{K}} (1 - \alpha_{1,k} \frac{\gamma_k - 1}{\gamma_k}) P_{1j,k}(\mathbb{P}_i; \mathbf{w}) Q_{1j,k}(\mathbb{P}_i; \mathbf{w}) = 0 \\ \vdots \\ f_N(\mathbb{P}_i; \mathbf{w}) \equiv w_N L_N - \sum_{j \in \mathbb{C}} \sum_{k \in \mathbb{K}} (1 - \alpha_{N,k} \frac{\gamma_k - 1}{\gamma_k}) P_{Nj,k}(\mathbb{P}_i; \mathbf{w}) Q_{Nj,k}(\mathbb{P}_i; \mathbf{w}) = 0 \end{cases} \tag{40}$$

Also, note that by Walras' law one equation is redundant so we can assign one element of \mathbf{w} as the numeraire:

$$\sum_n f_n(\mathbb{P}_i; \mathbf{w}) = 0. \quad [\text{Walras' Law}]$$

To characterize the term $\frac{d\mathbf{w}}{d\mathcal{P}_i}$ in the F.O.C., we can apply the Implicit Function Theorem to the above system as follows:

$$\frac{d \ln \mathbf{w}}{d \ln \mathcal{P}_i} = - \left(\frac{\partial f}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i}^{-1} \frac{\partial f}{\partial \ln \mathcal{P}_i}.$$

To characterize the matrix $\frac{\partial f}{\partial \mathbf{w}}$, let us briefly abstract from scale economies and abatement, which amounts to setting $\alpha_k \frac{\gamma_k - 1}{\gamma_k} = 0$ in System 40. This simplification helps us convey our main point succinctly; but it does not imply it. As we argue shortly, our main claim goes through without this simplification. Taking partial derivatives from System 40 w.r.t. \mathbf{w} holding \mathbb{P}_i fixed, yields

$$\left(\frac{\partial f}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i} = \begin{bmatrix} \frac{\partial f_1}{\partial \ln w_1} & \frac{\partial f_1}{\partial \ln w_2} & \cdots & \frac{\partial f_1}{\partial \ln w_N} \\ \frac{\partial f_2}{\partial \ln w_1} & \frac{\partial f_2}{\partial \ln w_2} & \cdots & \frac{\partial f_2}{\partial \ln w_N} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_N}{\partial \ln w_1} & \frac{\partial f_N}{\partial \ln w_2} & \cdots & \frac{\partial f_N}{\partial \ln w_N} \end{bmatrix} = \begin{bmatrix} 1 - \sum_{k,g} r_{11,k} \left(\eta_{11,k} + \epsilon_{11,k}^{(11,g)} \right) & \cdots & - \sum_{k,g} r_{1N,k} \left(\eta_{1N,k} + \epsilon_{1N,k}^{(NN,g)} \right) \\ 1 - \sum_{k,g} r_{21,k} \left(\eta_{21,k} + \epsilon_{21,k}^{(11,g)} \right) & \cdots & - \sum_{k,g} r_{2N,k} \left(\eta_{2N,k} + \epsilon_{2N,k}^{(NN,g)} \right) \\ \vdots & \ddots & \vdots \\ 1 - \sum_{k,g} r_{N1,k} \left(\eta_{N1,k} + \epsilon_{N1,k}^{(11,g)} \right) & \cdots & - \sum_{k,g} r_{NN,k} \left(\eta_{NN,k} + \epsilon_{NN,k}^{(NN,g)} \right) \end{bmatrix}.$$

Under Cobb-Douglas-CES preferences, the above matrix assumes the following parameterization:

$$\left(\frac{\partial f}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i} = \mathbf{I} - \underbrace{\begin{bmatrix} - \sum_k [r_{11,k} \epsilon_k (1 - \lambda_{11,k})] & \sum_k [r_{12,k} (1 + \epsilon_k \lambda_{22,k})] & \cdots & \sum_k [r_{1N,k} (1 + \epsilon_k \lambda_{NN,k})] \\ \sum_k [r_{21,k} (1 + \epsilon_k \lambda_{11,k})] & - \sum_k [r_{22,k} \epsilon_k (1 - \lambda_{22,k})] & \cdots & \sum_k [r_{2N,k} (1 + \epsilon_k \lambda_{NN,k})] \\ \vdots & \ddots & \ddots & \vdots \\ \sum_k [r_{N1,k} (1 + \epsilon_k \lambda_{11,k})] & \sum_k [r_{N2,k} (1 + \epsilon_k \lambda_{22,k})] & \cdots & - \sum_k [r_{NN,k} \epsilon_k (1 - \lambda_{NN,k})] \end{bmatrix}}_{\mathbf{A}}$$

Noting that $r_{ij,k}\epsilon_k(1 - \lambda_{jj,k}) \ll 1$ if $j \neq i$, we can produce the following approximation:²²

$$\begin{aligned} \left(\frac{\partial \mathbf{f}}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i}^{-1} &= (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \dots \approx \\ &\mathbf{I} + \sum_{m=1}^{\infty} \text{diag} \left(\left[- \sum_{k \in \mathbb{K}} r_{ii,k} \epsilon_k (1 - \lambda_{ii,k}) \right]^m \right) = \text{diag} \left(\left[\sum_{k \in \mathbb{K}} 1 + r_{ii,k} \epsilon_k (1 - \lambda_{ii,k}) \right]^{-1} \right). \end{aligned}$$

The above equation indicates that $\left(\frac{\partial \mathbf{f}}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i}$ is *nearly* diagonal with smaller-than-unity diagonal elements. Now, consider the case where $\mathcal{P}_i = \tilde{P}_{ji,k}$ and assign w_j as the numeraire. The derivative of f_{-j} (i.e., f excluding row j) w.r.t. $\tilde{P}_{ji,k}$ holding \mathbf{w} and $\mathbb{P}_i - \tilde{P}_{ji,k}$ fixed is given by:

$$\frac{\partial f_{-j}}{\partial \ln \tilde{P}_{ji,k}} = \begin{bmatrix} \frac{\partial f_1}{\partial \ln \tilde{P}_{ji,k}} \\ \frac{\partial f_2}{\partial \ln \tilde{P}_{ji,k}} \\ \vdots \\ \frac{\partial f_N}{\partial \ln \tilde{P}_{ji,k}} \end{bmatrix} = \begin{bmatrix} \sum_g r_{1i,g} \epsilon_{1i,g}^{(ji,k)} \\ \sum_g r_{2i,g} \epsilon_{2i,g}^{(ji,k)} \\ \vdots \\ \sum_g r_{Ni,g} \epsilon_{Ni,g}^{(ji,k)} \end{bmatrix} \xrightarrow{\text{Cobb-Douglas-CES}} = \begin{bmatrix} r_{1i} \\ \vdots \\ r_{j-1i} \\ r_{j+1i} \\ \vdots \\ r_{Ni} \end{bmatrix} \lambda_{ji,k} \epsilon_k$$

Given that (i) $\lambda_{ji,k} r_{ni} \approx 0$ if n and $j \neq i$, and (ii) $\left(\frac{\partial \mathbf{f}}{\partial \ln \mathbf{w}} \right)_{\mathbb{P}_i}$ is *nearly* diagonal with smaller-than-unity diagonal elements, it immediately follows that

$$\frac{d \ln \mathbf{w}_{-\{i,j\}}}{d \ln \tilde{P}_{ji,k}} = \left(\frac{\partial \mathbf{f}_{-j}}{\partial \ln \mathbf{w}_{-\{i,j\}}} \right)_{\mathbb{P}_i}^{-1} \frac{\partial f_{-j}}{\partial \ln \tilde{P}_{ji,k}} \approx 0,$$

where $\mathbf{w}_{-\{i,j\}}$ denotes the wage vector \mathbf{w} excluding elements i and j . The same steps can be taken with regards to any other price instrument in \mathbb{P}_i . Furthermore, the above argument goes through if we allow for a finite γ_k and a non-zero α_k .

²²The last line follows from the fact that for $a \in \mathbb{R}_+$,

$$\sum_{n=1}^{\infty} (-a)^n = -\frac{a}{1+a}.$$

A.5 Some Useful Relationships

Before turning to our derivations of optimal policy, we show two sets of useful relationships. The first one is the effects of policy instruments on emission levels. The second one is the effects of policy instruments on producer prices through industry-level scale economies.

Scale Effects in Emission. Recall that total emission, as a function of abatement and output, is given by

$$Z_{j,k}(a_{j,k}; Q_{j,k}) \equiv \bar{z}_{j,k}(1 - a_{j,k})^{\frac{1}{\alpha_{j,k}} + \frac{1}{\gamma_k} - 1} Q_{j,k}^{1 - \frac{1}{\gamma_k}}.$$

To track the policy response of emission we use two following partial derivatives. The first one, accounts for scale effects in emission:

$$\frac{\partial \ln Z_{j,k}(a_{j,k}, Q_{j,k})}{\partial \ln Q_{j,k}} = 1 - \frac{1}{\gamma_k}, \quad (41)$$

and, the second one accounts for abatement effects in emission:

$$\frac{\partial \ln Z_{j,k}(a_{j,k}, Q_{j,k})}{\partial \ln(1 - a_{j,k})} = \frac{1}{\alpha_{j,k}} + \frac{1}{\gamma_k} - 1. \quad (42)$$

Note that $a_{i,k}$ is directly chosen by the policy-maker in our reformulated optimal policy problem. $Q_{j,k}(\mathbb{P}_i; \mathbf{w}, Y_i)$ is implicitly determined by the policy-maker with respect to abatement and the remaining price instruments.

Scale Economies in Production and the Export Supply Elasticity. Scale Economies in Production and the Export Supply Elasticity.

Below, we define and characterize the export supply elasticity. To that end, we first introduce some intermediate partial derivatives that enter the export supply elasticity formula. These partial derivatives are also independently useful to our subsequent optimal analysis.

Following [Lashkaripour and Lugovskyy \(2016\)](#), the definition for the *general equilibrium* export supply elasticity can be expressed as follows

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k} \rho_{j,k}} \sum_g \left[\frac{w_i L_i}{w_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_i} + \sum_{n \neq i} \frac{w_n L_n}{w_j L_j} r_{ni,g} \rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_i} \right], \quad (43)$$

where $r_{ni,g} = P_{ni,g} Q_{ni,g} / \sum_{i \in C} (P_{ni,g} Q_{ni,g})$ and $\rho_{n,g} = \sum_{i \in C} (P_{ni,g} Q_{ni,g}) / w_n L_n$ respectively denote the

good ni, g -specific and industry-wide sales shares associated with origin $n \in \mathbb{C}$. The above expression accounts for the fact that a change in the export supply of good ji, k will affect the producer price of goods supplied by *origin j -industry k* as well as other suppliers via cross-demand effects.

Before unpacking and simplifying Equation 43, let us provide a brief description. Contracting the supply of good ji, k (i.e., $Q_{ji,k}$) increases the price of goods supplied by origin j -industry k through firm-entry (or scale) effects. Holding \mathbb{P}_i and \mathbf{w} constant, this change in price can affect the demand facing other suppliers via cross-demand effects in markets outside of i . Consumer prices in destination i are fixed by the government's choice vis-à-vis \mathbb{P}_i . So, once we fix \mathbb{P}_i , a change in $P_{in,g}$ has no bearing on the demand for other suppliers in market i . Outside of market i , however, a change in producer prices is completely passed on to consumer prices. Considering this, a change in $P_{jn,k}$ (which recall is triggered by a contraction in $Q_{ji,k}$) influences the demand for all suppliers serving market n . This change in demand, in turn, impacts the producer price of goods supplied by each international industry through scale effects. Equation 43 measures how these changes impact country i 's ToT.

We can follow the procedure in Lashkaripour and Lugovskyy (2016) to derive a simple first-order approximation for $\omega_{ji,k}$ in the absence of cross-industry demand effects. To this end, note that the producer price of good ni, g is given by $P_{ni,g} = \bar{d}_{ni,g} P_{nn,g}$, where $\bar{d}_{ni,g}$ denotes a constant iceberg trade cost and $P_{nn,g}$ denotes the price of goods supplied by *origin n -industry g* in the domestic market. As detailed in Section 2.2, $P_{nn,g}$ is an explicit function of *origin n -industry g 's* abatement, wage, and output schedule:

$$P_{nn,g}(a_{n,g}, w_n, Q_{n1,g}, \dots, Q_{nN,g}) = \bar{p}_{nn,g} w_n (1 - a_{n,g})^{\frac{1}{\gamma_g} - 1} \mathcal{Q}_{n,g}(Q_{n1,g}, \dots, Q_{nN,g})^{-\frac{1}{\gamma_g}}.$$

where $\mathcal{Q}_{n,g}(\cdot)$ is total effective output in origin j -industry k , as given by

$$\mathcal{Q}_{j,k}(Q_{j1,k}, \dots, Q_{jN,k}) = \bar{d}_{j1,k} Q_{j1,k} + \dots + \bar{d}_{jN,k} Q_{jN,k}.$$

Considering the above formulation, characterizing $\omega_{ji,k}$ requires that we first characterize $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_i} = \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_i}$ for each origin n -industry g . For this purpose, define the the following function for each *origin j -industry k* that treats \mathbb{P}_i and \mathbf{w} as given:

$$F_{j,g}(\mathbf{Q}_{i,g}, \mathbf{P}_g) \equiv P_{jj,g} - \bar{p}_{jj,g} w_j (1 - a_{j,g})^{\frac{1}{\gamma_g} - 1} \left[\bar{d}_{ji,g} Q_{ji,g} + \sum_{n \neq i} \bar{d}_{jn,g} Q_{jn,g} (\mathbf{d}_{-in,g} \odot \mathbf{P}_{-i,g}) \right]^{-\frac{1}{\gamma_g}} = 0$$

In the above formulation, $\mathbf{Q}_{i,g} \equiv \{Q_{1i,g}, \dots, Q_{Ni,g}\}$ denotes the vector of demand for industry g goods in destination i ; vector $\mathbf{P}_g \equiv \{P_{\ell,g}\}$ is the global vector of producer prices in industry g and $\mathbf{P}_{-i,g} = \mathbf{P}_g - \{P_{ii,g}\}$ encompasses the producer price for each origin aside from i . Correspondingly, $\tilde{\mathbf{P}}_{-in,g} \equiv \mathbf{d}_{-in,g} \odot \mathbf{P}_{-i,g}$ denotes the vector of consumer prices associated with non- i origins in destination $n \neq i$. Finally, the function $Q_{n\ell,g}(\tilde{\mathbf{P}}_{-in,g}) = \mathcal{D}_{n\ell,g}(\tilde{\mathbf{P}}_{-in,g}, \bar{P}_{in,g}, \bar{Y}_n)$ derives from the Marshallian demand function, treating $\bar{P}_{in,g} \in \bar{\mathbb{P}}_i$ and $w_n \in \mathbf{w}$ as given (with $\bar{Y}_n = \bar{w}_n \bar{L}_n$, accordingly). For any given \mathbb{P}_i and \mathbf{w} , the global vector of produce prices, \mathbf{P}_g , can be characterized as a function, $\mathbf{Q}_{i,g}$, based on the following system:

$$\begin{cases} F_{1,g}(Q_{1i,g}, \dots, Q_{Ni,g}, P_{11,g}, \dots, P_{NN,g}) = 0 \\ \vdots \\ F_{N,g}(Q_{1i,g}, \dots, Q_{Ni,g}, P_{11,g}, \dots, P_{NN,g}) = 0 \end{cases}$$

Applying the Implicit Function Theorem to the above system of equations, yields the following matrix of inverse export supply elasticities:

$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}} \right)_{\mathbf{w}, \mathbb{P}_i} & \dots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}} \right)_{\mathbf{w}, \mathbb{P}_i} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}} \right)_{\mathbf{w}, \mathbb{P}_i} & \dots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}} \right)_{\mathbf{w}, \mathbb{P}_i} \end{bmatrix} = - \underbrace{\begin{bmatrix} \frac{\partial F_{1,k}(\cdot)}{\partial \ln P_{11,k}} & \dots & \frac{\partial F_{1,k}(\cdot)}{\partial \ln P_{NN,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(\cdot)}{\partial \ln P_{11,k}} & \dots & \frac{\partial F_{N,k}(\cdot)}{\partial \ln P_{NN,k}} \end{bmatrix}^{-1}}_{\mathbf{A}_i} \begin{bmatrix} \frac{\partial F_{1,k}(\cdot)}{\partial \ln Q_{1i,k}} & \dots & \frac{\partial F_{1,k}(\cdot)}{\partial \ln Q_{Ni,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(\cdot)}{\partial \ln Q_{1i,k}} & \dots & \frac{\partial F_{N,k}(\cdot)}{\partial \ln Q_{Ni,k}} \end{bmatrix}, \quad (44)$$

Since function $F_{i,k}(\cdot)$ treats \mathbb{P}_i and \mathbf{w} as given, each element of the matrixes on the right-hand side can be specified as follows:

$$\frac{\partial F_{n,k}(\cdot)}{\partial \ln P_{jj,k}} = \mathbb{1}_{j=n} + \mathbb{1}_{j \neq i} \frac{1}{\gamma_g} \sum_{\ell \neq i} r_{n\ell,k} \varepsilon_{n\ell,k}^{(j\ell,k)}; \quad \frac{\partial F_{n,k}(\cdot)}{\partial \ln Q_{ji,k}} = \mathbb{1}_{j=n} \frac{1}{\gamma_g} r_{ji,k}.$$

Considering the above expression for $\partial F_{n,k}(\cdot) / \partial \ln P_{jj,k}$, it is straightforward to show that \mathbf{A}_i is diagonally-dominant. Hence, following [Lashkaripour and Lugovskyy \(2016\)](#), we can produce a simple first-order approximation for \mathbf{A}_i^{-1} around $r_{ji,k} \approx 0$ (for $j \neq i$), which yields the following:

$$\left(\frac{\partial \ln P_{m,g}}{\partial \ln Q_{ji,g}} \right)_{\mathbf{w}, \mathbb{P}_i} \approx \begin{cases} \frac{-\frac{1}{\gamma_g} r_{mi,g}}{1 + \frac{1}{\gamma_g} \sum_{\ell \neq i} r_{m\ell,g} \varepsilon_{m\ell,g}^{(mi,g)}} & n = j \\ \frac{\frac{1}{\gamma_g} r_{ji,g}}{1 + \frac{1}{\gamma_g} \sum_{\ell \neq i} r_{m\ell,g} \varepsilon_{m\ell,g}^{(mi,g)}} \left(\frac{1}{\gamma_g} \sum_{\ell \neq i} r_{m\ell,g} \varepsilon_{m\ell,g}^{(j\ell,g)} \right) & n \neq j \end{cases}$$

We can then plug the above expression back into Equation 43 to produce the following approximation for the export supply elasticity—noting that $r_{ni,g} \times r_{ji,g} \approx 0$ if $j \neq i$ and $n \neq i$:

$$\omega_{ji,k} \approx \frac{-\frac{1}{\gamma_k} r_{ji,k}}{1 + \frac{1}{\gamma_k} \sum_{l \neq i} r_{jl,k} \varepsilon_{jl,k}} \left[1 - \frac{1}{\gamma_k} \frac{w_i L_i}{w_j L_j} \sum_{n \neq i} \frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{ji,k}} \varepsilon_{in,k}^{(jn,k)} \right].$$

A.6 Multiplicity of Policy Schedules

First, we state the result concerning the multiplicity of optimal taxes. Then, we provide a formal proof. The present the proof we use $\mathbb{T} \equiv (\mathbb{I}_i, \mathbb{I}_{-i}, w_i, \mathbf{w}_{-i})$ to denote a global policy-wage combination, with $\mathbb{I}_i = (\mathbf{s}_i, \mathbf{t}_i, \mathbf{x}_i, \boldsymbol{\tau}_i)$. As before we use \mathbb{F}^w to denote the set of feasible policy-wage combinations—i.e., the combinations for which \mathbf{w} satisfies the labor market clearing condition given applied taxes, \mathbb{I} . Considering our choice of notation, we want to prove the following result, which is a basic extension of Lemma 1 in [Lashkaripour and Lugovskyy \(2016\)](#) to an economy with carbon externality and abatement.

Lemma 6. *For any a and $\tilde{a} \in \mathbb{R}_+$, the following two results hold: (1) if $\mathbb{T} = (\mathbf{1} + \mathbf{t}_i, \mathbf{t}_{-i}, \mathbf{1} + \mathbf{x}_i, \mathbf{x}_{-i}, \mathbf{1} + \mathbf{s}_i, \mathbf{s}_{-i}, \boldsymbol{\tau}_i, \boldsymbol{\tau}_{-i}; w_i, \mathbf{w}_{-i}) \in \mathbb{F}^w$, then $\mathbb{T}' = (a(\mathbf{1} + \mathbf{t}_i), \mathbf{t}_{-i}, a(\mathbf{1} + \mathbf{x}_i), \mathbf{x}_{-i}, \frac{1}{\tilde{a}}(\mathbf{1} + \mathbf{s}_i), \mathbf{s}_{-i}, \frac{a}{\tilde{a}}\boldsymbol{\tau}_i, \boldsymbol{\tau}_{-i}; \frac{a}{\tilde{a}}w_i, \mathbf{w}_{-i}) \in \mathbb{F}^w$; and (2) Welfare is preserved under policy-wage combination \mathbb{T} and \mathbb{T}' : $W_n(\mathbb{T}') = W_n(\mathbb{T})$ for all $n \in \mathbb{C}$.*

Proof. The proof closely follows the proof of Lemma 1 in [Lashkaripour and Lugovskyy \(2016\)](#). The only difference is that the labor market clearing condition must be adjusted to account for abatement activity. To restate the objective of the proof, consider two policy-wage combinations, $\mathbb{T} = (\mathbf{s}, \mathbf{t}, \mathbf{x}, \boldsymbol{\tau}; \mathbf{w})$, and $\mathbb{T}' = (\mathbf{s}', \mathbf{t}', \mathbf{x}', \boldsymbol{\tau}'; \mathbf{w}')$, that differ in uniform shifters a and $\tilde{a} \in \mathbb{R}_+$ with regards to country i 's taxes:

$$\mathbf{1} + \mathbf{x}'_i = a(\mathbf{1} + \mathbf{x}_i); \quad \mathbf{1} + \mathbf{t}'_i = a(\mathbf{1} + \mathbf{t}_i); \quad \mathbf{1} + \mathbf{s}'_i = (\mathbf{1} + \mathbf{s}_i) / \tilde{a}; \quad w'_i = (a/\tilde{a})w_i; \quad \boldsymbol{\tau}'_i = (a/\tilde{a})\boldsymbol{\tau}_i;$$

but consist of the same tax levels in the rest world (namely, $-i$):

$$\mathbf{1} + \mathbf{x}'_{-i} = \mathbf{1} + \mathbf{x}_{-i} \quad \mathbf{1} + \mathbf{t}'_{-i} = \mathbf{1} + \mathbf{t}_{-i} \quad \mathbf{1} + \mathbf{s}'_{-i} = \mathbf{1} + \mathbf{s}_{-i} \quad \mathbf{w}'_{-i} = \mathbf{w}_{-i}; \quad \boldsymbol{\tau}'_{-i} = \boldsymbol{\tau}_{-i}.$$

Our goal is to prove that (i) if $\mathbb{T} \in \mathbb{F}$ then $\mathbb{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbb{T}') = W_n(\mathbb{T})$ for all $n \in \mathbb{C}$. To this end, we establish to two intermediate claims. The first claim posits that—supposing equilibrium quantities are identical under \mathbb{T} and \mathbb{T}' (i.e., $\mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T})$)—the implied nominal income and price

levels under \mathbb{T} and \mathbb{T}' are the same up to a scale. Stated mathematically,

$$[\text{Claim 1}] \quad \mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T}) \implies \begin{cases} \tilde{\mathbf{P}}_i(\mathbb{T}') = a\tilde{\mathbf{P}}_i(\mathbb{T}); & \tilde{\mathbf{P}}_{-i}(\mathbb{T}') = \tilde{\mathbf{P}}_{-i}(\mathbb{T}) \\ Y_i(\mathbb{T}') = aY_i(\mathbb{T}); & \mathbf{Y}_{-i}(\mathbb{T}') = \mathbf{Y}_{-i}(\mathbb{T}) \end{cases}.$$

In the above formulation $\tilde{\mathbf{P}}_i = \{\tilde{\mathbf{P}}_{1i}, \dots, \tilde{\mathbf{P}}_{N_i}\}$ denotes the entire vector of consumer price indexes in destination i , and $\mathbf{Q} \equiv \{Q_{n\ell, g}\}_{n, \ell, g}$ is the entire vector of equilibrium quantities. To prove the above claim, we compute nominal income and consumer price indexes under \mathbb{T} and \mathbb{T}' starting from the assumption that $\mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T})$. First, consider nominal price indexes. To simplify the notation for prices, define

$$\delta_{jn,k}(\mathbb{T}) \equiv \bar{\rho}_{jn,k} (1 - a_{j,k}(\mathbb{T}))^{\frac{1}{\gamma_k} - 1} Q_{j,k}(\mathbb{T})^{-\frac{1}{\gamma_k}}.$$

By assumption, $a_{j,k}(\mathbb{T}') = a_{j,k}(\mathbb{T})$ and $Q_{j,k}(\mathbb{T}') = Q_{j,k}(\mathbb{T})$, indicating that $\delta_{jn,k}(\mathbb{T}) = \delta_{jn,k}(\mathbb{T}') = \bar{\delta}_{jn,k}$. Now, consider the price index of a generic good ji, k imported by i from origin $j \neq i$. Invoking Equations 2 and 5, the *consumer* price index of good ji, k under \mathbb{T}' and \mathbb{T} exhibit the following relationship:

$$\tilde{P}_{ji,k}(\mathbb{T}') = \bar{\delta}_{ji,k} \frac{1 + t'_{ji,k}}{(1 + x'_{ji,k})(1 + s'_{j,k})} w'_j = \bar{\delta}_{ji,k} \frac{a(1 + t_{ji,k})}{(1 + x_{ji,k})(1 + s_{j,k})} w_j = a\tilde{P}_{ji,k}(\mathbb{T}),$$

where the third equality follows from the fact that $1 + t'_{ji,k} = a(1 + t_{ji,k})$, while $w'_j = w_j$, $x'_{ji,k} = x_{ji,k}$, and $s'_{j,k} = s_{j,k}$ (since $w_j \in \mathbf{w}_{-i}$, $x_{ji,k} \in \mathbf{x}_{-i}$, and $s_{j,k} \in \mathbf{s}_{-i}$). Second, consider a typical good ii, k that is produced and consumed locally in country i . The consumer price of ii, k under combination \mathbb{T}' can be related to its price under \mathbb{T} as follows

$$\tilde{P}_{ii,k}(\mathbb{T}') = \bar{\delta}_{ii,k} \frac{1}{1 + s'_{i,k}} w'_i = \bar{\delta}_{ii,k} \frac{1}{\frac{1}{a}(1 + s_{i,k})} \times \frac{a}{a} w_i = a\tilde{P}_{ii,k}(\mathbb{T}),$$

where the third equality follows from the fact that $1 + s'_{i,k} = (1 + s_{i,k})/a$ and $w'_i = aw_i/a$. Third, consider the price of a typical good ij, k export by i to destination market $j \neq i$. The consumer price of ij, k under combination \mathbb{T}' can be related to its price under \mathbb{T} as follows:

$$\tilde{P}_{ij,k}(\mathbb{T}') = \bar{\delta}_{ij,k} \frac{1 + t'_{ij,k}}{(1 + x'_{ij,k})(1 + s'_{i,k})} w'_i = \bar{\delta}_{ij,k} \frac{1 + t_{ij,k}}{a(1 + x'_{ij,k}) \times \frac{1}{a}(1 + s'_{i,k})} \times \frac{a}{a} w_i = \tilde{P}_{ij,k}(\mathbb{T}),$$

where the third equality follows from the fact that $1 + x'_{ij,k} = a(1 + x_{ij,k})$, $1 + s_{i,k} = (1 + s'_{i,k})/a$, and $w'_i = aw_i/a$; while $t'_{ij,k} = t_{ij,k}$ since $t_{ij,k} \in \mathbf{t}_{-i}$. Lastly, it follows trivially that $\tilde{P}_{jn,k}(\mathbb{T}') = \tilde{P}_{jn,k}(\mathbb{T})$ if j

and $n \neq i$. Considering that $\tilde{\mathbf{P}}_i = \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\}$, the above equations establish that

$$\tilde{\mathbf{P}}_i(\mathbb{T}') = a\tilde{\mathbf{P}}_i(\mathbb{T}), \quad \tilde{\mathbf{P}}_{-i}(\mathbb{T}') = \tilde{\mathbf{P}}_{-i}(\mathbb{T}). \quad (45)$$

Next, we turn to our claim about nominal income levels. To simplify the presentation, we hereafter use $X \equiv X(\mathbb{T})$ and $X' \equiv X(\mathbb{T}')$ to denote the value of a generic variable X under policy-wage combinations \mathbb{T} and \mathbb{T}' . Keeping in mind this choice of notation, country i 's nominal income under \mathbb{T}' , i.e., $Y'_i \equiv Y_i(\mathbb{T}')$ is given by:

$$\begin{aligned} Y'_i &= w'_i L_i + \sum_k \sum_j \left(\alpha_k \frac{\gamma_k - 1}{\gamma_k} P'_{ij,k} Q'_{ij,k} \right) + \sum_k \left[\left(\frac{1}{1 + s'_{i,k}} - 1 \right) P'_{ii,k} Q'_{ii,k} \right] \\ &\quad + \sum_k \sum_{j \neq i} \left(\frac{t'_{ji,k}}{(1 + x'_{ji,k})(1 + s'_{j,k})} P'_{ji,k} Q'_{ji,k} + \left[\frac{1}{(1 + x'_{ij,k})(1 + s'_{i,k})} - 1 \right] P'_{ij,k} Q'_{ij,k} \right) \\ &= w'_i L_i + \sum_k \sum_j \left(\alpha_k \frac{\gamma_k - 1}{\gamma_k} \frac{(1 + x'_{ij,k})(1 + s'_{i,k})}{1 + t'_{ij,k}} \tilde{P}'_{ij,k} Q_{ij,k} \right) + \sum_k \left[(1 - [1 + s'_{i,k}]) \tilde{P}'_{ii,k} Q'_{ii,k} \right] \\ &\quad + \sum_k \sum_{j \neq i} \left(\left(1 - \frac{1}{1 + t'_{ji,k}} \right) \tilde{P}'_{ji,k} Q'_{ji,k} + \left[\frac{1}{1 + t'_{ij,k}} - \frac{(1 + x'_{ij,k})(1 + s'_{i,k})}{1 + t'_{ij,k}} \right] \tilde{P}'_{ij,k} Q'_{ij,k} \right). \end{aligned}$$

Note that—by assumption—policy-wage combinations \mathbb{T} and \mathbb{T}' exhibit the same output schedule, i.e., $Q'_{ii,k} = Q_{ii,k}$, $Q'_{ji,k} = Q_{ji,k}$, and $Q'_{ij,k} = Q_{ij,k}$. Also, recall that (\mathbb{T} and \mathbb{T}' are constructed such that) $1 + t'_{ji,k} = a(1 + t_{ji,k})$, $1 + x'_{ij,k} = a(1 + x_{ij,k})$, $1 + s_{i,k} = (1 + s'_{i,k})/\tilde{a}$, and $w'_i = aw_i/\tilde{a}$, $t'_{ij,k} = t_{ij,k}$. Considering these relationships and plugging our earlier result that (i) $\tilde{P}_{ii,k} = aP_{ii,k}$, (ii) $P'_{ji,k} = a\tilde{P}_{ji,k}$, and (iii) $\tilde{P}'_{ij,k} = \tilde{P}_{ij,k}$ into the above equation, yields the following expression for Y'_i :

$$\begin{aligned} Y'_i &= \frac{a}{\tilde{a}} w_i L_i + \sum_k \sum_j \left(\alpha_k \frac{\gamma_k - 1}{\gamma_k} \frac{1}{a} \frac{(1 + x'_{ij,k})^{\frac{1}{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) + \sum_k \left[\left(1 - \frac{1}{\tilde{a}} (1 + s_{i,k}) \right) a\tilde{P}_{ii,k} Q_{ii,k} \right] \\ &\quad + \sum_{j,k} \left[\left(1 - \frac{1}{a(1 + t_{ji,k})} \right) a\tilde{P}_{ji,k} Q_{ji,k} + \left[\frac{1}{1 + t_{ij,k}} - \frac{a(1 + x_{ij,k}) \times \frac{1}{\tilde{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \right] \tilde{P}_{ij,k} Q_{ij,k} \right]. \end{aligned}$$

Invoking the balanced trade condition, $\sum_k \sum_{j \neq i} \left(\frac{1}{1 + t_{ji,k}} \tilde{P}_{ji,k} Q_{ji,k} - \frac{1}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) = 0$, and observing that $(1 + s_{i,k})\tilde{P}_{ii,k} = P_{ii,k}$ and $\frac{(1 + x_{ij,k})(1 + s_{i,k})}{1 + t_{ij,k}} \tilde{P}_{ij,k} = P_{ij,k}$, the above equation reduces to

$$Y'_i = \frac{a}{\tilde{a}} w_i L_i + a \sum_k \left[\tilde{P}_{ii,k} Q_{ii,k} + \sum_{j \neq i} \tilde{P}_{ji,k} Q_{ji,k} \right] - \frac{a}{\tilde{a}} \sum_k \left[P_{ii,k} Q_{ii,k} + \sum_{j \neq i} \left(1 - \alpha_k \frac{\gamma_k - 1}{\gamma_k} \right) P_{ij,k} Q_{ij,k} \right].$$

Appealing to the labor market clearing condition, $w_i L_i - \sum_k \sum_j \left[\left(1 - \alpha_k \frac{\gamma_k - 1}{\gamma_k}\right) P_{ij,k} Q_{ij,k} \right] = 0$, the above equation further simplifies as follows

$$Y'_i = a \sum_k \left[\tilde{P}_{ii,k} Q_{ii,k} + \sum_{j \neq i} \tilde{P}_{ji,k} Q_{ji,k} \right] = a [w_i L_i + T_i] = a Y_i, \quad (46)$$

where $T_i \equiv T_i(\mathbb{T})$ denotes country i 's tax revenues under \mathbb{T} . To be clear, the third line, in the above equation, follows from country i 's balanced budget condition (i.e., total expenditure = total income). Turning to the rest of the world: The fact that $Y_n(\mathbb{T}') = Y_n(\mathbb{T})$ for all $n \neq i$ follows trivially from a similar line of arguments—hence, establishing our claim about nominal income levels:

$$Y_i(\mathbb{T}') = a Y_i(\mathbb{T}); \quad \mathbf{Y}_{-i}(\mathbb{T}') = \mathbf{Y}_{-i}(\mathbb{T})$$

Together Equations 45 and 46 establish Claim (1). Stepping back, Claim (1) starts from the assumption that $\mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T})$. Our next claim indicates that this assumption is validated by the nominal income and price levels implied by \mathbb{T}' and \mathbb{T} . Below, we state this lemma noting that it follows trivially from the Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, being homogeneous of degree zero. \square

$$[\text{Claim 2}] \quad \forall a \in \mathbb{R}_+ : \begin{cases} \tilde{\mathbf{P}}_i(\mathbb{T}') = a \tilde{\mathbf{P}}_i(\mathbb{T}) \\ Y_i(\mathbb{T}') = a Y_i(\mathbb{T}) \end{cases} \implies \mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T})$$

Together, Claims (1) and (2) establish that equilibrium quantities should be indeed identical under policy-wage combinations \mathbb{T} and \mathbb{T}' . That is, $\mathbf{Q}(\mathbb{T}') = \mathbf{Q}(\mathbb{T})$. Hence, if $\mathbb{T} \in \mathbb{F}$ it follows immediately that (i) $\mathbb{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbb{T}') = W_n(\mathbb{T})$ for all $n \in \mathbb{C}$, which is the claim of Lemma 1.

B Proofs and Derivations

B.1 Proof of Lemma 2

Step 1. We first show that $\frac{\partial \ln Y_i}{\partial \ln w_i} = 0$ if the policy vector \mathbb{P}_i is fixed and policy-wage is feasible $(\mathbb{P}_i; w_i) \in \mathbb{F}_i^w$. Using the income equation, and holding fixed $\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}\}_{j \neq i,k} \in \mathbb{P}_i$,

$$\frac{\partial Y_i}{\partial \ln w_i} = w_i \bar{L}_i - \sum_{k, j \neq i} \left[\frac{\partial \ln P_{ji,k}}{\partial \ln w_i} + \frac{\partial \ln Q_{ji,k}}{\partial \ln w_i} \right] P_{ji,k} Q_{ji,k} - \sum_{k, j} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) \left[\frac{\partial \ln P_{ij,k}}{\partial \ln w_i} + \underbrace{\frac{\partial \ln Q_{ij,k}}{\partial \ln w_i}}_{=0 \text{ if } j \neq i} \right] P_{ij,k} Q_{ij,k}$$

Notice that home's wage, w_i , affects price of a variety directly if that variety is produced at home, and also indirectly through scale economies,

$$\begin{aligned} \frac{\partial \ln Y_i}{\partial \ln w_i} &= \frac{w_i \bar{L}_i}{Y_i} - \sum_{k, j \neq i} \left[\frac{\partial \ln P_{ji,k}}{\partial \ln Q_{j,k}} \frac{\partial \ln Q_{j,k}}{\partial \ln Q_{ji,k}} \frac{\partial \ln Q_{ji,k}}{\partial \ln w_i} + \frac{\partial \ln Q_{ji,k}}{\partial \ln w_i} \right] \frac{P_{ji,k} Q_{ji,k}}{Y_i} \\ &\quad - \sum_{k, j} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) (1 + \frac{\partial \ln P_{ij,k}}{\partial \ln Q_{i,k}} \frac{\partial \ln Q_{i,k}}{\partial \ln Q_{ii,k}} \frac{\partial \ln Q_{ii,k}}{\partial \ln w_i}) \frac{P_{ij,k} Q_{ij,k}}{Y_i} - \sum_k (1 - \alpha_k \frac{\gamma_k - 1}{\gamma_k}) \frac{\partial \ln Q_{ii,k}}{\partial \ln w_i} \frac{P_{ii,k} Q_{ii,k}}{Y_i} \\ &= \frac{w_i \bar{L}_i}{Y_i} - \sum_{k, j \neq i} (1 - \frac{1}{\gamma_k} r_{ji,k}) \eta_{ji,k} \frac{\partial \ln Y_i}{\partial \ln w_i} \frac{P_{ji,k} Q_{ji,k}}{Y_i} \\ &\quad - \sum_{k, j} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) (1 - \frac{1}{\gamma_k} r_{ii,k} \eta_{ii,k} \frac{\partial \ln Y_i}{\partial \ln w_i}) \frac{P_{ij,k} Q_{ij,k}}{Y_i} - \sum_k (1 - \alpha_k \frac{\gamma_k - 1}{\gamma_k}) \eta_{ii,k} \frac{\partial \ln Y_i}{\partial \ln w_i} \frac{P_{ii,k} Q_{ii,k}}{Y_i} \end{aligned}$$

where $\frac{\partial \ln Q_{j,k}}{\partial \ln Q_{ii,k}} = r_{ji,k}$. Reorganizing terms,

$$\Lambda_i^Y \left(\frac{\partial \ln Y_i}{\partial \ln w_i} \right) - \frac{1}{Y_i} \underbrace{\left(w_i \bar{L}_i - \sum_{k, j} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k} Q_{ij,k} \right)}_{=0} = 0$$

where the second term equals zero since the policy-wage pair is feasible, $(\mathbb{P}_i; w_i) \in \mathbb{F}_i^w$, meaning that the labor market clearing condition (9) has to hold; and, Λ_i^Y summarizes the coefficient of the wage effect on income,

$$\Lambda_i^Y \equiv 1 + \sum_{k, j \neq i} (1 - \frac{r_{ji,k}}{\gamma_k}) \eta_{ji,k} \frac{P_{ji,k} Q_{ji,k}}{Y_i} - \sum_k \frac{r_{ii,k}}{\gamma_k} \eta_{ii,k} \frac{w_i \bar{L}_i}{Y_i} + \sum_k (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) \eta_{ii,k} \frac{\partial \ln Y_i}{\partial \ln w_i} \frac{P_{ii,k} Q_{ii,k}}{Y_i}$$

From the fact that Λ_i^Y is generically non-zero, it follows that:

$$\frac{\partial \ln Y_i}{\partial \ln w_i} = 0.$$

Step 2. Within the feasible set of policy-wage, $(\mathbb{P}_i; w_i) \in \mathbb{F}_i^w$, and holding fixed the policy vector \mathbb{P}_i , we can express the derivative of $W_i(\mathbb{P}_i; \mathbf{w})$ w.r.t. w_i as follows:

$$\frac{\partial W_i(\cdot)}{\partial w_i} = \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial Y_i}{\partial w_i} \right) - \frac{1}{Y_i} \sum_j \sum_k \left(\delta_{ji} Z_{jk} \frac{\partial \ln Z_{j,k}(\cdot)}{\partial \ln Q_{j,k}} \frac{\partial \ln Q_{j,k}(\cdot)}{\partial \ln Q_{j,k}} \frac{\partial \ln D_{ji,k}(\cdot)}{\partial \ln Y_i} \right) \left(\frac{\partial Y_i}{\partial w_i} \right) = 0$$

where we use $\frac{\partial \ln Y_i}{\partial \ln w_i} = 0$ from Step 1.

B.2 Proof of Lemma 3

Notice that we have already sketched a proof for Lemma 3 in the buildup to the formal statement of the lemma. However, here we prove this lemma using a somewhat different approach that allows us to provide more details.

Recall that Applying the chain rule to $W_i(\mathbb{P}_i; Y_i) = V_i(\bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i), \tilde{\mathbb{P}}_i) - \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)$, yields the following expression:

$$\frac{dW_i(\mathbb{P}_i; Y_i)}{d \ln \mathcal{P}} = \frac{\partial V_i(\cdot)}{\partial \ln \mathcal{P}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i} + \left(\frac{\partial W_i(\mathbb{P}_i; Y_i)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} \frac{d \ln Y_i}{d \ln \mathcal{P}}.$$

Before moving forward, let us emphasize two important details:

1. Following Lemma 2, we are treating the vector of wages, $\mathbf{w} = \bar{\mathbf{w}}$, as constant throughout our proof. So, the partial derivatives subject to Y_i can be more-broadly interpreted as partial derivatives subject to holding both Y_i and \mathbf{w} fixed, i.e., $\left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i} \sim \left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i, \mathbf{w}}$.
2. Every time we differentiated w.r.t. a $\mathcal{P} \in \mathbb{P}_i$, we are also fixing the remaining elements of \mathbb{P}_i . That is because the government is directly choosing every single element of \mathbb{P}_i . So, to be even more precise, we may interpret the partial derivative subject to Y_i as derivative subject to fixing Y_i , \mathbf{w} , and $\mathbb{P}_i - \{\mathcal{P}\}$, i.e., $\left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i} \sim \left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathcal{P}} \right)_{Y_i, \mathbf{w}, \mathbb{P}_i - \{\mathcal{P}\}}$.

Noting the above explanation, we now proceed with the proof in two steps.

Step #1: Characterizing $\left(\frac{\partial W_i}{\partial Y_i} \right)_{\mathbb{P}_i}$.

To characterize $\left(\frac{\partial W_i}{\partial Y_i} \right)_{\mathbb{P}_i}$, we can apply the chain rule, which implies

$$\left(\frac{\partial W_i(\mathbb{P}_i; Y_i)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} = \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln Y_i} \right)_{\mathbb{P}_i}. \quad (47)$$

As outlined in Appendix A.3 , $T_i(\cdot)$ and $Z_{j,k}(\cdot)$ are formulated as

$$\begin{cases} T_i(\mathbb{P}_i; Y_i) = \sum_k \left(\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} P_{ii,k}(\mathbb{P}_i; Y_i) Q_{i,k}(\mathbb{P}_i; Y_i) \right) \\ \quad + \sum_{k,j} [(\tilde{P}_{ij,k} - P_{ij,k}(\mathbb{P}_i; Y_i)) Q_{ij,k}(\mathbb{P}_i)] + \sum_{k,j \neq i} [(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{P}_i; Y_i)) Q_{ji,k}(\mathbb{P}_i; Y_i)] ; \\ Z_{j,k}(\mathbb{P}_i; Y_i) = \bar{z}_{j,k} (1 - a_{j,k})^{\frac{1}{\alpha_{j,k}} + \frac{1}{\gamma_k} - 1} Q_{j,k}(\mathbb{P}_i; Y_i)^{1 - \frac{1}{\gamma_k}} ; \end{cases}$$

with the equilibrium quantity and producer prices given by

$$\begin{aligned} Q_{jn,k}(\mathbb{P}_i; Y_i) &= \begin{cases} \mathcal{D}_{jn,k}(\tilde{\mathbf{P}}_{in}, \mathbf{P}_{-in}, \tilde{Y}_n) & \text{if } n \neq i \\ \mathcal{D}_{ji,k}(\tilde{\mathbf{P}}_i, Y_i) & \text{if } n = i \end{cases} ; \\ Q_{i,k}(\mathbb{P}_i; Y_i) &= \sum_j d_{ij,k} Q_{ij,k}(\mathbb{P}_i; Y_i); \\ P_{jn,k}(\mathbb{P}_i; Y_i) &= \bar{\rho}_{jn,k} (1 - a_{j,k})^{\frac{1}{\gamma_k} - 1} Q_{j,k}(\mathbb{P}_i; Y_i)^{-\frac{1}{\gamma_k}}. \end{aligned}$$

where $\bar{\rho}_{jn,k} \equiv \bar{d}_{jn,k} \bar{p}_{jj,k} \bar{\omega}_j$. Using our definition for the income elasticity of demand, we can produce the following partial derivatives for quantities and producer prices:

$$\begin{aligned} \left(\frac{\partial \ln Q_{ji,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} &= \frac{\partial \ln \mathcal{D}_{ji,k}(\cdot)}{\partial \ln Y_i} = \eta_{ji,k} \quad \left(\frac{\partial \ln Q_{jn,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} = 0, \quad (j \neq i) \\ \left(\frac{\partial \ln Q_{j,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} &= \left(\frac{\partial \ln Q_{j,k}}{\partial \ln Q_{ji,k}} \right)_{\mathbb{P}_i} \left(\frac{\partial \ln Q_{ji,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} = r_{ji,k} \eta_{ji,k} \\ \left(\frac{\partial \ln P_{ij,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} &= \left(\frac{\partial \ln P_{ii,k}(\cdot)}{\partial \ln Q_{i,k}} \right)_{\mathbb{P}_i} \left(\frac{\partial \ln Q_{i,k}}{\partial \ln Q_{ii,k}} \right)_{\mathbb{P}_i} \left(\frac{\partial \ln Q_{ji,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} = -\frac{1}{\gamma_k} r_{ii,k} \eta_{ii,k} \\ \left(\frac{\partial \ln P_{ji,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} &= \left(\frac{\partial \ln P_{jj,k}(\cdot)}{\partial \ln Q_{ji,k}} \right)_{\tilde{\mathbf{P}}_{ji,k}} \left(\frac{\partial \ln Q_{ji,k}(\cdot)}{\partial \ln Y_i} \right)_{\mathbb{P}_i} = \omega_{ji,k} \eta_{ji,k} \quad (j \neq i) \end{aligned}$$

where $\omega_{ji,k}$ denotes the *export supply elasticity* as defined in Appendix A.5. Using the above expressions and noting that

$$\begin{aligned} T_i(\mathbb{P}_i; Y_i) &= \sum_{k,j \neq i} [(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{P}_i; Y_i)) Q_{ji,k}(\mathbb{P}_i; Y_i)] \\ &\quad + \sum_{k,j} \left[(\tilde{P}_{ij,k} - (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\mathbb{P}_i; Y_i)) Q_{ij,k}(\mathbb{P}_i; Y_i) \right], \end{aligned}$$

produces the following formulation for $\left(\frac{\partial T_i(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i}$ and $\left(\frac{\partial Z_{j,k}(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i}$:

$$\begin{cases} \left(\frac{\partial T_i(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i} = -\sum_k \left[\left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}\right) \frac{1}{\gamma_k} \eta_{ii,k} P_{ii,k} Q_{ii,k} \right] \\ \quad + \sum_k \left[\left(\tilde{P}_{ii,k} - \left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}\right) P_{ii,k}\right) Q_{ii,k} \eta_{ii,k} \right] + \sum_{k,j \neq i} \left[\left(\tilde{P}_{ji,k} - (1 + \omega_{ji,k}) P_{ji,k}\right) Q_{ji,k} \eta_{ji,k} \right]; \\ \left(\frac{\partial Z_{j,k}(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i} = \left(1 - \frac{1}{\gamma_k}\right) Z_{j,k} r_{ji,k} \eta_{ji,k} = \frac{\gamma_k - 1}{\gamma_k} v_{j,k} P_{ji,k} Q_{ji,k} \eta_{ji,k}. \end{cases}$$

To provide more detail: The first line in the expression for $\left(\frac{\partial T_i(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i}$ derives from the following intermediate result:

$$\begin{aligned} \sum_{j=1}^N \left[\left(\frac{\partial \ln P_{ij,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbb{P}_i} P_{ij,g} Q_{ij,g} \right] &= \sum_{j=1}^N \left[\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbb{P}_i} \frac{\partial \ln Q_{i,g}(Q_{i1,g} \dots Q_{iN,g})}{\partial \ln Q_{ii,g}} P_{ij,g} Q_{ij,g} \right] \\ &= -\sum_{j=1}^N \left(\frac{1}{\gamma_g} r_{ii,g} P_{ij,g} Q_{ij,g} \right) = -\frac{1}{\gamma_g} r_{ii,g} \sum_{j=1}^N (P_{ij,g} Q_{ij,g}) = -\frac{1}{\gamma_g} P_{ii,g} Q_{ii,g} \end{aligned} \quad (48)$$

Plugging the expressions for $\left(\frac{\partial T_i(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i}$ and $\left(\frac{\partial Z_{j,k}(\cdot)}{\partial \ln Y_i}\right)_{\mathbb{P}_i}$ back into Equation 47, and noting that $\sum_j (P_{ij,k} Q_{ij,k} r_{ii,k}) = P_{ii,k} Q_{ii,k} r_{ii,k} = P_{ii,k} Q_{ii,k}$, yields

$$\begin{aligned} \left(\frac{\partial W_i}{\partial \ln Y_i}\right)_{\mathbb{P}_i} &= \sum_k \left[\left(\tilde{P}_{ii,k} - \frac{\gamma_k - 1}{\gamma_k} \left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} + \tilde{\delta}_{ii,k} v_{i,k}\right) P_{ii,k}\right) Q_{ii,k} \eta_{ii,k} + \sum_{j \neq i} \left(\left[\tilde{P}_{ji,k} - (1 + \omega_{ji,k} - \frac{\gamma_k - 1}{\gamma_k} \delta_{ji} v_{j,k}) P_{ji,k}\right] Q_{ji,k} \eta_{ji,k} \right) \right] \\ &= \sum_k \left[\left(1 - \frac{\gamma_k - 1}{\gamma_k} \left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} + \tilde{\delta}_{ii,k} v_{i,k}\right) \frac{P_{ii,k}}{\tilde{P}_{ii,k}}\right) e_{ii,k} \eta_{ii,k} + \sum_{j \neq i} \left(\left[1 - (1 + \omega_{ji,k} - \frac{\gamma_k - 1}{\gamma_k} \delta_{ji} v_{j,k}) \frac{P_{ji,k}}{\tilde{P}_{ji,k}}\right] e_{ji,k} \eta_{ji,k} \right) \right] Y_i. \end{aligned} \quad (49)$$

Step #2: Proving that $\left(\frac{\partial W_i}{\partial Y_i}\right)_{\mathbb{P}_i} = 0$ at the optimum.

This step establishes that if for all $\mathcal{P} \in \{\mathbf{a}_i, \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}$ if

$$\frac{\partial V_i(\cdot)}{\partial \ln \mathcal{P}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}, Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \mathcal{P}} \right)_{\mathbf{w}, Y_i} = 0,$$

then $\left(\frac{\partial W_i}{\partial Y_i}\right)_{\mathbb{P}_i} = 0$. For the sake of clarity, our notation indicates explicitly that the partial derivative w.r.t. \mathcal{P} are taken while holding both \mathbf{w} and Y_i (in the demand function) constant.

[Abatement Level: a_i] First, consider the case where $\mathcal{P} = 1 - a_{i,k}$. Keep in mind that the instrument set \mathbb{P}_i includes all consumers prices in the local economy. So, holding all instruments except $a_{i,k}$ (i.e., $\mathbb{P}_i - \{a_{i,k}\}$) fixed, then $a_{i,k}$ has no direct effect on $V_i(Y_i = \bar{w}_i \bar{L}_i + T_i, \tilde{\mathbf{P}}_i)$. However, $a_{i,k}$ does affect tax revenues and local emission levels as indicated below:

$$\begin{cases} \frac{\partial V_i(\cdot)}{\partial \ln(1+a_{i,k})} = 0 \\ \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln(1+a_{i,k})} \right)_{\mathbf{w}, Y_i} = - \sum_{j=1}^N \left((1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k} Q_{ij,k} \left(\frac{\partial \ln P_{ii,k}}{\partial \ln(1-a_{i,k})} \right)_{\mathbf{w}, Y_i} \right) = (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) \frac{\gamma_k - 1}{\gamma_k} \sum_{j=1}^N (P_{ij,k} Q_{ij,k}) \\ \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln(1+a_{i,k})} \right)_{\mathbf{w}, Y_i} = \delta_{ii} \left(\frac{\partial Z_{i,k}(\dots, 1-a_{i,k})}{\partial \ln(1-a_{i,k})} \right)_{\mathbf{w}, Y_i} = \left(\frac{1}{\alpha_{i,k}} - \frac{\gamma_k - 1}{\gamma_k} \right) \delta_{ii} Z_{i,k} \end{cases}$$

Combining the above equation yields (note that $P_{ii,k} Q_{i,k} = \sum_{j=1}^N P_{ij,k} Q_{ij,k}$)

$$\begin{aligned} & \frac{\partial V_i(\cdot)}{\partial \ln(1+a_{i,k})} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln(1+a_{i,k})} \right)_{\mathbf{w}, Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln(1+a_{i,k})} \right)_{\mathbf{w}, Y_i} \\ &= \frac{\partial V_i(\cdot)}{\partial Y_i} \left[1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \right] \frac{\gamma_k - 1}{\gamma_k} P_{ii,k} Q_{i,k} - \frac{1}{\alpha_k} \delta_{ii} Z_{i,k} \left[1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \right] = 0. \end{aligned} \quad (50)$$

[Domestic and Import Prices: $\tilde{\mathbf{P}}_{ii}$, and $\tilde{\mathbf{P}}_{ji}$] Next, consider the case where $\mathcal{P} = \tilde{P}_{ii,k}$ or $\tilde{P}_{ji,k}$ (where $j \neq i$). We are combining both instruments, as the partial derivative w.r.t. to both $\tilde{P}_{ii,k}$ and $\tilde{P}_{ji,k}$ produce similar-looking equation. So, we henceforth use n to denote the origin country with the understanding that either $n = i$ or $n = j$. For this case, we first detail the partial derivative of tax revenues, $T_i(\cdot)$, which is more involved:

$$\begin{aligned} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ni,k}} \right)_{\mathbf{w}, Y_i} &= \tilde{P}_{ii,k} Q_{ii,k} + \sum_g \left[\left(\tilde{P}_{ii,g} - [1 - \alpha_{i,g} \frac{\gamma_g - 1}{\gamma_g}] P_{ii,g} \right) Q_{ii,g} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ni,k}} \right)_{\mathbf{w}, Y_i} \right] \\ &\quad - \sum_g \sum_j \left[[1 - \alpha_{i,g} \frac{\gamma_g - 1}{\gamma_g}] P_{ij,g} Q_{ij,g} \left(\frac{\partial \ln P_{ij,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w}, Y_i} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ni,k}} \right)_{\mathbf{w}, Y_i} \right] \\ &\quad + \sum_{j \neq i} \sum_g \left[\left(\tilde{P}_{ji,g} - [1 + \left(\frac{\partial \ln P_{ji,g}}{\partial \ln Q_{ji,g}} \right)_{\mathbf{w}, Y_i}] P_{ji,g} \right) Q_{ji,g} \left(\frac{\partial \ln Q_{ji,g}}{\partial \ln \tilde{P}_{ni,k}} \right)_{\mathbf{w}, Y_i} \right]. \end{aligned}$$

As before, $\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ni,k}} \right)_{\mathbf{w}, Y_i} = \frac{\partial \ln \mathcal{D}_{ni,g}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,k}} = \varepsilon_{ni,g}^{(ii,k)}$. The second line can also be simplified the steps outlined under Equation 48. Accordingly, we can express the different elements in Equation

$$\left\{ \begin{array}{l} \frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ii,k}} = -P_{ii,k} Q_{ii,k} \frac{\partial V_i(\cdot)}{\partial Y_i} \quad \text{[Roy's identity]} \\ \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ii,k}} \right)_{\mathbf{w}, Y_i} = \sum_{n \neq i} \sum_g \left[\left(1 - (1 + \omega_{ni,g}) \frac{P_{ni,g}}{\tilde{P}_{ni,g}} \right) \tilde{P}_{ni,g} Q_{ni,g} \varepsilon_{ni,g}^{(ii,k)} \right] + \sum_g \left[\left(1 - \left(1 - \alpha_{i,g} \frac{\gamma_g - 1}{\gamma_g} \right) \frac{\gamma_g - 1}{\gamma_g} \frac{P_{ii,g}}{\tilde{P}_{ii,g}} \right) \tilde{P}_{ii,g} Q_{ii,g} \varepsilon_{ii,g}^{(ii,k)} \right] \\ \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ii,k}} \right)_{\mathbf{w}, Y_i} = \sum_g \sum_j \delta_{ji} \left(\frac{\partial Z_{j,g}(\dots; Q_{j,g})}{\partial \ln Q_{j,g}} \frac{\partial \ln Q_{j,g}(Q_{j1,k}, \dots, Q_{jN,k})}{\partial \ln Q_{j,g}} \frac{\partial \ln Q_{ji,g}}{\partial \ln \tilde{P}_{ii,k}} \right)_{\mathbf{w}, Y_i} = \sum_g \sum_j \left[\delta_{ji} \frac{\gamma_k - 1}{\gamma_k} v_{j,k} \varepsilon_{ji,g}^{(ni,k)} P_{ji,g} Q_{ji,g} \right] \end{array} \right.$$

where the last line follows from the fact that (1) $\frac{\partial Z_{j,g}(\dots; Q_{j,g})}{\partial \ln Q_{j,g}} = \frac{\gamma_k - 1}{\gamma_k} Z_{j,g}$, (2) $\frac{\partial \ln Q_{j,g}(Q_{j1,k}, \dots, Q_{jN,k})}{\partial \ln Q_{ji,g}} = r_{ji,g}$, and (3) $v_{j,k} \equiv Z_{j,k} / P_{j,k} Q_{j,k}$. Combining the above equations yields

$$\begin{aligned} & \frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ii,k}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i)}{\partial \ln \tilde{P}_{ii,k}} \right)_{\mathbf{w}, Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln \tilde{P}_{ii,k}} \right)_{\mathbf{w}, Y_i} = \\ & \sum_g \left[\sum_{j \neq i} \left(1 - \left(1 + \omega_{ji,g} + \tilde{\delta}_{ji} v_{j,g} \frac{\gamma_g - 1}{\gamma_g} \right) \frac{P_{ji,g}}{\tilde{P}_{ji,g}} \right) e_{ji,g} \varepsilon_{ji,g}^{(ni,k)} \right] Y_i + \sum_g \left[\left(1 - \left(1 - \alpha_{i,g} \frac{\gamma_g - 1}{\gamma_g} + \tilde{\delta}_{ii} v_{i,g} \right) \frac{\gamma_g - 1}{\gamma_g} \frac{P_{ii,g}}{\tilde{P}_{ii,g}} \right) e_{ii,g} \varepsilon_{ii,g}^{(ni,k)} \right] Y_i \end{aligned} \quad (51)$$

For Equation 50 to hold it should be that $\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} P_{ii,k} Q_{i,k} - \tilde{\delta}_{ii} Z_{i,k} = 0$. Plugging this expression into Equation 51 yields

$$\sum_g \left[\sum_{j \neq i} \left(1 - \left(1 + \omega_{ji,g} + \tilde{\delta}_{ji} v_{j,g} \frac{\gamma_g - 1}{\gamma_g} \right) \frac{P_{ji,g}}{\tilde{P}_{ji,g}} \right) e_{ji,g} \varepsilon_{ji,g}^{(ni,k)} \right] + \sum_g \left[\left(1 - \frac{\gamma_g - 1}{\gamma_g} \frac{P_{ii,g}}{\tilde{P}_{ii,g}} \right) e_{ii,g} \varepsilon_{ii,g}^{(ni,k)} \right] = 0.$$

The above equation specifies the optimality condition for $N \times K$ different price instruments, $\tilde{P}_{ni,k}$. Simultaneously solving the above equation for all $\tilde{P}_{ni,k}$ amounts to solving the following matrix equation.

$$\begin{bmatrix} e_{1i,1} \varepsilon_{1i,1}^{(1i,1)} & \cdots & e_{Ni,1} \varepsilon_{Ni,1}^{(1i,1)} & \cdots & e_{1i,K} \varepsilon_{1i,K}^{(1i,1)} & \cdots & e_{Ni,K} \varepsilon_{Ni,K}^{(1i,1)} \\ \vdots & & \ddots & \ddots & & & \vdots \\ e_{1i,1} \varepsilon_{1i,1}^{(Ni,K)} & \cdots & e_{Ni,1} \varepsilon_{Ni,1}^{(Ni,K)} & \cdots & e_{1i,K} \varepsilon_{1i,K}^{(Ni,K)} & \cdots & e_{Ni,K} \varepsilon_{Ni,K}^{(Ni,K)} \end{bmatrix} \begin{bmatrix} \frac{\tilde{P}_{1i,k}^*}{P_{1i,1}} - \left(1 + \omega_{1i,k} + \tilde{\delta}_{1i,k} v_{1,k} \frac{\gamma_k - 1}{\gamma_k} \right) \\ \vdots \\ \frac{\tilde{P}_{ii,k}^*}{P_{ii,k}} - \frac{\gamma_k - 1}{\gamma_k} \\ \vdots \\ \frac{\tilde{P}_{Ni,k}^*}{P_{Ni,k}} - \left(1 + \omega_{Ni,k} + \tilde{\delta}_{Ni,k} v_{N,k} \frac{\gamma_k - 1}{\gamma_k} \right) \end{bmatrix}_k = \mathbf{0}.$$

As discussed in Section 3.1 and proven in the following appendix, is non-singular. So, the unique solution to the above equation is

$$\frac{\tilde{P}_{ji,k}^*}{P_{ji,1}} = 1 + \omega_{ji,k} + \tilde{\delta}_{ji} v_{j,k} \frac{\gamma_k - 1}{\gamma_k}; \quad \frac{\tilde{P}_{ii,k}^*}{P_{ii,k}} = \frac{\gamma_k - 1}{\gamma_k}, \quad (52)$$

which when plugged into Equation 49, trivially implies $\left(\frac{\partial W_i}{\partial \ln Y_i}\right)_{\mathbb{P}_i} = 0$.

B.3 Proof of Lemma 5

Following Proposition 2.E.2 in Mas-Colell et al. (1995) the Walrasian demand function satisfies $e_{ji,k} = |e_{ji,k} \varepsilon_{ji,k}^{(ji,k)}| - \sum_{n,g \neq j,k} |e_{ni,g} \varepsilon_{ni,g}^{(ji,k)}|$. Hence, since there exists a ji,k such that $e_{ji,k} > 0$, the matrix Ξ is strictly diagonally dominant. The Lèvy-Desplanques Theorem (Horn and Johnson (2012)), therefore, ensures that Ξ is non-singular. The lower bound on $\det(\Xi)$ follows trivially from Gerschgorin's circle theorem. Specifically, following Ostrowski (1952),

$$|\det(\Xi)| \geq \prod_j \prod_k \left(|e_{ji,k} \varepsilon_{ji,k}^{(ji,k)}| - \sum_{n,g \neq j,k} |e_{ni,g} \varepsilon_{ni,g}^{(ji,k)}| \right) = \prod_j \prod_k e_{ji,k} > 0.$$

B.4 Proof of Theorem 1

As discussed in Section 3.2, the expression for emission taxes follows from combining cost minimization with the optimal tax condition (refer to Equation 20). Domestic and import taxes were also implicitly derived in Appendix B.2 under Equation 52. Combining these expressions, we have:

$$\tau_{i,k}^* = \tilde{\delta}_{ii}, \quad 1 + s_{i,k}^* = \frac{P_{ii,k}^*}{\tilde{P}_{ii,k}} = \frac{\gamma_k}{\gamma_k - 1}; \quad 1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}} = 1 + \omega_{ji,k} + \tilde{\delta}_{ji} v_{j,k} \frac{\gamma_k - 1}{\gamma_k}.$$

To determine the export tax we can appeal to Proposition 1, whereby the necessary condition for optimality w.r.t. $\tilde{P}_{ij,k}$ ($j \neq i$) is

$$\frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} = 0. \quad (53)$$

First note that $\tilde{P}_{ij,k}$ does not directly enter the indirect utility function, so $\frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ij,k}} = 0$. Recalling the expression for $T_i(\mathbb{P}_i; Y_i, \mathbf{w})$ we can express the term corresponding to tax revenue effects as

$$\begin{aligned} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} &= \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g}] P_{ij,g} \right) Q_{ij,g} \left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} \right] \\ &\quad - \sum_g \sum_j \left[[1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g}] P_{ij,g} Q_{ij,g} \left(\frac{\partial \ln P_{ij,g}}{\partial \ln Q_{ij,g}} \right)_{\mathbf{w}, Y_i} \left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} \right] \\ &\quad - \sum_{n \neq i} \sum_g \left[P_{ni,g} Q_{ji,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{nj,g}} \right)_{\mathbf{w}, Y_i} \left(\frac{\partial \ln Q_{nj,g}}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} \right] = 0. \end{aligned}$$

To simplify the above equation, we can appeal to Equation 48 (Appendix B.2) and the following relationship:

$$\begin{aligned} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{nj,g}} \right)_{\mathbf{w}, Y_i} P_{ni,g} Q_{ni,g} &= \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{nj,g}} \right)_{\mathbf{w}, Y_i} P_{ni,g} Q_{ni,g} \\ &= \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, Y_i} P_{ni,g} Q_{ni,g} \equiv \omega_{ni,g} P_{ni,g} Q_{ni,g}. \end{aligned}$$

Doing so yields the following equation:

$$\left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} = \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g}] \frac{\gamma_k - 1}{\gamma_k} P_{ij,g} \right) Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)} \right] - \sum_g \sum_j \left[\omega_{ni,g} P_{ni,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)} \right].$$

Likewise the last term in Equation 53 (that accounts for emission effects) can be specified as

$$\left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} = \sum_{n,g} \left[\delta_{ni} \left(\frac{\partial Z_{n,g}(\dots, Q_{n,g})}{\partial \ln Q_{nj,g}} \frac{\partial \ln Q_{n,g}(Q_{n1,g}, \dots, Q_{nN,g})}{\partial \ln Q_{nj,g}} \frac{\partial \ln Q_{nj,g}}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} \right] = \frac{\partial V_i(\cdot)}{\partial Y_i} \sum_{n,g} \left[\tilde{\delta}_{ni} v_{n,g} \frac{\gamma_g - 1}{\gamma_g} P_{ni,g} Q_{nj,g} \right].$$

Plugging the above expressions back into Equation 53 (and dividing everything by $\frac{\partial V_i(\cdot)}{\partial Y_i} \tilde{P}_{ij,k} Q_{ij,k}$) yields the following optimality condition:

$$\begin{aligned} \left\{ \frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(\cdot)}{\partial Y_i} \left(\frac{\partial T_i(\mathbb{P}_i; Y_i, \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} - \left(\frac{\partial \delta_i \cdot \mathbf{Z}(\mathbb{P}_i; Y_i)}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w}, Y_i} \right\} \left[\frac{\partial V_i(\cdot)}{\partial Y_i} P_{ij,k} Q_{ij,k} \right]^{-1} &= \\ 1 + \sum_g \left[\left(1 - \left(1 - \alpha_g \frac{\gamma_g - 1}{\gamma_g} + \tilde{\delta}_{ni} v_{i,g} \right) \frac{\gamma_g - 1}{\gamma_g} \frac{P_{ij,g}}{\tilde{P}_{ij,g}} \right) \frac{e_{ij,g}}{e_{ij,k}} \varepsilon_{ij,g}^{(ij,k)} \right] - \sum_{n \neq i} \sum_g \left[\left(\omega_{ni,g} + \tilde{\delta}_{ni} z_{n,k} \frac{\gamma_k - 1}{\gamma_k} \right) \frac{e_{nj,g}}{e_{ij,k}} \varepsilon_{nj,g}^{(ij,k)} \right] &= 0 \end{aligned}$$

To detect the optimal export taxes, we guess the following formulation:

$$(1 + x_{ij,k}) \equiv \frac{P_{ij,k}/\tilde{P}_{ij,g}}{P_{ii,g}/\tilde{P}_{ii,g}} = \frac{\gamma_k}{\gamma_k - 1} \frac{\tilde{P}_{ij,k}}{\tilde{P}_{ij,g}} = \frac{1 + \varepsilon_{ij,k}}{\varepsilon_{ij,k}} \chi_{ij,k}$$

Plugging the above guess back into the F.O.C. yields the following:

$$1 + \sum_g \left[\left(1 - \chi_{ij,k} \frac{1 + \varepsilon_{ij,k}}{\varepsilon_{ij,k}} \right) \frac{e_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}} \right] - \sum_{n \neq i} \sum_g \left[t_{ni,g}^* \frac{e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}} \right] = 0; \quad [\tilde{P}_{ij,k}]$$

Noting that $1 + \sum_g \left[\frac{e_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}} \right] = - \sum_{n \neq i} \sum_g \left[\frac{e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}} \right]$ and dividing the above equation by $1 + \varepsilon_{ij,k}$,

$$- \sum_g \left[\chi_{ij,k} \frac{e_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k} \varepsilon_{ij,k}} \right] - \sum_{n \neq i} \sum_g \left[\frac{(1 + t_{ni,g}^*) e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k} (1 + \varepsilon_{ij,k})} \right] = 0$$

Noting that $(1 + \varepsilon_{ij,k}) e_{ij,k} = - \sum_{n \neq i} \sum_g e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}$, we can write the above equation in matrix as

$$\underbrace{\begin{bmatrix} \frac{e_{ij,1} \varepsilon_{ij,1}^{(ij,1)}}{e_{ij,1} \varepsilon_{ij,1}} & \dots & \frac{e_{ij,K} \varepsilon_{ij,K}^{(ij,1)}}{e_{ij,K} \varepsilon_{ij,1}} \\ \dots & \dots & \dots \\ \frac{e_{ij,1} \varepsilon_{ij,1}^{(ij,K)}}{e_{ij,1} \varepsilon_{ij,1}} & \dots & \frac{e_{ij,K} \varepsilon_{ij,K}^{(ij,K)}}{e_{ij,K} \varepsilon_{ij,K}} \end{bmatrix}}_{\mathbf{E}_{ij}} \begin{bmatrix} \chi_{ij,1} \\ \dots \\ \chi_{ij,K} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\sum_{n \neq i} \sum_g t_{ni,g}^* e_{nj,g} \varepsilon_{nj,g}^{(ij,1)}}{\sum_{n \neq i} \sum_g e_{nj,g} \varepsilon_{nj,g}^{(ij,1)}} \\ \dots \\ 1 + \frac{\sum_{n \neq i} \sum_g t_{ni,g}^* e_{nj,g} \varepsilon_{nj,g}^{(ij,K)}}{\sum_{n \neq i} \sum_g e_{nj,g} \varepsilon_{nj,g}^{(ij,K)}} \end{bmatrix}.$$

Since $| e_{ij,k} \varepsilon_{ij,k}^{(ij,k)} | - \sum_{k \neq j} e_{ij,g} \varepsilon_{ij,g}^{(ij,k)} = e_{ij,k} + \sum_{n \neq i} \sum_g e_{ij,g} \varepsilon_{nj,g}^{(ij,k)} > 0$, then $\mathbf{E}_{ij} \equiv \left[\frac{e_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k} \varepsilon_{ij,k}} \right]_{k,g}$ is strict diagonally dominant. Hence, following the Lèvy-Desplanques Theorem, \mathbf{E}_{ij} is invertible ([Horn and Johnson \(2012\)](#)) and we can compute the vector χ_{ij} as

$$\chi_{ij} = \left[\frac{e_{ij,g} \varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k} \varepsilon_{ij,k}} \right]_{k,g}^{-1} \left(\mathbf{1}_K + \left[\frac{\sum_{n \neq i} t_{ni,g}^* e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}}{\sum_{n \neq i} \sum_g e_{nj,g} \varepsilon_{nj,g}^{(ij,k)}} \right]_k \right). \quad (54)$$

Combining the above result with the previously-derived formulas for emission, domestic, and import taxes yields

$$\begin{cases} 1 + s_{i,k}^* = \frac{\gamma_k}{\gamma_k - 1}; & \tau_{i,k}^* = \tilde{\delta}_{ii} \\ 1 + t_{ji,k}^* = (1 + \omega_{ni,k}) + \tilde{\delta}_{ni} \left(\frac{\gamma_k - 1}{\gamma_k} \right) v_{n,k} \\ 1 + x_{ij,k}^* = \left(1 + \frac{1}{\varepsilon_{ij,k}} \right) \chi_{ij,k} \end{cases} \quad , \quad (55)$$

where $\chi_{ij,k}$ is given by Equation 54.

C Optimal Emission Policy when Other Taxes are Banned

The following definition puts the case for second-best scenarios formally.

Definition. The *Second-best Unilateral Policy* for country i is achieved by choosing a *subset* of policy instruments to maximize \mathcal{W}_i (equation 15) subject to equilibrium conditions (1)-(9).

We consider three cases: (1) Emission taxes are unavailable, (2) Export subsidies are unavailable, (3) All taxes but emission taxes are unavailable.

Case #1: Emission taxes are unavailable As the emission elasticity approaches zero, i.e., $\alpha_k \rightarrow 0$, our model collapses to a model with exogenous emission intensity à la [Markusen \(1975\)](#) (See footnote (8) As such, emission taxes can be dropped from the model as firms do not undertake abatement. In this case, the optimal production tax will include the markup-correcting term $\frac{\gamma_k - 1}{\gamma_k}$ plus an extra term that taxes high-emission (high- v) industries. Namely,

$$1 + s_{i,k}^{**} = \frac{\gamma_k}{\gamma_k - 1} \left(1 + \tilde{\delta}_{ii,k} v_{i,k} \right)^{-1}$$

As before, the emission-correcting term depends on $\frac{\gamma_k - 1}{\gamma_k}$ because there are scale economies in emission. For instance, it may be optimal to subsidize a high-returns-to-scale industry that exhibits a high emission intensity. That is because subsidizing such an industry may lower emission through scale effects that dominate the higher firm-level emission intensity.

Alternatively, maintaining the assumption that $\alpha_k \in (0, 1)$, we could examine second-best production taxes in cases where the government is not afforded choices of emission taxes. Suppose $v_{i,k}$ is the emission intensity under some emission tax that is different from the unilaterally optimal. This might be either because emission is unabated, or quite the opposite, because home country has set its emission tax in line with international agreements at a higher level compared to the unilaterally

first best. In either case, production taxes must correct emission externalities that are too little or too much from the unilateral point of view:

$$1 + s_{i,k}^{**} = \frac{\gamma_k}{\gamma_k - 1} [1 + \tilde{\delta}_{ii,k}(v_{i,k} - v_{i,k}^*)]^{-1}$$

where v_k^* is the emission intensity attainable under the first-best unilateral policy schedule. Consider a country with sub-optimal emission. In that case, production subsidies/taxes reflect a trade off between promoting scale economies and reducing emissions. More interestingly, consider a country whose $v_{i,k}$ is smaller than $v_{i,k}^*$ because the country is abiding with an international climate agreement. In that case, $(1+s_{i,k}^{**})$ includes an extra subsidy that promotes domestic production.

Case #2: Export Taxes are unavailable In this case, the optimal emission tax remains uniform and follows the same rule as the first-best. Derivations for this case are similar to the ones in the build up to equation (22) in Theorem 1. Specifically, in lemma 4, we consider only the first two sets of FOCs related to abatement and prices faced by home consumers (both domestic purchases and imports). The resulting optimal tax schedule is given by:

$$\begin{cases} 1 + t_{ji,k}^* = (1 + \bar{t}_i)(1 + \omega_{ji,k}) + \tilde{\delta}_{ji,k} v_{j,k} \frac{\gamma_k - 1}{\gamma_k} & \forall j, k \\ 1 + s_{i,k}^* = \frac{\gamma_k}{\gamma_k - 1} & \forall k \\ \tau_{i,k}^* = \tau_i^* = \tilde{\delta}_{ii,k} & \forall k \end{cases} \quad (56)$$

Here, $\bar{t}_i \equiv \left(\frac{\partial \ln Y_i}{\partial \ln w_i} \right) / \left(\frac{\partial B_i}{\partial \ln w_i} \right)$ where B_i is the trade balance condition. \bar{t}_i is zero only when export taxes are available.

Case #2: Emission Taxes are used as protection in disguise Suppose that all tax instruments but emission taxes are banned. In that case, optimal emission taxes will be no longer uniform. Instead, it is optimal for country i to apply a higher emission tax on industries where it possesses more export market power. To make this point succinctly, consider a perfectly competitive economy ($f_{i,k}^e = 0, \gamma_k \rightarrow \infty$) in which $\alpha_k = \alpha$ is uniform across industries and preferences have a Cobb-Douglas-CES parameterization given by equation (25). Then, as shown below, the optimal emission tax is given by

$$\tau_{i,k}^* = \left(\frac{\alpha(1 - \sigma_k)(1 - \lambda_{ii,k} r_{ii,k}) + 1}{\tilde{\alpha}_i(1 - \sigma_k)(1 - \lambda_{ii,k} r_{ii,k}) + r_{ii,k}} \right) \tilde{\delta}_{ii} \quad (\text{only } \tau \text{ available}) \quad (57)$$

where $\tilde{\alpha}_i > \alpha$ is a country-wide term that depends on the industry-composition of country i 's production. The above formula suggests that it is optimal to tax emission above the first-best level in low- σ industries. We continue to show the derivation of equation (57).

The F.O.C. w.r.t. $1 - a_{i,k}$ can be expressed as ($Z_i \equiv \sum_{n,k} \delta_{ni} Z_{n,k}$):

$$\begin{aligned} \frac{\partial V_i(\cdot)}{\partial Y_i} \frac{\partial Y_i(\mathbf{w}, \mathbf{a})}{\partial \ln(1 - a_{i,k})} + \frac{\partial V_i(\cdot)}{\partial \ln \tilde{P}_i} \frac{\partial \ln \tilde{P}_i(\mathbf{w}, \mathbf{a})}{\partial \ln(1 - a_{i,k})} + \\ \frac{\partial Z_i}{\partial Y_i} \frac{\partial Y_i(\mathbf{w}, \mathbf{a})}{\partial \ln(1 - a_{i,k})} + \frac{\partial Z_i}{\partial \tilde{P}_i} \frac{\partial \tilde{P}_i(\mathbf{w}, \mathbf{a})}{\partial (1 - a_{i,k})} + \frac{\partial \mathcal{V}_i(\cdot)}{\partial \ln \mathbf{w}} \frac{d \ln \mathbf{w}}{d \ln(1 - a_{i,k})} = 0 \end{aligned}$$

To simplify the above problem, we impose the following additional assumptions:

1. Preferences are given by the Cobb-Douglas-CES specification;
2. Country i is a small open economy with $\delta_{-ii,k} = 0$; and
3. All industries are perfectly competitive, i.e., $\gamma_k \rightarrow \infty$.

Noting that $\partial \ln P_{in,k} / \partial \ln(1 - a_{i,k}) = -1$ and noting that $Z_{i,k} = v_{i,k} P_{ii,k} Q_{i,k}$, it follows that:

$$\begin{aligned} \frac{\partial Z_i}{\partial \ln(1 - a_{i,k})} = \frac{\partial \delta_{ii,k} Z_{i,k}}{\partial \ln(1 - a_{i,k})} = -\delta_{ii,k} v_{i,k} \sum_j [P_{ij,k} Q_{ij,k} \varepsilon_{ij,k}] \\ + \left(\frac{1}{\alpha_{i,k}} - 1 \right) \delta_{ii,k} v_{i,k} P_{ii,k} Q_{i,k} + \delta_{ii,k} v_{i,k} P_{ii,k} Q_{ii,k} \frac{\partial Y_i}{\partial \ln(1 - a_{i,k})} \end{aligned}$$

Wage effects can be characterized by applying the $D_i(\mathbf{a}_i, w_i) = \sum_{j \neq i} \sum_g (P_{ji,g} Q_{ji,g} - P_{ij,g} Q_{ij,g})$

$$\frac{d \ln w_i}{d \ln(1 - a_{i,k})} = - \left(\sum_{j \neq i} [P_{ji,k} Q_{ji,k} \varepsilon_{ji,k}^{ii} - P_{ij,k} Q_{ij,k} (1 + \varepsilon_{ij,k})] + \sum_{j \neq i} \sum_g (P_{ji,g} Q_{ji,g}) \frac{\partial Y_i}{\partial \ln(1 - a_{i,k})} \right) \left(\frac{\partial D_i}{\partial \ln w_i} \right)^{-1}$$

Using the above expression, invoking Roy's identity, and noting that $Y_i = w_i L_i + \sum_k \alpha_{i,k} P_{ii,k} Q_{i,k}$, yields the following formulation of the F.O.C.

$$\begin{aligned}
& P_{ii,k}Q_{ii,k} - \alpha_{i,k} \sum_j [P_{ij,k}Q_{ij,k} (1 + \varepsilon_{ij,k})] + \tilde{\delta}_{ii}v_{i,k} \sum_j [P_{ij,k}Q_{ij,k}\varepsilon_{ij,k}] \\
& - \left(\frac{1}{\alpha_k} - 1\right) \tilde{\delta}_{ii,k}v_{i,k}P_{ii,k}Q_{i,k} - \sum_g ([\alpha_g - \tilde{\delta}_{ii,g}v_{i,g}] P_{ii,g}Q_{ii,g}) \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,k})} \\
& - \bar{\Delta}_i \left[\sum_{j \neq i} [P_{ji,k}Q_{ji,k}\varepsilon_{ji,k}^{ii} - P_{ij,k}Q_{ij,k} (1 + \varepsilon_{ij,k})] + \sum_{j \neq i} \sum_g (P_{ji,g}Q_{ji,g}) \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,k})} \right] = 0. \quad (58)
\end{aligned}$$

where $\bar{\Delta}_i \equiv \frac{\partial \mathcal{V}_i / \partial \ln w_i}{\partial D_i / \partial \ln w_i}$ is a uniform term without industry subscript. Dividing Equation 58 by $R_{i,k} = \sum_n P_{in,k}Q_{in,k}$ and defining $\mathcal{E}_{i,k} = \sum_j [r_{ij,k} (1 + \varepsilon_{ij,k})] = -\epsilon_k (1 - r_{ii,k}\lambda_{ii,k})$, we can simplify the F.O.C.

$$\begin{aligned}
& r_{ii,k} - \alpha_{i,k}\mathcal{E}_{i,k} + \alpha_k \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} (\mathcal{E}_{i,k} - 1) - (1 - \alpha_{i,k}) \frac{\tilde{\delta}_{ii,k}}{\tau_{i,k}} \\
& + \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g}r_{i,g} \right) \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,k})} r_{i,k}^{-1} - \bar{\Delta}_i \left[\mathcal{E}_{i,k} + (1 - \lambda_{ii}) \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,k})} r_{i,k}^{-1} \right] = 0. \quad (59)
\end{aligned}$$

$\partial Y_i / \partial \ln(1 - a_{i,k})$, in the above expression, can be obtained by applying the Implicit Function Theorem to $Y_i = w_i L_i + \sum_k \alpha_{i,k} P_{ii,k} Q_{i,k}$, while noting that $\eta_{in,k} = 1$ given our parametric assumption with regards to preferences. Namely,

$$\frac{\partial Y_i}{\partial \ln(1 - a_{i,k})} = \frac{-\alpha_{i,k} \sum_j [P_{ij,k}Q_{ij,k} (1 + \varepsilon_{ij,k})]}{Y_i - \sum_g \alpha_g \eta_{ii,g} P_{ii,g} Q_{ii,g}} = \frac{-\alpha_{i,k} \mathcal{E}_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} r_{i,k}$$

Plugging the above equation back into the F.O.C. implies

$$\frac{\tilde{\delta}_{ii,k}}{\tau_{i,k}} - 1 = \frac{(\bar{\alpha}_{i,k} - \alpha_{i,k}) \mathcal{E}_{i,k} + 1 - r_{ii,k}}{\alpha_{i,k} \mathcal{E}_{i,k} - 1} \implies \tau_{i,k} = \left(\frac{\alpha_{i,k} \mathcal{E}_{i,k} - 1}{\bar{\alpha}_{i,k} \mathcal{E}_{i,k} - r_{ii,k}} \right) \tilde{\delta}_{ii,k}$$

where

$$\bar{\alpha}_{i,k} - \alpha_{i,k} \equiv \bar{\Delta}_i \left[\frac{1 - \alpha_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} \right] - \frac{\alpha_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii,k}}{\tau_{i,g}} \right] r_{ii,g}r_{i,g} \right).$$

To finalize the proof, we need to characterize $\bar{\Delta}_i$, which will in turn pin down $\tilde{\alpha}_{i,k}$. To this end, we can

appeal to the definition $\bar{\Delta}_i \equiv \frac{\partial Y_i / \partial \ln w_i}{\partial D_i / \partial \ln w_i}$, which implies that

$$\bar{\Delta}_i = \frac{(1 - \bar{\alpha}_i) - \lambda_{ii} + \sum_k \left(\left[\alpha_{i,k} \mathcal{E}_{i,k} - \alpha_k \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} (\mathcal{E}_{i,k} - 1) \right] r_{i,k} \right) + \sum_k \left([\alpha_{i,k} - \delta_{ii,k} v_{i,k}] r_{ii,k} r_{i,k} \right) \frac{\partial Y_i}{\partial \ln w_i}}{(1 - \lambda_{ii}) \frac{\partial Y_i}{\partial \ln w_i} - \mathcal{E}_i}$$

We can replace for $\alpha_k \mathcal{E}_{i,k} - \alpha_{i,k} \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} (\mathcal{E}_{i,k} - 1)$ from the F.O.C. (Equation 59), which implies

$$\begin{aligned} \bar{\Delta}_i &= \frac{(1 - \bar{\alpha}_i) - \lambda_{ii} + \sum_g \left(\left[r_{ii,g} - (1 - \alpha_g) \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{i,g} \right) + \sum_g \left(\alpha_g \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \left[\frac{\partial Y_i}{\partial \ln w_i} + \sum_g \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,g})} \right]}{(1 - \lambda_{ii}) \left[\frac{\partial Y_i}{\partial \ln w_i} + \sum_k \frac{\partial Y_i}{\partial \ln(1 - a_{i,k})} \right]} \\ &= \frac{\sum_g \left[(1 - \alpha_g) \left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right) r_{i,g} \right] + \sum_g \left(\alpha_g \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \left[\frac{\partial Y_i}{\partial \ln w_i} + \sum_g \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,g})} \right]}{(1 - \lambda_{ii}) \left[\frac{\partial Y_i}{\partial \ln w_i} + \sum_k \frac{\partial Y_i}{\partial \ln(1 - a_{i,k})} \right]} \end{aligned} \quad (60)$$

Reapplying the Implicit Function Theorem to $Y_i = w_i L_i + \sum_k \alpha_{i,k} P_{i,k} Q_{i,k}$ implies that

$$\frac{\partial \ln Y_i}{\partial \ln w_i} + \sum_k \frac{\partial \ln Y_i}{\partial \ln(1 - a_{i,k})} = \frac{1 - \sum_k (\alpha_{i,k} r_{i,k}) + \sum_k (\alpha_{i,k} \mathcal{E}_i r_{i,k})}{1 - \bar{\alpha}_i \lambda_{ii}} - \sum_k \frac{\alpha_{i,k} \mathcal{E}_i r_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} = \frac{1 - \bar{\alpha}_i}{1 - \bar{\alpha}_i \lambda_{ii}}.$$

Combining the above expression with Equation 60 and assuming that $\alpha_{i,k} = \alpha$ for all k , yields the following:

$$(1 - \lambda_{ii}) \frac{1 - \alpha}{1 - \alpha \lambda_{ii}} \bar{\Delta}_i = (1 - \alpha) \sum_g \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right) r_{i,g} \right] + \frac{1 - \alpha}{1 - \alpha \lambda_{ii}} \sum_g \left(\alpha \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right),$$

Finally, noting the definition for $\tilde{\alpha}_{i,k} - \alpha$, delivers the following expression

$$\begin{aligned} \tilde{\alpha}_{i,k} - \alpha &= \left[(1 - \alpha) \sum_g \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right) r_{i,g} \right] + \alpha \sum_g \left(\left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \right] (1 - \lambda_{ii})^{-1} \\ &= \sum \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} \right) \frac{1 - \alpha(1 - r_{ii,g})}{1 - \lambda_{ii}} r_{i,g} \right] = - \sum_g \left[\left(\frac{(\tilde{\alpha}_{i,g} - \alpha) \mathcal{E}_{i,g} + 1 - r_{ii,g}}{\alpha \mathcal{E}_{i,g} - 1} \right) \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} r_{i,g} \right]. \end{aligned}$$

The above system implies that $\tilde{\alpha}_{i,k} = \tilde{\alpha}_i$ is uniform. So, given that $\mathcal{E}_{i,g} = -\epsilon_g (1 - r_{ii,g} \lambda_{ii,g})$, we can solve for $\tilde{\alpha}_i$ as

$$\tilde{\alpha}_i - \alpha = \frac{\sum_g \left[\frac{1 - r_{ii,g}}{\epsilon_g (1 - r_{ii,g} \lambda_{ii,g}) + 1} \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} r_{i,g} \right]}{\sum_g \left[\left(1 + \frac{\epsilon_g (1 - r_{ii,g} \lambda_{ii,g})}{\epsilon_g (1 - r_{ii,g} \lambda_{ii,g}) + 1} \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} \right) r_{i,g} \right]} > 0$$

D Data and Calibration

Expenditures, Revenues, and Emissions. Given data on expenditure, emission, and applied tariff levels $\{\tilde{P}_{ji,k}Q_{ji,k}, Z_{i,k}, t_{ji,k}\}_{ji,k}$, and estimated parameters, $\{\gamma_k, \sigma_k, \alpha_{n,k}\}$, we can construct the vector of observables, $\mathcal{B}_v \equiv \{\lambda_{ni,k}, r_{ni,k}, \rho_{i,k}, \tilde{\delta}_{ni}, e_{n,k}, w_n \bar{L}_n, Y_n\}_{ni,k}$, needed to implement Proposition 3. Values for emission intensities, $v_{i,k}$ can be calculated as follows

$$v_{i,k} = \frac{Z_{i,k}}{\sum \frac{1}{1+t_{in,k}} \tilde{P}_{in,k} Q_{in,k}}$$

Total national expenditure Y_i and expenditure share variables, $\lambda_{ji,k}$, and $e_{i,k}$ can be recovered from variety-level expenditure and tariff data as follows:

$$Y_i = \sum_{j=1}^{15} \sum_{k=1}^{19} \tilde{P}_{ji,k} Q_{ji,k}, \quad \lambda_{ji,k} = \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\sum_{n=1}^{15} \tilde{P}_{ni,k} Q_{ni,k}}, \quad e_{i,k} = \frac{\sum_{n=1}^{15} \tilde{P}_{ni,k} Q_{ni,k}}{Y_i}.$$

Finally, the national wage bill, $w_i \bar{L}_i$, industry-level labor shares, $\rho_{i,k}$, and revenue shares, $r_{ji,k}$, can be constructed as follows, given variety-level expenditure and tariff data and the estimated structural parameters:

$$w_i \bar{L}_i = \sum_{j=1}^{15} \sum_{k=1}^{19} \left[\left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \right) \frac{1}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right];$$

$$\rho_{i,k} = \frac{\sum_{j=1}^{15} \left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \right) \frac{\tilde{P}_{ij,k} Q_{ij,k}}{1 + t_{ij,k}}}{w_i \bar{L}_i}; \quad r_{ji,k} = \frac{\frac{1}{1 + t_{in,k}} \tilde{P}_{in,k} Q_{in,k}}{\sum \frac{1}{1 + t_{in,k}} \tilde{P}_{in,k} Q_{in,k}}.$$

Emission Taxes. We have collected emission tax data from EUROSTAT and OECD-PINE. Both of these data report environmentally-related taxes for four categories of energy, transport (except fuels), pollution, and resources. Below, we explain more about the coverage of these tax data, and data availability issues that remain to be addressed.

According to EUROSTAT data, out of total environmentally-related taxes paid by all industries in the EU in 2009, 81.6% are for energy, 3.1% pollution, 0.7% resources, and 14.5% transport taxes. These numbers, when paid by households are 69.3% for energy, 2.5% pollution, 0.4% resources, and 27.8% transport. These data cover only European countries and are reported for every industry, and for the category of households as final consumption.

According to OECD-PINE data that cover non-European countries, out of total environmentally-related taxes paid in the economy (both industries and households), on average 64.1% is paid for

energy, 3.1% for pollution, 3.8% for resources, and 30.5% for transport. These data cover many countries (and, not only European ones) but they are reported at the level countries, meaning that in these data we observe neither disaggregated industry-level records nor the distinction between firms and households.

We take energy taxes in these data as emission taxes in our model. This is consistent with our interpretation that emission-intensive input in our model can be thought of as energy (i.e., fossil fuels) in the data. The transport category does not include taxes on fuels, and reflects taxes such as those on motor vehicle sales, motor vehicle registrations, and flight tickets. Environmentally-related taxes on pollution and resources are not meant to target energy use, either. Specifically, pollution taxes target (1) a number of emissions that matter for the local environment (2) water pollution, and (3) waste management. Tax on resources include (1) water abstraction, (2) harvesting of biological resources such as fisheries, (3) extraction of raw material such as oil and gas, (4) landscape changes and cutting of trees.

We face two issues in mapping our model to emission tax data. First, we do not observe emission (energy) taxes by industry disaggregation in non-European countries. Second, our model allows emission taxes only on production while in the data, a portion of them are paid by households. We continue to explain how we re-calibrate the data to make our model quantification consistent with the accounting of taxes and emissions.

For a generic variable x , let $x_{i,k}$ be that variable in country-industry ik , and x_i be the country-level aggregate. In addition, let x^P be that variable from production side, and x^H from household consumption side, with $x = x^P + x^H$. Specifically, we have: $T_{i,k}^E = \tau_{i,k} Z_{i,k}$, where $T_{i,k}^E$ refers to emission tax paid by country-industry ik , $Z_{i,k}$ measures tonnes of CO2 emission in country-industry ik , and $\tau_{i,k}$ is the associated tax rate. First, we explain how we scale the data from production side to make them consistent with the national accounting of taxes and emissions. For European countries, we observe variables by industry disaggregation, but for household consumption, we only observe variables at the aggregate country level. Starting with CO2 emissions, let c_i^Z be an adjustment scalar that brings emission data into country-level aggregates: $Z_i = c_i^Z Z_{i,k}^P$ where $c_i^Z \equiv \frac{Z_i}{Z_{i,k}^P}$. Similarly, let c_i^τ be an adjustment to bring emission tax data into country-level aggregate, $\tau_i = c_i^\tau \tau_{i,k}^P$ where $c_i^\tau \equiv \frac{T_i^E}{\sum_k \tau_{i,k}^P Z_{i,k}^P}$. Using these two adjustments, we re-scale the emission and tax data of the production side of an economy to incorporate those of households.

Next, we explain how we proceed in the face of the issue that we do not observe emission taxes by industry disaggregation in non-European countries. For country i , we observe from OECD-PINE

the emission taxes collected as percentage of GDP. From here, we calculate total emission taxes T_i in country i . We observe these taxes by industry disaggregation, $T_{i,k}^E$, only for the EU. Let us then make the assumption that $\tau_{i,k}/\tau_{i,k_0} = \tau_{EU,k}/\tau_{EU,k_0}$ —that is, the relative emission tax rate of industry k to that of a reference industry is the same between the EU and other countries. Then, for a non-EU country i , the accounting of taxes and emissions require that: $T_i^E = \sum_k \tau_{i,k} Z_{i,k}$, which then implies $T_{i,k}^E = \frac{\tau_{EU,k} Z_{i,k}}{\sum_k \tau_{EU,k} Z_{i,k}} T_i^E$. Using this proportionality assumption, we construct industry-level emission taxes for non-European countries in a way that is consistent with their total emission taxes which we observe in OECD-PINE data.

Emission Disutility Parameters. Our calibration of countries' perceived CPI-adjusted disutility from emissions is based on two assumptions: (a) unilaterally optimal domestic emission tax equals the currently-applied energy tax in a country; (b) the globally optimal CO2 tax equals world's disutility from CO2 emissions,

$$\begin{cases} T_i^E = \sum_k (\tilde{\phi}_{i,k} + \tilde{\phi}_i) Z_{i,k} & \text{(a)} \\ SCC = \sum_i \tilde{\phi}_i & \text{(b)} \end{cases} \quad (61)$$

We let $\tilde{\phi}_i$ be proportional to country i 's share of world GDP adjusted for differences in energy tax rates across countries. Specifically, we recover relative values of $\tilde{\phi}_i$ across countries, by making two assumptions: (i) If every individual person cares equally about global warming, the aggregate care of a larger country will be proportionally larger. To reflect the importance of size, it is more plausible to denote the damage from climate change as a percentage of countries' real GDP. This means that, $\frac{\phi_i}{\phi_j} \propto \frac{Y_i/\bar{P}_i}{Y_j/\bar{P}_j}$, which is equivalent to $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{Y_i}{Y_j}$. (ii) Countries do not care equally about carbon taxes. We take a stand that countries' care about climate change is reflected in their current policy toward the environmental damage of burning fossil fuels. As such, we make the assumption that country i relative to j 's care about climate change is proportional to their observed emission taxes per tonne of CO2, which means: $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{(T_i^E/Z_i)}{(T_j^E/Z_j)}$. Putting these two assumptions together, we can specify $\tilde{\phi}_i$ as:

$$\tilde{\phi}_i = \bar{h} y_i (T_i^E/Z_i), \quad y_i \equiv Y_i/Y_W$$

Equation (39)-b requires that if countries could act cooperatively, they would target the social cost of carbon with their domestic CO2 taxes, $\sum_i \tilde{\phi}_i = SCC$. This pins down scalar \bar{h} in the above equation,

$\bar{h} = \frac{SCC}{\sum_i y_i (T_i^E / Z_i)}$, and delivers $\tilde{\phi}_i$ as:

$$\tilde{\phi}_i = \frac{y_i T_i^E / Z_i}{\sum_i y_i (T_i^E / Z_i)} SCC$$

Using equation (39)-a, we can next calibrate the local component of emission taxes, $\tilde{\phi}_{i,k}$, by attributing the difference between observed emission taxes and the calibrated $\tilde{\phi}_i$ to the local component of emission taxes, $\tilde{\phi}_{i,k}$, $T_i^E = \sum_k (\tilde{\phi}_i + \tilde{\phi}_{i,k}) Z_{i,k} \Rightarrow \sum_k \tilde{\phi}_{i,k} Z_{i,k} = T_i^E - \tilde{\phi}_i Z_i$. Notice that, $\tilde{\phi}_{i,k} \equiv \tilde{\phi}_i^0 \bar{\zeta}_{i,k}$ where we observe $\bar{\zeta}_{i,k}$ in the data as the amount of local pollutants generated in country-industry i, k when burning fuels there generates one tonne of CO2. Putting together, this delivers $\tilde{\phi}_{i,k}$ as:

$$\tilde{\phi}_{i,k} = \frac{\zeta_{i,k} (1 - h_i) T_i^E}{\sum_k \zeta_{i,k} Z_{i,k}}, \quad h_i = \frac{y_i SCC}{\sum_i y_i (T_i / Z_i)}$$

E Additional Figures and Tables

Table 4: Countries and their Select Characteristics

Country	Share of World GDP	Share of World CO2	CO2 Intensity (\bar{v}_i)	Energy Tax Rate ($\bar{\tau}_i$)	Disutility from CO2 ($\tilde{\phi}_i$)
AUS	1.7%	1.4%	100.00	20.19	0.95
EU	27.1%	12.1%	53.54	58.90	43.33
BRA	2.4%	2.4%	121.15	7.27	0.48
CAN	2.0%	1.7%	99.85	16.43	0.91
CHN	13.6%	23.8%	209.42	3.67	1.36
IDN	1.0%	1.8%	221.48	3.87	0.10
IND	2.2%	6.4%	347.71	2.99	0.18
JPN	8.4%	2.9%	41.17	41.88	9.54
KOR	1.9%	1.6%	100.49	20.37	1.05
MEX	1.2%	1.4%	132.20	7.77	0.26
RUS	2.0%	6.0%	354.46	2.29	0.13
TUR	1.0%	0.9%	113.48	37.37	0.99
TWN	0.7%	0.9%	143.02	5.29	0.10
USA	21.2%	14.8%	83.56	12.13	6.98
RoW	13.6%	22.0%	194.09	4.46	1.64

Note: This table shows for every of the 15 regions (13 countries + the EU + the RoW), their share of world GDP, share of world CO2 emissions, CO2 emission intensity (CO2 emissions per dollar of output) normalized by that of Australia, Emission tax rate (dollar per tonne of CO2), and calibrated CPI-adjusted disutility parameter from one tonne of CO2 emission ($\tilde{\phi}_i$). All CO2 measures are CO2 equivalent.

Table 5: Non-cooperative and Cooperative Outcomes - CRS

Country	Constant Returns to Scale					
	Non-Cooperative			Global Cooperation		
	$\Delta\text{CO}_2\text{e}$	ΔV	ΔW	$\Delta\text{CO}_2\text{e}$	ΔV	ΔW
AUS	-6.4%	-1.8%	-1.7%	-82.9%	-0.8%	1.2%
EU	-0.1%	-1.0%	-0.9%	-33.8%	-0.2%	4.7%
BRA	-6.3%	-1.1%	-1.1%	-90.4%	-0.7%	0.0%
CAN	-11.2%	-4.0%	-4.0%	-79.5%	-0.8%	0.9%
CHN	1.1%	-0.8%	-0.8%	-94.5%	-0.7%	-0.3%
IDN	-3.5%	-1.4%	-1.4%	-93.2%	-0.7%	-0.2%
IND	-2.5%	-3.6%	-3.6%	-95.2%	-1.1%	-0.7%
JPN	-0.0%	-1.0%	-0.9%	-63.4%	-0.3%	3.3%
KOR	-0.4%	-2.7%	-2.7%	-81.0%	-0.9%	1.1%
MEX	-3.6%	-3.4%	-3.4%	-89.5%	-0.8%	0.0%
RUS	-8.7%	-4.1%	-4.1%	-91.9%	-0.9%	-0.6%
TUR	-6.3%	-3.4%	-3.4%	-71.8%	-1.8%	1.8%
TWN	5.5%	-4.6%	-4.6%	-90.8%	-0.3%	0.4%
USA	-1.1%	-1.4%	-1.4%	-83.0%	-0.4%	0.6%
RoW	-6.9%	-2.6%	-2.6%	-89.7%	-0.8%	-0.3%
Global	-2.7%	-1.6%	-1.5%	-82.3%	-0.5%	1.7%