Term Limits and Bounds on Policy Responsiveness in Dynamic Elections

PRELIMARY DRAFT

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Abstract

I analyze equilibria in an infinite-horizon model of elections with a two-period term limit in the presence of adverse selection and moral hazard. The focus is on responsiveness of policy choices of first-term politicians. For a given level of office benefit, the commitment problem of voters imposes a bound on equilibrium effort exerted by politicians that holds uniformly across of the rate of time discounting. In addition, I prove existence of equilibrium, pointing out and correcting an error in the proof of Banks and Sundaram (1998).

1 Introduction

An essential feature of representative democracy is the periodic reconsideration of political agents by their principal, the electorate. Elections allow voters to express approval or disapproval of their elected delegates, and at the same time they provide politicians with incentives to shun parochial interests in favor of the public good. The operation of these incentives is complicated by informational asymmetries—in the form of adverse selection and moral hazard—and by the extended time horizon over which interaction takes place. We would expect these electoral incentives to be mitigated by term limits but enhanced if the benefit of holding office per se is larger or the weight placed by politicians on short-term gains is decreased. These issues are properly addressed in a dynamic framework that explicitly accounts for

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informational asymmetries, but doing so requires precise attention to the ensuing technical difficulties.

This paper considers elections in a dynamic environment similar to that analyzed by Banks and Sundaram (1998). In this framework, an office holder's choice is unobserved by voters and stochastically determines an outcome that is observable; moreover, politicians' preferences are indexed by their types, which are unobserved by voters. Once an outcome is generated by the choice of a first-term office holder, voters must decide whether to replace the incumbent with a challenger, whereas politicians are automatically removed from office after their second term; and this process is repeated ad infinitum. The paper provides a partial characterization of stationary electoral equilibria for arbitrary parameterizations, along with sharper results for the case of highly office-motivated politicians. Of interest is the possibility that in response to electoral incentives, elected politicians decline the opportunity to shirk, by choosing policies close to their own ideal points, and instead choose policies that are good for voters; this phenomenon is referred to as *policy responsiveness.* It is shown that office holders choose policies strictly higher than their ideal policies, and as politicians become highly motivated, the highest politician type mixes with positive probability on arbitrarily high policies. Thus, a minimal level of policy responsiveness is achieved in equilibrium. However, the main substantive result of the paper, presented in Theorems 3 and 4, is that the voters' equilibrium payoff is bounded above by the expected utility from the ideal policy of the highest politician type. The upper bound holds generally across all parameter values, and for a given level of office benefit, the bound holds strictly regardless of the level of citizens' patience.

Following the literature on electoral accountability, and consistent with the citizen-candidate approach of Osborne and Slivinski (1996) and Besley and Coate (1997), it is assumed that neither candidates nor voters can make binding promises about future behavior. In particular, candidates cannot commit to policy platforms before an election, and thus they do not compete for votes in the manner of the Downsian electoral model; this is especially natural in the current framework, as actions of politicians are unobservable to voters, so that they would have no way of verifying that a platform was implemented. Rather, electoral incentives arise from the desire of a first-term office holder to signal that she is a high type, in which case voters would prefer to re-elect the incumbent over the prospect of a relatively unknown challenger. This highlights the well-known *commitment problem of politicians,* which is featured in, e.g., Acemoglu et al. (2005).

The bound on policy responsiveness described above is due, however, to the *commitment problem of voters.* It is assumed that the electorate cannot write a binding contract to re-elect an incumbent following policy outcomes above a predetermined level. Because second-term office holders simply choose their ideal point, this means that the voters' continuation value of a challenger can never exceed the expected utility from the ideal point of the highest politician type: if it did exceed this amount, then voters would always have an incentive to remove an incumbent after her first term to insert a more productive challenger, but then first-term office holders would have no incentive to depart from their ideal points. In fact, it is shown that for a given level of office benefit, the value of a challenger must fall strictly below this level. Thus, the incentives of voters imply a general bound on the possibility of policy responsiveness.

A key contribution of the paper, on a technical level, is the proof of existence of a perfect Bayesian equilibrium that is stationary, in the sense that the policy choices of first-term office holders are history independent and the voters' decision to re-elect a first-term incumbent depends only on the observed policy outcome generated by the office holder's unobserved action in office. An existence result is stated by Banks and Sundaram (1998), but as discussed following the existence proof below, their argument is problematic: they define a correspondence and attempt to apply Glicksberg's fixed point theorem to deduce existence of equilibrium, but the domain of their correspondence is not convex; thus, Glicksberg's theorem cannot be applied. The non-convexity arises because the authors attempt to incorporate a monotonicity property of policy strategies into the domain of their correspondence: they assume that the supports of policy strategies are weakly ordered by politician type. This is helpful to them because given policy strategies with this property, the voters' optimal responses are cutoff strategies that comprise a compact interval. The approach of this paper is to leave monotonicity out of the domain of the correspondence and to obtain the property ex post, after a fixed point is derived.

Section 2 contains a review of the related literature. The dynamic electoral model is described in Section 3, the concept of stationary electoral equilibrium is defined in Section 4, and preliminary results are set forth in Section 5. Existence of equilibrium and a partial characterization are provided in Section 6, and Section 7 sharpens the characterization for the case of highly office motivated politicians. Section 8 contains two results establishing bounds on the possibility of policy responsiveness.

2 Related literature

The literature on electoral accountability traces to Barro (1973), in which there is a single politician type and policy choices are directly observable by voters, and Ferejohn (1986), who considers agency problems in which policy choices are subject to imperfect monitoring. Duggan (2000) analyzes elections under pure adverse selection, where policy choices are observable but politicians are privately informed about their preferences, and Bernhardt et al. (2004) consider the model with term limits and pork barrel spending.[1] The infinite-horizon model with a two-period term limit and combined adverse selection and moral hazard is the subject of Banks and Sundaram (1998), who show that when politicians' strategies are monotone in type and voters use a cutoff re-election rule, each politician type chooses higher effort in the first term of office than the second (if re-elected), and re-elected politicians are more productive on average than an untried challenger due to selection effects.

In the infinite-horizon model without term limits, Banks and Sundaram (1993) depart from the restriction to stationary electoral equilibria and show existence of equilibria in the class of trigger strategies, in which voters and politicians use history-dependent strategies that condition on past outcomes generated by an incumbent in addition to the voters' posterior beliefs. In particular, if the realized policy outcome falls below a given cutoff level during a politician's term, the politician shirks (i.e., chooses zero effort) thereafter, and voters remove the incumbent from office. This approach has several shortcomings. First, even if the incumbent is a good type with arbitrarily high probability, there is always a positive probability that a bad outcome will be realized and voters will replace the incumbent; while optimal given the anticipated actions of the incumbent, this behavior may run counter to intuition. Second, the exact value of the trigger is not pinned down in the model, and in fact a continuum of levels can be supported in equilibrium. Third, the analysis relies on the assumption that all politician types are equivalent when they shirk; without this assumption, the trigger strategy construction breaks down, as voters may have an incentive to re-elect an incumbent who is a good type with high probability, even if it is known that she will shirk in the future.[2]

The two-period version of the model is analyzed in Duggan and Martinelli (2015). Although policy choices in the second period of this model are trivial due to endgame effects, the existence of equilibrium still requires a fixed point argument, because of the interaction between optimal policy choices in the first period and the updating of voter beliefs. The authors establish existence of equilibrium in which each type of politician mixes between at most two policies ("taking it easy" and "going for broke"), and they show that increasing office motivation leads to

[1] The adverse selection model is extended to allow for multiple dimensions by Banks and Duggan (2008), partisan challenger selection by Bernhardt et al. (2009), and valence by Bernhardt et al. (2011).

[2] Related work on dynamic elections with an endogenous state variable includes Duggan and Forand (2014), who give conditions under which electoral equilibria solve the dynamic programming problem of a representative voter, and Battaglini (2014), who assumes parties can commit to fiscal platforms prior to each election and that these choices affect the level of public debt.

arbitrarily high expected policy outcomes in the first period. This finding contrasts with the results of the current paper, where the commitment problem of voters implies an upper bound on expected policy outcomes, and it points to a discrepancy between the two-period model and the infinite-horizon model with two-period term limit.[3] Thus, a byproduct of the analysis is the perhaps surprising observation that the two-period model, which is common in applications, is qualitatively different than the infinite-horizon model with a two-period term limit.

3 Dynamic political agency model

This paper analyzes repeated elections to fill a political office that is subject to a two-period term limit. Elections are held over an infinite horizon, with periods indexed $t = 1,2,\ldots$. In each period t, an incumbent office holder makes a policy choice $x_t \in X = \mathbb{R}_+$, a policy outcome $y_t \in Y = \mathbb{R}$ is drawn according to the distribution $F(\cdot|x_t)$, and a challenger is drawn without replacement.[4] If the incumbent is in her first term, then an election is held, and the winner takes office next period; and otherwise, if the incumbent is in her second term, then the challenger assumes office automatically. The preferences of politicians are represented by a type $j \in T = \{1,\ldots,n\}$ and are private information; voters do not observe the politicians' types. The policy choice x_t is also not directly observed by voters, but the outcome y_t is publicly observed. The types of politicians are identically and independently distributed, with $p_j > 0$ denoting the prior probability that a politician is type j. Consistent with the citizen-candidate approach, politicians and voters cannot make binding commitments regarding future actions, and thus political campaigns are suppressed from the analysis.

Voters and politicians who are out of office receive a payoff $u(y_t)$ from policy outcome y_t in period t, so that for simplicity, the electorate is homogeneous. A type j politician in office receives payoff $w_j(x_t) + \beta$ from policy choice x_t, where $\beta \geqslant 0$ is a non-negative office benefit. Citizens have a common rate of time discounting, which is represented by the discount factor $\delta \in [0,1)$. Given a sequence $x_1, x_2, \ldots$ of choices and a sequence $y_1, y_2, \ldots$ of outcomes, the total payoff of a type j citizen

[3] That paper contains detailed discussion of related work in the two-period framework by Fearon (1999), Besley (2006), Ashworth and Bueno de Mesquita (2008), Persson and Tabellini (2000), and Ashworth (2005).

[4] The set of possible challengers can be modeled as a separate pool, or if the electorate is a continuum, then challengers may be drawn from a continuous distribution over voters. To simplify the calculations of voters, it is assumed that the probability that any given voter is selected as challenger is zero.

is the discounted sum of per period payoffs,

$$\sum_{t=1}^{\infty} \delta^{t-1}[I_t(w_j(x_t)+\beta)+(1-I_t)u(y_t)],$$

where $I_t \in \{0,1\}$ is an indicator variable that takes a value of one if the citizen holds office in period t and takes a value of zero otherwise.

Assume voter preferences over policy outcomes are monotonic and continuous, so that $u\colon Y \to \mathbb{R}$ is continuous and strictly increasing. Write $\mathbb{E}[u(y)|x]$ for the voters' expected payoff from the distribution $F(\cdot|x)$ over policy outcomes determined by policy choice x, and without loss of generality normalize payoffs so that the expected payoff of voters from $x = 0$ is zero, i.e., $\mathbb{E}[u(y)|0] = 0$. Since the electorate is homogeneous, the analysis assumes a representative voter in the sequel. Assume that while in office, the payoffs w_j of each politician type are continuously differentiable, quasi-concave, and satisfy the following supermodularity assumption.

$$\text{(C1)} \qquad \begin{array}{c} \text{for all } x, x' \in X \text{ with } x > x' \text{ and all } j < n, \\ w_{j+1}(x) - w_{j+1}(x') > w_j(x) - w_j(x'). \end{array}$$

Informally, the latter condition means that differences in payoffs are strictly monotone in politician type. In addition, assume that for each politician type j, the payoff function w_j has a unique maximizer $\hat{x}_j$, that this ideal point is the unique critical point of w_j, and that at most the type 1 politicians have ideal point equal to zero. It follows from supermodularity that the ideal points of office holders are strictly ordered according to type: $\hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n$. Lastly, assume that office holders' payoff is unbounded below when policy choices are arbitrarily high.

$$\text{(C2)} \qquad \text{for all } j = 1, \ldots, n, \lim_{x \to \infty} w_j(x) = -\infty.$$

Under our remaining assumptions, (C2) is implied by concavity of office holders' payoffs.

Given the structure imposed on payoffs, it is natural to view the policy choice x as an effort level, which stochastically determines an outcome y that can be viewed as a level of public good. Then a minimal criterion for policy responsiveness is that a type j office holder exert positive effort, i.e., she chooses $x > \hat{x}_j$, in her first term of office. A more demanding criterion is that the expected effort of a newly elected challenger be large; of special interest are the effect of varying model parameters such as office benefit and, in particular, the possibility that policy responsiveness grows when office holders are highly office motivated.

A special case of the model that highlights its potential applicability is that in which politicians incur a cost for higher policy choices, with higher types weighting cost less. That is, we capture the case in which the payoff of a type j office holder has the simple form

$$w_j(x) \quad = \quad \lambda_j\left(v(x) - \frac{1}{\theta_j}c(x)\right) + \kappa_j,$$

where $v{:}X \to \mathbb{R}$ is continuously differentiable, concave, and strictly increasing, and $c{:}X \to \mathbb{R}_+$ is continuously differentiable, strictly convex, and has positive derivative, $\kappa_j, \lambda_j, \theta_j$ are type-dependent parameters, with $0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n$ and $0 < \theta_1 < \theta_2 < \cdots < \theta_n$. Thus, higher policy choices are more costly for higher politician types. The functional form for politicians' payoffs admits two simple specifications that are worthy of note. One common specification is *quadratic payoffs,* in which case $w_j(x) = -(x-\hat{x}_j)^2 + K_j$, where K_j is a constant. To obtain this, we set

$$v(x) = 2x,\ c(x) = x^2,\ \kappa_j = -\hat{x}_j^2 + K_j,\ \lambda_j = \theta_j = \hat{x}_j.$$

Another specification of interest is is *exponential payoffs,* whereby $w_j(x) = -e^{x-\hat{x}_j} + x + K_j$, which is obtained by setting

$$v(x) = x,\ c(x) = e^x,\ \kappa_j = K_j,\ \lambda = 1,\ \theta_j = e^{\hat{x}_j}.$$

Although politician preferences are defined over policy choices, rather than outcomes, we can reconcile this apparent difference from voters by setting the term $v(x) = \mathbb{E}[u(y)|x]$ equal to the expected payoff from policy outcomes generated by the choice x, so that politicians share the voter's preferences over policy outcomes. In this case, an office holder differs from other citizens only by the cost term $(1/\theta_j)c(x)$.

Assume that the outcome distribution $F(\cdot|x)$ has a jointly differentiable density $f(y|x)$ with full support on $Y = \mathbb{R}$ for all x. For simplicity, take the policy choice x to be a shift parameter on the density of outcomes, so, abusing notation slightly, the density can be written $f(y|x) = f(y-x)$ for some fixed density $f(\cdot)$, and the probability that the realized outcome is less than y given policy x is simply $F(y-x)$. We assume that f satisfies the standard monotone likelihood ratio property (MLRP), i.e.,

(C3) for all $x > x'$ and all $y > y'$, $\dfrac{f(y-x)}{f(y-x')} \ > \ \dfrac{f(y'-x)}{f(y'-x')}$

This implies that greater policy outcomes induce voters to favorably update their beliefs about the policy adopted by the incumbent in the first period. As is well-known, the MLRP implies that the density function is unimodal, and that both the density and the distribution functions are strictly log-concave. Moreover, we assume

$$\text{(C4)} \qquad \text{for all } x > x', \ \lim_{y \to -\infty} \frac{f(y-x)}{f(y-x')} = \lim_{y \to +\infty} \frac{f(y-x')}{f(y-x)} = 0,$$

so that arbitrarily extreme signals become arbitrarily informative. As an example, $f(\cdot)$ may be a normal density.

4 Stationary electoral equilibrium

The analysis focusses on perfect Bayesian equilibria of the political agency model. Strategies in this dynamic game are potentially highly complex, as policy choices and votes could conceivably depend arbitrarily on observed histories of policy outcomes and electoral outcomes. To preclude implausible behavior by voters and politicians, we impose refinements that strengthen perfect Bayesian equilibrium by limiting the extent of history dependence specified by citizens' strategies. A *stationary strategy for a type j politician* is a pair (π_j^1, π_j^2), where $\pi_j^1 \in \Delta(X)$ specifies the politician's mixture over policy choices in her first term, and $\pi_j^2 \in \Delta(X)$ specifies the mixture of policy choices in her second term.[5] A *stationary strategy the voter* is a measurable function $\rho : Y \to [0,1]$ specifying the probability that a first-term incumbent is re-elected as a function of policy outcomes. Note that π_j^2 could conceivably depend on the politician's policy choice x and the outcome y in her first term, and π_j^1, π_j^2, and ρ could conceivably depend on histories of previous office holders, but stationarity isolates strategies in which such conditioning does not occur. A *belief system* is a function $\mu : Y \to \Delta(T \times X)$, where $\mu(\cdot|y)$ represents the voter's posterior beliefs about an incumbent's type and policy choice conditional on policy outcome y in her first term of office. Then the marginal $\mu_T(\cdot|y)$ gives the voter's posterior beliefs about the incumbent's type.

A strategy profile $\sigma = (\pi_j^1, \pi_j^2, \rho)$ is *sequentially rational* given belief system μ if no politician can gain by deviating to a different policy choice in any term of office, and for all policy outcomes y, the voter votes for the candidate that offers the highest expected discounted payoff conditional on her information. Beliefs μ are *consistent* with σ if for all $y \in Y$, $\mu(\cdot|y)$ is derived using Bayes rule. A *stationary perfect Bayesian equilibrium* is a pair $\psi = (\sigma, \mu)$ such that σ is sequentially rational given μ and such that μ is consistent with σ.

We focus on a refinement of perfect Bayesian equilibrium reflecting two additional behavioral simplifications beyond stationarity. First, write $V^I(y|\psi)$ for the

[5] The notation $\Delta(\cdot)$ represents the set of Borel probability measures on a given measurable subset of the real line.

voter's expected discounted payoff from re-electing a first-term incumbent conditional on policy outcome y, and write $V^C(\psi)$ for the voter's continuation value of electing a challenger. A pair $\psi = (\sigma, \mu)$ is *deferential* if the voter favors the incumbent when indifferent, or more formally, given any policy outcome y, the voter votes for the incumbent if and only if $V^I(y|\psi) \geqslant V^C(\psi)$. Second, ψ is *monotonic* if there is some utility cutoff $\underline{u} \in \mathbb{R} \cup \{\infty\}$ such that for all policy outcomes y, the voter votes to re-elect the incumbent if and only if the payoff from y meets or exceeds that cutoff, i.e.,

$$\rho(y) \quad = \quad \begin{cases} 1 & \text{if } u(y) \geqslant \underline{u}, \\ 0 & \text{else.} \end{cases}$$

Since u is strictly increasing and continuous, this entails that there is a cutoff outcome $\overline{y} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that a first term office holder is re-elected if and only if the policy outcome meets or exceeds $\overline{y}$. Finally, $\psi = (\sigma, \mu)$ is a *stationary electoral equilibrium* if it is a stationary perfect Bayesian equilibrium that is deferential and monotonic.

Whereas stationarity implies that the continuation value of a challenger is history independent, the continuation value of re-electing a first-term incumbent depends, via the voter's updated beliefs, on the policy outcome realized. Clearly, stationarity also implies that the policy choice of a second-term office holder, if elected, is simply her ideal policy. Using the latter observation, the continuation values $V^I(y|\psi)$ and $V^C(\psi)$ are the unique solutions to the recursions

$$V^I(y|\psi) \quad = \quad \sum_k \mu_T(k|y) \Big[\mathbb{E}[u(y)|\hat{x}_k] + \delta V^C(\psi) \Big] \tag{1}$$

and

$$\begin{aligned} V^C(\psi) \quad = \quad \sum_j p_j \int_x \Big[& \mathbb{E}[u(y)|x] + \delta[(1 - F(\overline{y} - x))(\mathbb{E}[u(y)|\hat{x}_j] \\ & + \delta V^C(\psi)) + F(\overline{y} - x) V^C(\psi)] \Big] \pi_j^1(dx). \end{aligned}$$

Solving for $V^C(\psi)$ explicitly, we have

$$V^C(\psi) \quad = \quad \frac{\sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta(1 - F(\overline{y} - x)) \mathbb{E}[u(y)|\hat{x}_j] \Big] \pi_j^1(dx)}{1 - \delta \sum_j p_j \int_x [(1 - F(\overline{y} - x))\delta + F(\overline{y} - x)] \pi_j^1(dx)}. \tag{2}$$

Stationary electoral equilibria then satisfy four necessary conditions. *First,* as noted above, a second-term office holder's mixture π_j^2 over policy choices puts probability one on her ideal point $\hat{x}_j$, i.e., for all j, $\pi_j^2(\{\hat{x}_j\}) = 1$. *Second,* if the cutoff outcome is finite, then it must satisfy the indifference condition that, conditional on observing $\overline{y}$, the voter is indifferent between re-electing a first-term incumbent and electing a challenger. Formally, this condition is

$$\sum_j \mu_T(j|\overline{y}) \Big[\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi) \Big] \quad = \quad V^C(\psi).$$

Simplifying, this means that the expected payoff from the incumbent's policy choice in the second term is equal to the (normalized) continuation value of a challenger:

$$\sum_j \mu_T(j|\overline{y}) \mathbb{E}[u(y)|\hat{x}_j] \quad = \quad (1-\delta)V^C(\psi). \tag{3}$$

If $\overline{y} = \infty$, then an incumbent is never re-elected, and the assumption of deferential voting implies that the inequality

$$\sum_j \mu_T(j|\overline{y}) \mathbb{E}[u(y)|\hat{x}_j] \quad < \quad (1-\delta)V^C(\psi),$$

must hold; and if $\overline{y} = -\infty$, then a first-term office holder is always re-elected, and the inequality must hold weakly in the opposite direction. *Third,* each politician type, knowing that she is re-elected after the first term of office if and only if $y \geqslant \overline{y}$, mixes over optimal actions in her first term of office, i.e., π_j^1 puts probability one on solutions to

$$\max_{x \in X} w_j(x) + \delta \Big[(1 - F(\overline{y} - x))[w_j(\hat{x}_j) + \beta + \delta V^C(\psi)] + F(\overline{y} - x) V^C(\psi) \Big]. \tag{4}$$

An implication is that every policy choice x in the support of π_j^1 must solve the first order condition for this problem, i.e.,

$$w_j'(x) \quad \leqslant \quad -f(\overline{y} - x)\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)], \tag{5}$$

with equality for $x > 0$. *Fourth,* updating of voter beliefs follows Bayes rule: conditional on policy outcome y, the posterior probability the politician is type j is

$$\mu_T(j|y) \quad = \quad \frac{p_j \int_x f(y-x) \pi_j^1(dx)}{\sum_k p_k \int_x f(y-x) \pi_k^1(dx)}.$$

Note that stationary electoral equilibria refine perfect Bayesian equilibrium, so that after all histories, no citizen can increase her expected discounted payoff by deviating to another strategy (stationary or non-stationary). And although we allow in principle for behavior as a general function of updated priors, the two additional restrictions we impose capture some intuitive ideas. The assumption of deferential strategies is a form of *prospective voting,* in which the representative voter acts as though pivotal in the election, and the assumption of monotonicity formalizes *retrospective voting,* in which a voter asks, "What have you done for me lately?" and votes to re-elect the incumbent if the policy outcome delivered by the politician satisfies a certain threshold. Thus, in a stationary electoral equilibrium, prospective and retrospective voting are compatible and both describe the behavior of voters, and the choices of office holders are optimal given these voting strategies.

5 Preliminary analysis

To facilitate the analysis, it is assumed that first-term office holders are in principle interested in re-election, a condition formalized as follows.

(C5) $$w_1(\hat{x}_1) + \beta \quad > \quad \mathbb{E}[u(y)|\hat{x}_n],$$

Thus, a first-term office holder prefers to remain in office, even if she can return to the electorate and in all future periods, outcomes are determined by the ideal point of the highest type. We will see that $\mathbb{E}[u(y)|\hat{x}_n]$ does indeed bound the (normalized) continuation value of a challenger in equilibrium, so (C5) means that all first-term office holders view re-election as inherently desirable. Note that by (C5), the right-hand side of the first order condition in (5) is negative, and it follows that for arbitrary cutoff $\overline{y}$ and challenger continuation value $V \leqslant \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n]$, an office holder exerts positive effort in the first term, i.e., if x_j^* is optimal for the type j office holder in her first term, then $x_j^* > \hat{x}_j$. In particular, the marginal disutility in the current period from increasing the policy choice is just offset by the marginal utility in the second period, owing to the politician's increased chance of re-election.

We can gain some insight into a first-term office holder's policy choice problem by reformulating it in terms of optimization subject to an inequality constraint. Define the new objective function

$$W_j(x,r;V) \quad = \quad w_j(x) + r\delta[w_j(\hat{x}_j) + \beta - (1-\delta)V],$$

which is the expected payoff if the politician makes policy choice x and is re-elected with probability r, minus a constant term. Here, the objective function is parameterized by an arbitrary value V with $0 \leqslant V \leqslant \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n]$, which represents the voter's continuation value of a challenger. Note that W_j is concave in (x,r) and quasi-linear in r. Of course, given x, there is only one possible re-election probability consistent with a cutoff $\overline{y}$, namely, $1-F(\overline{y}-x)$. Defining the constraint

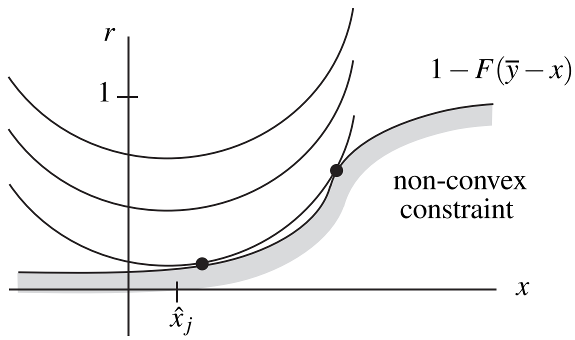

Figure 1: Politician's optimization problem

function

$$g(x,r) \quad = \quad 1-F(\bar{y}-x)-r,$$

the type j office holder's problem (4) in her first term of office can then be reformulated as

$$\max_{(x,r)} W_j(x,r;V) \quad \text{s.t. } g(x,r) \leqslant 0,$$

which has the general form depicted in Figure 1. Here, the objective function is well-behaved, but the constraint set inherits the natural non-convexity of the distribution function F, leading to the possibility of multiple solutions. This, in turn, can lead to multiple optimal policy choices, as in the figure.

Assuming $0 \leqslant V \leqslant \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n]$, condition (C5) implies that it is not optimal for a first-term office holder to make policy choices below her ideal point, and by (C2) it is not optimal for office holders to choose arbitrarily high policies, so each type of office holder has an optimal policy choice in the first term of office. We use x_j^* to denote the greatest optimal policy choice of a first-term office holder of type j, and we denote by $x_{*,j}$ the least optimal policy choice. Here, we suppress dependence of these optimal policy choices on V for notational simplicity.

The next proposition establishes that the politicians' objective functions satisfy the important property that differences in payoffs are monotone in type and allows us to conclude that optimal policy choices are in fact ordered by type. We say that $W_j(x,1-F(\bar{y}-x);V)$ is *supermodular* in (j,x) if for all (j,x) and all (k,z) with $j>k$ and $x>z$, we have

$$\begin{aligned} & W_j(x,1-F(\bar{y}-x);V)-W_j(z,1-F(\bar{y}-z);V) \\ > \quad & W_k(x,1-F(\bar{y}-x);V)-W_k(z,1-F(\bar{y}-z);V). \end{aligned}$$

A well-known implication is that given an arbitrary value $\overline{y}$ of the cutoff, the optimal policy choices of the types are strictly ordered by type, i.e., for all $j < n$, if x_j solves

$$\max_{x \in X} W_j(x, 1 - F(\overline{y} - x); V)$$

and x_{j+1} solves the corresponding problem for type $j+1$, then $x_j < x_{j+1}$; in terms of the above conventions for greatest and least optimal policy choices, this is $x_j^* < x_{*,j+1}$. This ordering property will, in turn, be critical for establishing existence of equilibrium. The proof follows the lines of Lemma A.3 of Banks and Sundaram (1998) and is omitted.

Proposition 1 *Assume (C1)–(C5), and let* $0 \leqslant V \leqslant \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n]$. *The politicians' objective function,* $W_j(x, 1 - F(\overline{y} - x); V)$*, is super modular in* (j, x).

Now define the voter's ex ante expected discounted payoff from re-electing the incumbent using the cutoff $\overline{y}$, given mixed policy strategies $(\pi_1^1, \ldots, \pi_n^1)$ and challenger continuation value V, as

$$\begin{aligned} & U(\overline{y}, \pi_1^1, \ldots, \pi_n^1; V) \\ = \; & \sum_j p_j \int_x \Bigg[\int_{y \leqslant \overline{y}} (u(y) + \delta V) f(y - x) dy \\ & + \int_{y > \overline{y}} (u(y) + \delta \mathbb{E}[u(y)|\hat{x}_j] + \delta^2 V) f(y - x) dy \Bigg] \pi_j^1(dx). \end{aligned}$$

That is, each type j politician mixes according to π_j, and depending on the choice x, an outcome is realized from the density $f(y - x)$. If this falls below the cutoff, then the voter receives utility from y and subsequently receives the continuation value of a challenger; and if it falls above the cutoff, then the voter additionally receives the expected payoff from the type j politician's ideal point before electing a challenger. Note that this function is jointly continuous in its arguments.

Let $\pi_1^1, \ldots, \pi_n^1$ be mixed policy strategies with supports that are strictly ordered according to type. Then arguments of Lemmas A.5 and A.6 of Banks and Sundaram (1998) establish that there is a unique solution to

$$\max_{\overline{y} \in Y \cup \{-\infty, \infty} U(\overline{y}, \pi_1^1, \ldots, \pi_n^1; V),$$

and we denote this by $y^*(\pi_1^1, \ldots, \pi_n^1; V)$. Note that by the theorem of the maximum, this function is jointly continuous in its arguments. Indeed, differentiating and examining the necessary first order condition for a finite optimal cutoff,

$$\sum_j p_j \int_x ((1 - \delta)V - \delta \mathbb{E}[u(y)|\hat{x}_j]) f(\overline{y} - x) \pi_j^1(dx) \quad = \quad 0,$$

we see immediately that if the optimal cutoff is finite, then it satisfies the indifference condition

$$\sum_k \mu_T(k|\overline{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad (1-\delta)V, \tag{6}$$

where $\mu_T(k|\overline{y})$ is derived by Bayes rule. In particular, when the continuation value V satisfies $\mathbb{E}[u(y)|\hat{x}_1] < (1-\delta)V < \mathbb{E}[u(y)|\hat{x}_n]$, the unique optimal cutoff is given by the unique solution to (6).

The next result states this finding and in addition to uniqueness, it establishes that the cutoff lies between the choices of the type 1 and type n politicians, shifted by the mode of the density of $f(\cdot)$, denoted $\hat{z}$. The proof of the latter claim follows the lines of the proof of Proposition 3 of Duggan and Martinelli (2015) and is omitted.

Proposition 2 *Assume (C1)–(C5), and let V be such that $\mathbb{E}[u(y)|\hat{x}_1] < (1-\delta)V < \mathbb{E}[u(y)|\hat{x}_n]$. For all mixed policy strategies $\pi^1_1,\ldots,\pi^1_n$ with supports that are strictly ordered by type and all belief systems μ derived via Baye's rule, there is a unique optimal cutoff $y^*(\pi^1_1,\ldots,\pi^1_n,V)$ for the voter, this cutoff is finite, and it is the solution to the voter's indifference condition (6). Moreover, the optimal cutoff is jointly continuous as a function of its arguments, and it lies between the extreme policy choices shifted by the mode of the outcome density, i.e.,*

$$\min \operatorname{supp}(\pi^1_1)+\hat{z} \;\leqslant\; y^*(\pi^1_1,\ldots,\pi^1_n,V) \;\leqslant\; \max \operatorname{supp}(\pi^1_n)+\hat{z}.$$

Combining Propositions 1 and 2, it follows that the four necessary conditions listed at the end of Section 4 are in fact sufficient. In particular, given the re-election standard $\overline{y}$, if the strategies π^1_j put probability one on solutions to the maximization problem in (4), then Proposition 1 implies that they are strictly ordered according to type; and then by Proposition 2, the cutoff is optimal for the voter.

6 Existence of stationary electoral equilibria

The first main result of the paper establishes existence of stationary electoral equilibrium, along with a partial characterization of equilibria.

Theorem 1 *Assume (C1)–(C5). Then there is a stationary electoral equilibrium, and every stationary electoral equilibrium is given by policy strategies $\pi^*_1,\ldots,\pi^*_n$ and a finite cutoff y^* such that:*

(i) each type j politician mixes over policy choices using π^*_j, and the least optimal policy choice is greater than the ideal point of the politician, i.e., $x_{*,j} > \hat{x}_j$.

(ii) the supports of policy strategies are strictly ordered by type, and in fact optimal policy choices are ordered by type, i.e., for all $j < n$, we have $x_j^* < x_{*,j+1}$,

(iii) the voter re-elects the incumbent if and only if $y \geqslant y^*$, where the cutoff lies between the extreme policies shifted by the mode of the outcome density, i.e., $\min \operatorname{supp}(\pi_1^*) + \hat{z} \leqslant y^* \leqslant \max \operatorname{supp}(\pi_n^*) + \hat{z}$.

Proof: The existence proof follows from an application of Glicksberg's fixed point theorem to an appropriately defined correspondence. To define the domain of this correspondence, first note that by (C2), there exists $\tilde{x}$ sufficiently large that for all types j, all cutoffs $\overline{y}$, and all values V with $0 \leqslant V \leqslant \frac{1}{1-\delta}\mathbb{E}[u(y)|\hat{x}_n]$, and all policy choices $x \geqslant \tilde{x}$, we have

$$W_j(\hat{x}_j, 1-F(\overline{y}-\hat{x}_j);V) \quad \geqslant \quad W_j(x, 1-F(\overline{y}-x);V).$$

Let $\Delta([0,\tilde{x}])$ denote the set of Borel probability measures on $[0,\tilde{x}]$, endowed with the weak* topology.

Next, I claim that the distance between optimal policy choices of different politician types as the voter's cutoff $\overline{y}$ varies has a positive lower bound, say $\varepsilon > 0$. That is, for all $\overline{y}$ and all $j < n$, we have $x_j^*(\overline{y}) + \varepsilon \leqslant x_{*,j+1}(\overline{y})$. Indeed, given any cutoff $\overline{y}$ and any type j politician, note that every optimal policy choice for the politician satisfies the first order condition (5) with equality. Also note that $f(\overline{y}-x) \to 0$ uniformly on $[0,\tilde{x}]$ as $|\overline{y}| \to \infty$, and from the first order condition, this implies that the optimal policy choices of the type j politician converge to the ideal point, i.e., $x_j^*(\overline{y}) \to \hat{x}_j$ and $x_{*,j}(\overline{y}) \to \hat{x}_j$. Thus, we can choose a sufficiently large interval $[y_L, y_H]$ and $\varepsilon' > 0$ such that for all $\overline{y}$ outside the interval, optimal policy choices differ by at least ε', i.e., for all $j < n$, we have $|x_{*,j+1}(\overline{y}) - x_j^*(\overline{y})| > \varepsilon'$. By upper semi-continuity of $x_j^*(\cdot)$ and lower semi-continuity of $x_{*,j+1}(\cdot)$, the function $|x_{*,j+1}(\overline{y}) - x_j^*(\overline{y})|$ attains its minimum on $[y_L, y_H]$, and this minimum is positive. Thus, there exists $\varepsilon'' > 0$ such that for all $\overline{y} \in [y_L, y_H]$, optimal policy choices differ by at least ε''. Finally, we set $\varepsilon = \min\{\varepsilon', \varepsilon''\}$ to establish the desired lower bound.

Let $\Delta_\varepsilon^n([0,\tilde{x}])$ denote the set of mixed policy profiles $(\pi_1, \ldots, \pi_n)$ such that for all j, π_j has support in $[0,\tilde{x}]$, and such that for all $j < n$ the support of π_j is separated from the support of π_{j+1} by a distance of at least ε, i.e.,

$$\max \operatorname{supp}(\pi_j) + \varepsilon \quad \leqslant \quad \min \operatorname{supp}(\pi_{j+1}).$$

Because it is defined by weak inequalities, the set $\Delta_\varepsilon^n([0,\tilde{x}])$ is compact in the relative topology inherited from $\Delta([0,\tilde{x}])^n$, but it is not convex; we return to this observation in discussion following the proof. Letting

$$\mathcal{V} \quad = \quad \left[\frac{\mathbb{E}[u(y)|\hat{x}_1]}{1-\delta}, \frac{\mathbb{E}[u(y)|\hat{x}_n]}{1-\delta}\right],$$

the function $y^*(\cdot)$ attains a minimum and maximum on $\Delta^n_\varepsilon([0,\tilde{x}]) \times \mathcal{V}$. Denote these by a and b, respectively, and let $\Delta([a,b])$ be the set of Borel probability measures with support in $[a,b]$, with elements denoted ρ.

As a last step before defining the fixed point correspondence, we use the expression for the voter's continuation value of a challenger in (2) to define the value induced by policy strategies $\pi_1,\ldots,\pi_n$ and cutoff $\overline{y}$:

$$\tilde{V}(\pi_1,\ldots,\pi_n,\overline{y}) = \frac{\sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta(1-F(\overline{y}-x))\mathbb{E}[u(y)|\hat{x}_j]\Big]\pi^1_j(dx)}{1-\delta\sum_j p_j\int_x[(1-F(\overline{y}-x))\delta + F(\overline{y}-x)]\pi^1_j(dx)}.$$

Then we bound this quantity below by the expected discounted payoff from the ideal point of the lowest and highest types:

$$V(\pi_1,\ldots,\pi_n,\overline{y}) = \max\left\{\frac{\mathbb{E}[u(y)|\hat{x}_1]}{1-\delta}, \min\left\{\frac{\mathbb{E}[u(y)|\hat{x}_n]}{1-\delta}, \tilde{V}(\pi_1,\ldots,\pi_n,\overline{y})\right\}\right\}.$$

Note that this expression is jointly continuous in its arguments.

We are now ready to define the fixed point correspondence $\Phi\colon\Delta([0,\tilde{x}])^n \times \Delta([a,b]) \rightrightarrows \Delta([0,\tilde{x}])^n \times \Delta([a,b])$. Given type j and $(\pi_1,\ldots,\pi_n,\rho)$ in the domain, we let $\phi_j(\pi_1,\ldots,\pi_n,\rho)$ be the set of optimal policy choices for the type j politician:

$$\phi_j(\pi_1,\ldots,\pi_n,\rho) = \arg\max_{x\in[0,\tilde{x}]} W_j(x, 1-F(\mathbb{E}[\rho]-x); V(\pi_1,\ldots,\pi_n,\mathbb{E}[\rho])).$$

Three features of this construction are noteworthy. First, the probability measure ρ is reduced to its mean, so that no other information about this distribution is used. Second, the continuation value used by the politician is imputed by the policy strategies and cutoff $\mathbb{E}[\rho]$. Third, the objective function used in the definition of ϕ_j is jointly continuous in x and $(\pi_1,\ldots,\pi_n,\rho)$, and it follows from the theorem of the maximum that ϕ_j has non-empty values and closed graph. Then define

$$\Phi_j(\pi_1,\ldots,\pi_n,\rho) = \Delta(\phi_j(\pi_1,\ldots,\pi_n,\rho))$$

as the set of mixtures over optimal policy choices of type j politicians. The correspondence Φ_j then has convex values, and it inherits non-empty values and closed graph from ϕ_j.

To complete the construction, given $(\pi_1,\ldots,\pi_n,\rho)$ in the domain, we define $\phi_{n+1}(\pi_1,\ldots,\pi_n,\rho)$ to be the set of optimal cutoffs for the voter:

$$\phi_{n+1}(\pi_1,\ldots,\pi_n,\rho) = \arg\max_{\overline{y}\in[a,b]} U(\overline{y},\pi_1,\ldots,\pi_n; V(\pi_1,\ldots,\pi_n,\mathbb{E}[\rho]))$$

Then let

$$\Phi_{n+1}(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \Delta(\phi_{n+1}(\pi_1,\ldots,\pi_n,\rho))$$

be the set of mixtures over optimal cutoffs. This correspondence has non-empty, convex values and closed graph.

Finally, we define the fixed point correspondence Φ as the product of the correspondences defined above:

$$\Phi(\pi_1,\ldots,\pi_n,\rho) \quad = \quad \prod_{i=1}^{n+1} \Phi_i(\pi_1,\ldots,\pi_n,\rho).$$

This correspondence inherits non-empty, convex values and closed graph from its components, and Glicksberg's fixed point theorem implies that Φ admits a fixed point, i.e., there exists $(\pi_1^*,\ldots,\pi_n^*,\rho^*)$ in the domain such that $\Phi(\pi_1^*,\ldots,\pi_n^*,\rho^*) = (\pi_1^*,\ldots,\pi_n^*,\rho^*)$.

By Proposition 1, setting $\overline{y} = \mathbb{E}[\rho^*]$, it follows that the supports of $\pi_1^*,\ldots,\pi_n^*$ are ordered according to type and separated by a distance of ε, i.e., $(\pi_1^*,\ldots,\pi_n^*) \in \Delta_\varepsilon^n([0,\tilde{x}])$. Then Proposition 2 establishes that the voter has a unique optimal cutoff, say y^*. This implies that ρ^* is the unit mass on y^*, so that π_j^* has support on solutions to the office holder's best response problem with $y^* = \mathbb{E}[\rho^*]$. Writing $V^* = V(\pi_1^*,\ldots,\pi_n^*,y^*)$, I claim that $(1-\delta)V^* < \mathbb{E}[u(y)|\hat{x}_n]$, for suppose otherwise. Note that the derivative of $U(y^*,\pi_1^*,\ldots,\pi_n^*;V^*)$ with respect to y^* is

$$\begin{aligned} & \sum_j p_j \int_x (u(y^*) + \delta V^* - u(y^*) - \delta\mathbb{E}[u(y)|\hat{x}_j] + \delta^2 V^*) f(y^*-x)\pi_j(dx) \\ = \quad & \delta \sum_j p_j \int_x ((1-\delta)V^* - \mathbb{E}[u(y)|\hat{x}_j]) f(y^*-x)\pi_j(dx) \\ > \quad & 0, \end{aligned}$$

where the inequality follows by supposition. But this implies that y^* is not optimal, a contradiction. It can also be shown that $\mathbb{E}[u(y)|\hat{x}_1 < (1-\delta)V^*$. We conclude that $\mathbb{E}[u(y)|\hat{x}_1] < (1-\delta)V^* < \mathbb{E}[u(y)|\hat{x}_n]$, and it follows that $V^* = \tilde{V}(\pi_1^*,\ldots,\pi_n^*,y^*)$.

To obtain a stationary electoral equilibrium, we then specify $\psi = (\sigma,\mu)$ so that: for each $j = 1,\ldots,n$, $\pi_j^1 = \pi_j^*$ and π_j^2 places probability one on $\hat{x}_j$; that $\rho = \rho^*$; and that μ is derived via Bayes rule. In particular, $V^* = V^C(\psi)$, and ψ satisfies the necessary and sufficient conditions for equilibrium described in Section 4. This completes the proof of existence. Given an arbitrary stationary electoral equilibrium ψ, Theorem 3 in Section 8 implies that $\mathbb{E}[u(y)|\hat{x}_1] < (1-\delta)V^C(\psi) < \mathbb{E}[u(y)|\hat{x}_n]$, and thus properties (i), (ii), and (iii) follow from the first order condition (5) with (C5), from Proposition 1, and from Proposition 2, respectively. ∎

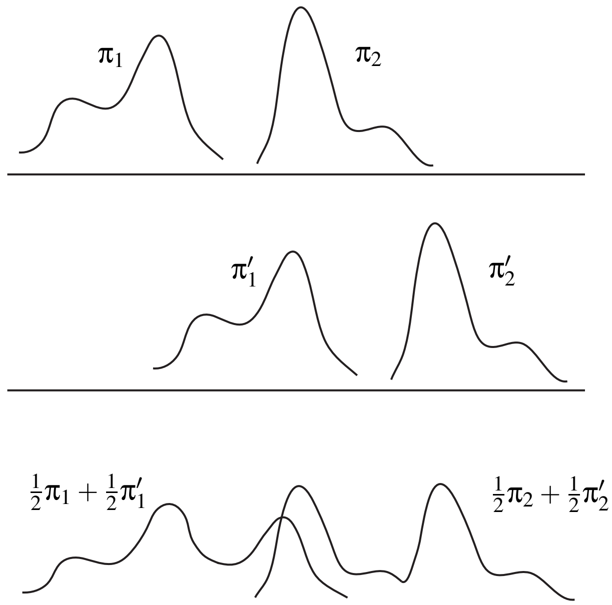

Figure 2: Non-convexity of Banks-Sundaram domain

In the proof of Theorem 1, the cutoff $y^*(\cdot)$ attains a minimum and maximum over the compact set $\Delta^n_\varepsilon([0,\tilde{x}]) \times \mathcal{V}$, where $\Delta^n_\varepsilon([0,\tilde{x}])$ is the set of profiles $(\pi_1,\ldots,\pi_n)$ such that the support of π_j is separated from the support of π_{j+1} by at least ε. It was noted there that this set is not convex; this is illustrated in Figure 2, where policy mixtures are represented for two types by density functions. In the top panel, the supports of π_1 and π_2 are separated by a positive amount, as are the supports of π'_1 and π'_2 in the middle panel. But taking convex combinations of π_1 and π'_1, and of π_2 and π'_2, the supports of the resulting mixtures overlap. This observation is problematic for the existence argument put forward by Banks and Sundaram (1998), as they take the domain of their fixed point correspondence to be the set of profiles $(\pi_1,\ldots,\pi_n)$ such that for all $j < n$, the support of π_j is weakly less than the support of π_{j+1}; as this set of profiles is not convex, the assumptions of Glicksberg's fixed point theorem are not satisfied, and their application of Glicksberg is invalid.

The approach of the current paper is to relinquish monotonicity of policy mixtures in the domain in order to satisfy convexity. Monotonicity is then recaptured after the acquisition of a fixed point, through the ordering of best responses implied by supermodularity of the politicians' payoffs. This approach relies on the expansion of the domain to include the cutoff used by voters; it is the politicians' response to this cutoff that generates the desired ordering of policy mixtures. But since the domain of policy mixtures no longer incorporates the ordering of supports, there may be multiple optimal cutoffs for the voter, and we must technically expand the domain to include *mixtures* over cutoffs; such a mixture ρ does not have a behavioral interpretation in the model, and in fact the mixing is extracted from the fixed point obtained by virtue of politicians responding only to the expectation of ρ. Here, monotonicity of policy mixtures is like love: we let it go, and it comes back to us in the end.

7 Asymptotic policy choices

Theorem 1 provides a partial characterization of policy choices for arbitrary model parameters. More can be said when politicians are highly office motivated, i.e., fixing a discount factor δ, we let $\beta \to \infty$. The following result shows that faced with highly office-motivated politicians, voters become arbitrarily demanding, in the sense that the cutoff for re-election goes to infinity. Furthermore, the type n politicians mix between arbitrarily high policy choices and policy choices close to her ideal point, while all other types shirk by making policy choices close to their ideals. In fact, the result is more general than described above, because we can let the discount factor vary with office benefit, as long as the product $\beta\delta$ diverges to infinity; thus, we capture the case of impatient citizens, as long as the increasing office benefit compensates for decreasing discount factors. The result assumes that voter payoffs are unbounded,

$$\text{(C6)} \qquad \lim_{x \to \infty} u(x) \;=\; \infty,$$

which of course holds in the risk neutral case.

Theorem 2 *Assume (C1)–(C6). Let the office benefit* $\beta \geqslant 0$ *and* $\delta \in [0,1)$ *vary arbitrarily subject to* $\lim \beta\delta = \infty$. *Then for every selection of stationary electoral equilibria* ψ*, the voters' cutoff diverges to infinity; the type n politicians in their first term mix between policy choices that are close to their ideal point and ones that are arbitrarily high, with small, positive probability on the latter; and all other type* $j < n$ *politicians make policy choices close to their ideal points in the first term, i.e.,*

(i) $y^* \to \infty$,

(ii) $x_n^* \to \infty$ *and* $x_{*,n} \to \hat{x}_n$,

(iii) for all $j < n$, $x_j^* \to \hat{x}_j$,

(iv) for all $\eta > 0$ *and for* $\beta\delta$ *large enough, we have* $\pi^1_n([\eta,\infty)) > 0$*,*

(v) for all $\eta > 0$*, we have* $\pi^1_n((\hat{x}_n,\eta)) \to 1$ *and* $\pi^1_n([\eta,\infty)) \to 0$*.*

Proof: We first prove (i). Indeed, suppose there is a subsequence such that y^* is bounded, so going to a subsequence if needed, we can assume that $y^* \to \overline{y}$. We claim that for each type j politician, the least optimal policy choice diverges to infinity, i.e., $x_{*,j} \to \infty$. Otherwise, we can go to a subsequence if needed to suppose that $x_{*,j} \to \tilde{x}_j < \infty$. By the first order condition in (5), we then have

$$w'_j(\tilde{x}_j) \;=\; \lim w'_j(x_{*,j}) \quad \leqslant \quad -f(\overline{y}-\tilde{x}_j)\lim\beta\delta - C,$$

where C is the limit infimum of $f(\overline{y}-x)\delta[w_j(\hat{x}_j)-(1-\delta)V^C(\psi)]$, which, by Theorem 3 in Section 8, is finite. Since $\lim\beta\delta = \infty$, the right-hand side of the above inequality has infinite magnitude, a contradiction. Thus, we have $x_{*,j} \to \infty$ for each politician type j, as claimed. But by (C6), the voter's expected payoff from the policy choices of first-term office holders then diverges to infinity, contradicting Theorem 3. We conclude that $|y^*| \to \infty$. Now suppose there is a subsequence such that $y^* \to -\infty$. Because policy choices are strictly ordered by type, it follows that for all $x_1,\ldots,x_n$ in the support of the politicians' policy strategies, we have

$$f(y^*-x_1) \;>\; f(y^*-x_2) \;>\; \cdots \;>\; f(y^*-x_n).$$

Therefore, the coefficients on prior beliefs from Bayes rule are ordered by type, i.e.,

$$\frac{\sum_x f(y^*-x)\pi^1_1(x)}{\sum_k p_k\sum_x f(y^*-x)\pi^1_k(x)} \;>\; \cdots \;>\; \frac{\sum_x f(y^*-x)\pi^1_n(x)}{\sum_k p_k\sum_x f(y^*-x)\pi^1_k(x)},$$

and we conclude that voter's prior first order stochastically dominates the posterior distribution $\mu_T(\cdot|p,y^*)$, which implies

$$\sum_j p_j\mathbb{E}[u(y)|\hat{x}_j] \quad > \quad \sum_j \mu_T(j|p,y^*)\mathbb{E}[u(y)|\hat{x}_j].$$

From the argument in the proof of Theorem 3, inequality (9) holds, and since $\hat{x}_j < x_{*,j}$ for all j, we have $V^C(\psi) > \frac{1}{1-\delta}\sum_j p_j\mathbb{E}[u(y)|\hat{x}_j]$. Using (1), we then have

$$V^I(y^*|\psi) \;<\; \sum_j p_j\left[\mathbb{E}[u(y)|\hat{x}_k]+\delta V^C(\psi)\right] \;<\; V^C(\psi),$$

contradicting the voter's indifference condition. Therefore, $y^* \to \infty$, as desired.

To prove (ii) and (iii), we next show that for all types j, there is no subsequence of greatest optimal policy choices x^*_j that converge to a finite policy choice greater than the ideal point; by the same argument, the least optimal policy choices $x_{*,j}$ also cannot converge to a finite policy choice greater than the ideal point. Indeed, suppose that there is some type j such that $x_j^* \to \tilde{x}_j$ with $\hat{x}_j < \tilde{x}_j < \infty$. Then for sufficiently large $\beta\delta$, we have $\hat{x}_j < x_j^*$. For these parameters, the current gain to the type j politician from choosing $\hat{x}_j$ instead of x_j^* is non-positive, and thus we note that

$$\delta(F(y^* - \hat{x}_j) - F(y^* - x_j^*))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \quad \geqslant \quad w_j(\hat{x}_j) - w_j(x_j^*).$$

That is, the current gains from choosing the ideal point are offset by future losses. Since $y^* \to \infty$, the limit of

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)}$$

as $\beta\delta$ becomes large is indeterminate, and by L'Hôpital's rule, the limit is equal to

$$\lim \frac{f(y^* - x_j^*) - f(y^* - \tilde{x}_j - 1)}{f(y^* - \hat{x}_j) - f(y^* - x_j^*)} \; = \; \lim \frac{f(y^* - \tilde{x}_j - 1)\left(\frac{f(y^* - x_j^*)}{f(y^* - \tilde{x}_j - 1)} - 1\right)}{f(y^* - x_j^*)\left(\frac{f(y^* - \hat{x}_j)}{f(y^* - x_j^*)} - 1\right)} \; = \; \infty,$$

where we use (C3) and (C4). Then, however, the future gain from choosing $\tilde{x}_j + 1$ instead of x_j^* strictly exceeds current losses, i.e.,

$$\begin{aligned} &\delta(F(y^* - x_j^*) - F(y^* - \tilde{x}_j - 1))[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \\ &\quad > \quad w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned} \tag{7}$$

for some parameters (β', δ'). To be specific, let

$$\begin{aligned} A &= \delta[w_j(\hat{x}_j) + \beta - (1-\delta)V^C(\psi)] \\ B &= w_j(\hat{x}_j) - w_j(x_j^*) \\ C &= w_j(x_j^*) - w_j(\tilde{x}_j + 1), \end{aligned}$$

where A is evaluated at β and δ with $\beta\delta$ sufficiently large. Note that since $\hat{x}_j < \tilde{x}_j < \infty$, we have $\lim B > 0$ and $\lim C < \infty$. We have noted that $(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A \geqslant B$ for sufficiently large $\beta\delta$, and we have shown that

$$\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)} \quad > \quad \frac{C}{B}$$

for sufficiently large $\beta\delta$. Combining these facts, we have

$$(F(y^* - \hat{x}_j) - F(y^* - x_j^*))A\left(\frac{F(y^* - x_j^*) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x_j^*)}\right) \quad > \quad B\left(\frac{C}{B}\right),$$

which yields (7) for some (β', δ'). This gives the type j politician a profitable deviation from x_j^*, a contradiction.

Now, suppose there is a subsequence such that the greatest optimal policy choice x_n^* of the type n politicians is bounded above by some policy choice, say $\bar{x}$. It follows that for all politician types j, we have $x_j^* \to \hat{x}_j$, so that the probability of re-electing an incumbent goes to zero, i.e., for all politician types j, we have $\int_x F(y^* - x)\pi_j^1(dx) \to 1$. Re-writing (2), we have

$$\begin{aligned}(1-\delta)V^C(\psi) \quad &= \quad \left(\frac{1-\delta}{1 - \delta \sum_j p_j \sum_x [(1 - F(\bar{y} - x))\delta + F(\bar{y} - x)]\pi_j^1(x)}\right) \\ &\quad \cdot \left(\sum_j p_j \sum_x \left[\mathbb{E}[u(y)|x] + \delta(1 - F(\bar{y} - x))\mathbb{E}[u(y)|\hat{x}_j]\right]\pi_j^1(x)\right).\end{aligned}$$

Taking limits as $y^* \to \infty$, and using L'Hôpital's rule in case $\delta \to 1$, we see that $(1-\delta)V^C(\psi) \to \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]$. But using $y^* \to \infty$, we also have

$$\begin{aligned}\mu_T(n|p, y^*) \quad &= \quad \frac{p_n \int_x f(y^* - x)\pi_n^1(dx)}{\sum_k p_k \sum_x f(y^* - x)\pi_k^1(x)} \\ &= \quad \frac{p_n \int_x \frac{f(y^* - x)}{f(y^* - \hat{x}_n)}\pi_n^1(dx)}{\sum_{k<n} p_k \int_x \frac{f(y^* - x)}{f(y^* - \hat{x}_n)}\pi_k^1(dx) + p_n \int_x \frac{f(y^* - x)}{f(y^* - \hat{x}_n)}\pi_n^1(dx)} \\ &\to \quad 1.\end{aligned}$$

By the indifference condition (3), we then also have $(1-\delta)V^C(\psi) \to \mathbb{E}[u(y)|\hat{x}_n]$, a contradiction. We conclude that $x_n^* \to \infty$. By Theorem 3, it cannot be that the type n politicians place probability one on arbitrarily high policy choices as $\beta\delta$ becomes large, and it follows that $x_{*,n} \to \hat{x}_n$, which proves (ii). Moreover, since policy choices are ordered by type, this implies that for all $j < n$, we have $x_j^* \to \hat{x}_j$. This proves (iii).

Next, consider $\eta > 0$, and suppose there is a subsequence such that $\pi_n^1([\eta, \infty)) = 0$ for arbitrarily large $\beta\delta$. Then (2) again yields the implication that $(1-\delta)V^C(\psi) \to \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j]$, and choosing any $\bar{x} > \hat{x}_n$, we obtain a contradiction as in the previous paragraph. This proves (iv).

Finally, consider $\eta > 0$, and suppose there exist $c > 0$ and a subsequence such that $\pi_n^1([\eta, \infty)) > c$. With (C6), Theorem 3 implies that $\lim_{\theta \to \infty} \pi_n^1([\theta, \infty)) \to 0$, and in particular there exists $\theta > \eta$ such that $\pi_n^1([\eta,\theta]) > \frac{c}{2}$ for $\beta\delta$ large. This in turn implies that there is a subsequence of optimal policy choices for the type n politicians converging to a finite policy choice greater than the ideal point $\hat{x}_n$, and this generates a contradiction as in the first part of the proof of (ii). Thus, we have $\pi_n^1([\eta,\infty)) \to 0$. Moreover, since $\pi_n^1((\hat{x}_n,\eta)) \cup \pi_n^1([\eta,\infty)) = 1$, we have $\pi_n^1((\hat{x}_n,\eta)) \to 1$. This proves (v), as required. ∎

The asymptotic analysis reveals that as politicians become more office motivated, the policy choices of all politician types but the highest converge to their ideal point, i.e., their behavior approximates shirking in the limit. In contrast, the type n politicians mix over arbitrarily high policy choices, but with probability going to zero. Thus, to the extent that policy is responsive to voter preferences, this is driven by the choices of type n politicians in their first term of office. This leaves open the possibility that the probability of high policy choices decreases slowly relative to the increase in the type n politician's effort, with the net effect of these forces being that the voter's discounted payoff becomes arbitrarily high. We will see in the next section that this possibility is not realized.

8 Bounds on policy responsiveness

The results of this section show that as politicians become office motivated, the voter's expected payoff from the policy choices of first-term office holders—and therefore the (normalized) continuation value of a challenger—cannot exceed the expected payoff from the ideal point of the highest type. The reason is that second-term politicians exert zero effort in equilibrium, and so the voter's expected payoff from electing a politician to a second term is bounded above by the payoff generated by the highest ability type choosing zero effort; and if first-term politicians' effort levels are too high, then the voter would rather elect a challenger, but then politicians will exert zero effort in equilibrium. The first result holds for general parameterizations of the model.

Theorem 3 *Assume (C1)–(C5). For all levels of office benefit $\beta \geqslant 0$ and all discount factors $\delta \in [0,1)$, in every stationary electoral equilibrium ψ, the expected payoff to the voter from policy choices of by first-term office holders is no more than the expected payoff from the ideal point of the type n politician, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad < \quad \mathbb{E}[u(y)|\hat{x}_n].$$

Proof: To prove the result, suppose that for some parameterization of the model, we have a stationary electoral equilibrium such that

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \geqslant \quad \mathbb{E}[u(y)|\hat{x}_n].$$

Recall that the continuation value of a challenger satisfies

$$\begin{aligned} V^C(\psi) \quad = \quad & \sum_j p_j \int_x \Big[\mathbb{E}[u(y)|x] + \delta[(1-F(\bar{y}-x))(\mathbb{E}[u(y)|\hat{x}_j] \\ & + \delta V^C(\psi)) + F(\bar{y}-x)V^C(\psi)] \Big] \pi_j^1(dx). \end{aligned} \tag{8}$$

Note that

$$\begin{aligned} & \sum_j p_j \int_x (1-F(y^*-x))(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi))\pi_j^1(dx) \\ = \quad & \int_{y^*}^{\infty} \left[\sum_j \left(p_j \int_x f(\tilde{y}-x)\pi_j^1(dx) \right) (\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi)) \right] d\tilde{y} \\ = \quad & \int_{y^*}^{\infty} \left(\sum_k p_k \int_x f(\tilde{y}-x)\pi_k^1(dx) \right) \left[\sum_j \mu_T(j|p,\tilde{y})(\mathbb{E}[u(y)|\hat{x}_j] + \delta V^C(\psi)) \right] d\tilde{y} \\ = \quad & \int_{y^*}^{\infty} \left(\sum_k p_k \int_x f(\tilde{y}-x)\pi_k^1(dx) \right) V^I(\tilde{y}|\psi) d\tilde{y} \\ \geqslant \quad & V^C(\psi) \sum_k p_k \int_x (1-F(y^*-x))\pi_k^1(dx), \end{aligned}$$

where the second equality follows by multiplying and dividing by the denominator in the expression for Baye's rule. Thus, we infer from (8) that the voter's (normalized) continuation value of a challenger is at least equal to the expected payoff from the policy choices of first-period office holders, i.e.,

$$(1-\delta)V^C(\psi) \quad \geqslant \quad \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx). \tag{9}$$

Combining our observations, we have $(1-\delta)V^C(\psi) \geqslant \mathbb{E}[u(y)|\hat{x}_n]$, but the indifference condition (3) yields

$$V^C(\psi) \quad = \quad \frac{\sum_j \mu_T(j|y^*)\mathbb{E}[u(y)|\hat{x}_j]}{1-\delta} \; < \; \frac{\mathbb{E}[u(y)|\hat{x}_n]}{1-\delta},$$

a contradiction. This establishes the result. ∎

Theorem 3 implies a general limit on policy responsiveness stemming from the commitment problem of voters, regardless of the office benefit or rate of discount. The next result gives a partial strengthening by showing that for a given level of office benefit, the voter's expected payoff from the policy choices of first-term office holders—and therefore the continuation value of a challenger—is bounded *strictly* below the expected payoff from the ideal point $\hat{x}_n$ as we vary the discount factor. In fact, the statement of the result is stronger that this, in that we can allow the office benefit to become large, as long as the discount factor eventually offsets the increase in office motivation. For the result, we impose the additional Inada-type condition:

$$\text{(C7)} \qquad \text{for all } j = 1,\ldots,n, \ \lim_{x\to\infty} w_j'(x) \ = \ -\infty.$$

This standard condition is obviously satisfied in the quadratic and exponential special cases.

Theorem 4 *Assume (C1)–(C5) and (C7). For every constant $c > 0$, there is a bound $\overline{u} < \mathbb{E}[u(y)|\hat{x}_n]$ such that for all levels of office benefit $\beta \geqslant 0$ and all discount factors $\delta \in [0,1)$ satisfying $\beta\delta \leqslant c$, in every stationary electoral equilibrium ψ for parameters (β,δ), the expected payoff to the voter from policy choices of first-term office holders is below this bound, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \overline{u}.$$

Proof: To deduce a contradiction, suppose there is a constant $c > 0$ and a sequence of parameters (β,δ) such that $\beta\delta \leqslant c$ and for which the voter's expected payoff from the choices of first-term office holders approaches the expected payoff from the ideal point of the type n politician, i.e.,

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \ \rightarrow \ \mathbb{E}[u(y)|\hat{x}_n].$$

Note that the right-hand side of the first order condition in (5) is bounded, and thus by (C7), we can bound the optimal policy choices of the politicians along the sequence by some $\overline{x}$. From the argument in the proof of Theorem 3, inequality (9) holds, and thus the voter's (normalized) continuation value of a challenger has limit infimum at least equal to the expected payoff from the ideal point of the type n politicians, as

$$(1-\delta)V^C(\psi) \ \geqslant \ \sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \ \rightarrow \ \mathbb{E}[u(y)|\hat{x}_n].$$

The indifference condition (3) then implies that the posterior probability that the incumbent is type n conditional on observing y^* goes to one, i.e.,

$$\frac{p_n \int_x f(y^* - x)\pi_n^1(dx)}{\sum_k p_k \int_x f(y^* - x)\pi_k^1(dx)} \;=\; \mu_T(n|y^*) \;\to\; 1.$$

Because the equilibrium policy choices of the politicians belong to the compact interval $[0,\overline{x}]$, this implies that $y^* \to \infty$, and thus the probability of re-electing an incumbent goes to zero, i.e., for all politician types j, we have $\int_x F(y^* - x)\pi_j^1(dx) \to 1$. Letting x_j^* be the maximum of the support of π_j^1, we have $f(y^* - x_j^*) \to 0$ for each type j, and thus the right-hand side of the first order condition converges to zero when evaluated at x_j^*. It follows that $x_j^* \to \hat{x}_j$ for each type j, but then

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \;\to\; \sum_j p_j \mathbb{E}[u(y)|\hat{x}_j],$$

a contradiction. This establishes the result. ∎

To apply the previous result for a given level of office benefit, say β, we simply set $c = \beta$. The implication is that for a given benefit β, the voter's loss is not ameliorated by increased patience of citizens.

Corollary 1 *Assume (C1)–(C6), and fix the office benefit $\beta \geqslant 0$. Then there is a bound $\overline{u} < \mathbb{E}[u(y)|\hat{x}_n]$ such that for all discount factors $\delta \in [0,1)$ and every stationary electoral equilibrium ψ, the expected payoff to the voter from policy choices of first-term office holders is below this bound, i.e.,*

$$\sum_j p_j \int_x \mathbb{E}[u(y)|x]\pi_j^1(dx) \quad \leqslant \quad \overline{u}.$$

We conclude that the possibility of policy responsiveness in dynamic elections is subject to a strict bound, owing to the commitment problem of voters: if first-term office holders generated utility greater than the ideal point of the type n politicians, then voters would have an incentive to continually replace office holders after their first term in order to reap the benefit from the effort of newly elected politicians. This contrasts with the responsiveness result of Duggan and Martinelli (2015), who analyze a two-period model of elections and show that increasing office motivation leads to arbitrarily high expected policy outcomes in the first period. There, an office holder in the first period has an incentive to choose high levels of policy to increase the likelihood of a high outcome, which signals to voters that she is a high type. At the same time, the bar for re-election increases, reflecting that voters impose an arbitrarily demanding standard for re-election. These forces operate simultaneously and are balanced in such a way that all "above average" politician types choose high policies in the first period. In the two-period model, the voters do not suffer from the same commitment problem as in the infinite-horizon model: because the game ends and office holders simply choose their ideal points in the second period, there is no temptation to replace the incumbent with a fresh candidate.

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