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Asymmetric and State Dependent Dynamics in Credit and Labor Markets

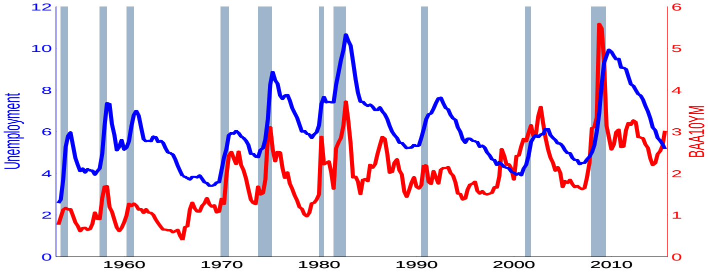
Petrosky-Nadeau, Tengelsen, and Wasmer

FRB San Francisco, CMU, and Sciences Po

Penn State - April 13, 2016

Overview - Time series asymmetries

- Unemployment and BAA- 10Y Treasury interest rate spread



  - High contemporaneous business cycle correlation
  - Similar time series asymmetry:
    - Skewness and kurtosis of deviations from trend

Overview - Time series asymmetries in the literature

Steep, deep or delayed ? - focus here on deepness, asymmetry in levels

- Hanson and Prescott (2000): capacity constraints limit booms
- Kocherlakota (2000): financial constraint amplify downturns
- Acemoglu and Scott (1997): learning by doing amplifies trough

In this paper: search and matching in labor and credit markets

- Congestion property of matching markets limits booms and amplifies downturns

Steepness - asymmetry in growth rates

- Boldrin and Levine (2001), Jovanovic (2003), Van Nieuwerburgh and Veldkamp (2006),

Overview - State dependent dynamics

Unemployment response to 1 p.p. increase in BAA-10 yr Treasury spread

- Unemployment forecast regressions with business cycle indicator
  - Unemployment moves twice as much during a recession compared to normal times
  - Little response during expansions

Overview - State dependence in the literature

U.S. time series evidence following various approaches:

- Smooth Transition VARs: Govermnent spending (Auerbach and Gorodnichenko 2012)
- Local projection approach: Jorda (2005), Ramey and Zubairy (2015)
- Overview of the empirical literature: Ramey (2016)

In this paper

- Empirics: local projection
- Model (Today): Theoretical IRFs increasing in unemployment both due to concavity of matching functions

Overview - Theory needed to account for facts

Search in the labor market:

- Diamond-Mortensen-Pissarides in a rep. agent DSGE model
- Congestion in matching:
  - Elasticity of matching to change in vacancy increases with unemployment
  - Asymmetry in hiring over the business cycle

Search in the credit market:

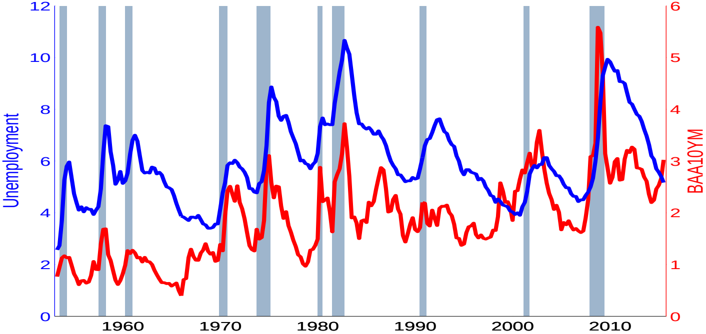
- New projects search for financial institutions
  - Additional cost to job creation
- Share the rents of production
  - Reduce the surplus of a labor match
- Financial multiplier:
  - Amplifies shocks to productivity and credit market
  - Increasing in search costs in the credit market

Overview - Taking the model to the data

- Solution method and estimation on U.S. data
  - Non-linear model solved by projection algorithm
  - Parameters estimated by Simulated Method of Moments
  - Particle filter to obtain model implied histories of productivity and credit shocks
- Quantitative results: (Preliminary)
  - Present model moments
  - State dependent IRFs
  - Shock histories and counterfactuals

Outline

Credit and labor markets - time series asymmetries



- Credit spread: Annualized return on BAA corporate bond - 10 year treasuries
- Unemployment rate: civilian population over 16

Credit and labor markets - time series asymmetries

1953:I - 2015:III	U	Spread
Standard moments		
Mean (raw, %)	6.4	2.2
Standard deviation	0.12	0.12
Higher order moments		
Skewness	**0.57**	**1.69**
Kurtosis	**3.14**	**9.77**

- Measure moments removing a HP trend
- Skewness: evidence of "deepness"
- Kurtosis: importance of rare event far from mean

Credit and labor markets - state dependence

Local projection approach, Jorda 2005:

- Run forecast resgression of different horizons h
- Horizon h regression coefficients on the variable of interest map out an empirical impulse response
- Approach permits the inclusion of an interaction term to test for state dependence
- Advantage: flexible and transparent

State dependence - local projection

$$\mathcal{U}_{t+h} = \beta_0 + \beta_{R,h}(L)R_t + \beta_{D,h}(L)D_t + \beta_{DR,h}(L)DR_t + \beta_X(L)X_t + \varepsilon_{t+h}$$

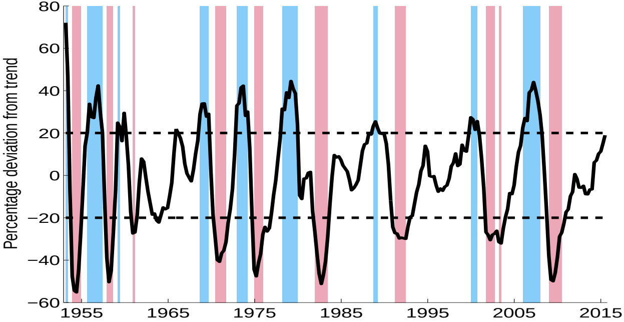
- $\mathcal{U}_{t+h}$: $h > 0$ periods ahead unemployment rate
- R_t: measure of credit spread
- D_t: dummies for state of economy
- DR_t: interaction terms between D_t and R_t
- X_t: vector of controls

Coefficients of interest: $\beta_{DR}(L)$

- Indicates whether or not credit markets move symmetrically with unemployment over the business cycle
- Trace out an empirical impulse response function

Business cycle indicator

Cyclical component of labor market tightness, $\tilde{\theta}$



- Expansion threshold: $\tilde{\theta} > 80\text{th pctl}$
- Recession threshold: $\tilde{\theta} < 20\text{th pctl}$

Alternative indicators and thresholds [Link]

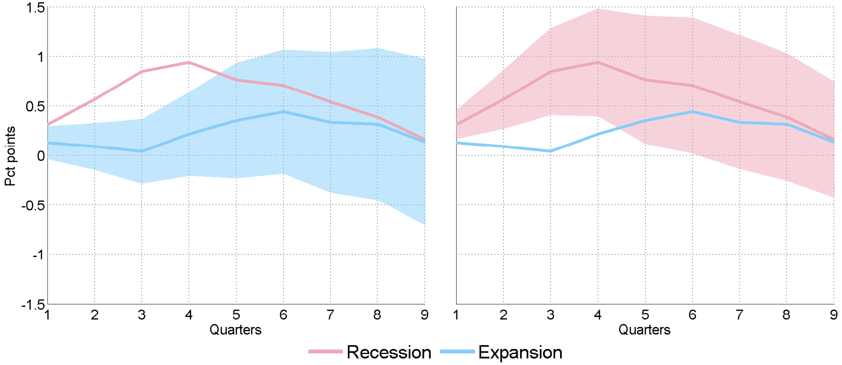
Regression results - Unemployment rate

Table: Regression results - credit market shocks and unemployment at different forecast horizons (R=BAA10YM)

Horizon:	h=1	h=2	h=3	h=4	h=5	h=6
β_R	0.255***	0.396***	0.493**	0.619***	0.552**	0.561**
	(0.082)	(0.170)	(0.220)	(0.259)	(0.327)	(0.312)
β_{R_REC}	0.053	0.166*	0.351**	0.317*	0.208	0.143
	(0.065)	(0.120)	(0.182)	(0.207)	(0.231)	(0.245)
β_{R_EXP}	-0.129	-0.307**	-0.454**	-0.405**	-0.201	-0.122
	(0.109)	(0.165)	(0.211)	(0.239)	(0.258)	(0.262)
Obs.	248	247	246	245	244	243
R^2	.98	.92	.84	.75	.69	.66

Additional Regressions [Link], F-tests [Link]

State dependence - unemployment rate



Unemployment response to a 1 p.p. increase in the credit spread

- Twice as large during a recession compared to normal times
- Results robust to alternative measures of credit spread [Link] and recession indicator [Link] or forecasting the V-U ratio [Link]

Summary

Asymmetry:

- Unemployment and measures of the spread have longer right tails (skewness) and a significant portion of the variance is attributable to infrequent large deviation (kurtosis).

State dependence:

- Labor market response when the spread increases 1 p.p. much larger if the labor market is slack.

Outline

Empirical facts

Model

Estimate and analyze the model

Discussion

Model

Three types of agents

1. Workers in a representative households - supply labor
2. Firms - produce with labor
3. Financial institutions - supply funds to firms

Two search frictional markets

1. Labor market: matching unemployed $\mathcal{U}$ and vacant jobs $\mathcal{V}$
2. Credit market: matching new projects $\mathcal{N}_c$ and financial institutions $\mathcal{B}_c$

Two measures of market tightness and Nash bargained prices:

1. Labor: tightness θ and wage W with share $\alpha_L \in (0, 1)$ to worker
2. Credit: tightness ϕ and repayment Ψ with share $\alpha_C \in (0, 1)$ to creditor

Search and matching - financial market

Firms: add production capacity (job) matching with a creditor

- Place $\mathcal{N}_{ct}$ potential projects to search at unit cost $\kappa_I > 0$
- Match with a creditor at rate p_t
- Receive funds when job is vacant to cover costs
- Share revenues during production

Financial institutions: search and manage credit market matches

- Place $\mathcal{B}_{ct}$ units of effort to search at unit cost $\kappa_{Bt} > 0$
- Match with a creditor at rate $\bar{p}_t$
- Provide funds when job is vacant to cover costs
- Receive payment from jobs in production

Search and matching - financial market

Meetings in the financial market: CRS matching function $M_c(\mathcal{B}_{ct}, \mathcal{N}_{ct})$

Contact rates - function of credit tightness $\phi_t = \mathcal{N}_{ct}/\mathcal{B}_{ct}$:

- Project meets creditor:

$$p_t = \frac{M_c(\mathcal{B}_{ct}, \mathcal{N}_{ct})}{\mathcal{N}_{ct}} = p(\phi_t) \text{ with } p'(\phi_t) < 0$$

- Creditor meets project:

$$\bar{p}_t = \frac{M_c(\mathcal{B}_{ct}, \mathcal{N}_{ct})}{\mathcal{B}_{ct}} = \bar{p}(\phi_t) \text{ with } \bar{p}_t'(\phi_t) > 0$$

Credit match separate at exogenous rate $s^C \in (0, 1)$

Search and matching - labor market

Vacant positions $\mathcal{V}$ search for the unemployed, funded by creditors :

- Have search costs $\gamma > 0$ per vacancy per period of time
- Find a worker with probability $q_t \in (0, 1)$

Unemployment workers $\mathcal{U}$ search for vacant jobs

- Enjoy leisure l and receive UI benefits b
- Find a job with probability $f_t \in (0, 1)$

Matching governed by CRS function $M_l(\mathcal{V}_t, \mathcal{U}_t)$, with tightness $\theta = \frac{\mathcal{V}}{\mathcal{U}}$

$$q_t = \frac{M_l(\mathcal{V}_t, \mathcal{U}_t)}{\mathcal{V}_t} = q(\theta_t) \text{ with } q'(\theta_t) < 0,$$
$$f_t = \frac{M_l(\mathcal{V}_t, \mathcal{U}_t)}{\mathcal{U}_t} = f(\theta_t) \text{ with } f'(\theta_t) > 0.$$

Labor market - turnover and laws of motion

Two types of turnover:

1. Labor matches separate at rate $s^L \in (0, 1)$
   - Worker becomes unemployed
   - Job becomes a vacant position
2. Credit matches separate at rate $s^C \in (0, 1)$
   - Job is destroyed and worker becomes unemployed

Law of motion for unemployment:

$$\mathcal{U}_{t+1} = \mathcal{U}_t + \left[s^C + \left(1 - s^C\right) s^L \right] \mathcal{N}_t - M_l(\mathcal{V}_t, \mathcal{U}_t)$$

Law of motion for vacancies:

$$\mathcal{V}_t = \left(1 - s^C\right) \left[(1 - q(\theta_{t-1})) \mathcal{V}_{t-1} + s^L \mathcal{N}_{t-1} \right] + M_c(\mathcal{B}_{ct}, \mathcal{N}_{ct})$$

Firm's decision problem

Choose new projects to maximize the value of the firm S_t:

$$S_t = \max_{\mathcal{N}_{ct}} \left[X_t \mathcal{N}_t - W_t \mathcal{N}_t - \Psi_t \mathcal{N}_t - \kappa_I \mathcal{N}_{ct}\right] + \mathbb{E}_t M_{t+1} \left[S_{t+1}\right]$$

$$\text{subject to :}$$

$$\mathcal{V}_t = \left(1 - s^C\right) \left[\left(1 - q(\theta_{t-1})\right) \mathcal{V}_{t-1} + s^L \mathcal{N}_{t-1}\right] + p(\phi_t) \mathcal{N}_{ct}$$

$$\mathcal{N}_{t+1} = \left(1 - s^C\right) \left[\left(1 - s^L\right) \mathcal{N}_t + q(\theta_t) \mathcal{V}_t\right]$$

- X_t: labor productivity
- W_t: wage for each $\mathcal{N}_t$ worker
- Ψ_t: repayment to each credit match currently generating revenue
- M_{t+1}: Household's stochastic discount factor between t and $t+1$

Firm marginal values: [Link]

Financial institution's decision problem

Choose effort in finding new projects to maximize its equity value B_t:

$$B_t = \max_{\mathcal{B}_{ct}} \left[\Psi_t \mathcal{N}_t - \gamma \mathcal{V}_t - \kappa_{Bt} \mathcal{B}_{ct} \right] + \mathbb{E}_t M_{t+1} \left[B_{t+1} \right]$$

$$\text{subject to :}$$

$$\mathcal{V}_t = \left(1 - s^C\right) \left[\left(1 - q\left(\theta_{t-1}\right)\right) \mathcal{V}_{t-1} + s^L \mathcal{N}_{t-1} \right] + \bar{p}(\phi_t) \mathcal{B}_{ct}$$

$$\mathcal{N}_{t+1} = \left(1 - s^C\right) \left[\left(1 - s^L\right) \mathcal{N}_t + q(\theta_t) \mathcal{V}_t \right]$$

- Ψ_t: repayment to each credit match currently generating revenue
- M_{t+1}: Household's stochastic discount factor between t and $t+1$

Marginal values: [Link]

Representative Household's decision problem

Choose consumption C_t and holding of risk free bonds A_t:

$$H_t = \max_{C_t, A_t} \left[u(C_t) + l\mathcal{U}_t \right] + \beta \mathbb{E}_t \left[H_{t+1} \right]$$

subject to :

$$W_t \mathcal{N}_t + b\mathcal{U}_t + A_{t-1}(1 + r_{t-1}) + D_t^S + D_t^B = C_t + T_t + A_t$$

Laws of motion of employed and unemployed

- β: time discount factor
- r_t : risk-free interest rate
- $D_t^S = X_t \mathcal{N}_t - W_t \mathcal{N}_t - \Psi_t \mathcal{N}_t - \kappa_I \mathcal{N}_{Ct}$: firm dividends
- $D_t^B = \Psi_t \mathcal{N}_t - \gamma \mathcal{V}_t - \kappa_{Bt} \mathcal{B}_{ct}$: financial institution dividends
- T_t: lump sum taxes

Marginal values: [Link]

Bargaining and Equilibrium in the Financial Market

First order condition of the firm and financial institution:

$$S_{ct} = 0 \quad \rightarrow \quad \frac{\kappa_I}{p(\phi_t)} = S_{lt}$$

$$B_{ct} = 0 \quad \rightarrow \quad \frac{\kappa_{B_t}}{\bar{p}(\phi_t)} = B_{lt}$$

- Value of a vacant position to each side of the credit market equal to creation (search) costs

Define the joint value of a vacant position to the firm and the creditor:

$$K_t = \frac{\kappa_I}{p(\phi_t)} + \frac{\kappa_{B_t}}{\bar{p}(\phi_t)}$$

- Increasing in the cost of search in the credit market
- In anticipation of wage bargaining: firm's outside option in bargaining with worker

Bargaining and Equilibrium in the Financial Market

Bargaining over the joint match surplus $(B_{lt} - B_{ct}) + (S_{lt} - S_{ct})$:

- Share of surplus to the creditor: $\alpha_C \in (0, 1)$
- Solve $\mathbb{E}_t [\Psi_{t+1}] = argmax\, (B_{lt} - B_{ct})^{\alpha_C} (S_{lt} - S_{ct})^{1-\alpha_C}$
- Sharing rule : $(1 - \alpha_C) B_{l,t} = \alpha_C S_{l,t}$

Equilibrium credit market tightness:

$$\phi_t = \frac{1 - \alpha_C}{\alpha_C} \frac{\kappa_{Bt}}{\kappa_I}$$

- ϕ_t decreasing in α_C: relatively more entry of creditors
- ϕ_t increasing in search costs κ_{Bt}

Bargaining and Equilibrium in the Financial Market

Equilibrium expected repayment:

$$\mathbb{E}_t\left[\Psi_{t+1}\right] = \alpha_C \mathbb{E}_t\left[X_{t+1} - W_{t+1}\right] + (1-\alpha_C)\left[\frac{\gamma}{q_t}\left(\frac{1+r_t}{1-s^C}\right) - \left(1-s^L\right)\mathbb{E}_t\left[\frac{\gamma}{q_{t+1}}\right]\right].$$

Creditor receives:

- α_C of the profit flow from labor
- more if the current costs γ/q_t - paid by the creditor in the period of price setting - are large relative to expected in the future

Bargaining and Equilibrium in the Financial Market

Expected return on loans:

$$R_t \quad = \quad \frac{\mathbb{E}_t\left[\Psi_{t+1}\right]}{\gamma/q(\theta_t)} - \left(s^C + \left(1 - s^C\right)s^L\right)$$

- Rate which sets the expected discounted value of a loan, $\frac{\gamma}{R_t+q(\theta)}$ equal to the expected discounted repayment $\frac{q(\theta)}{R_t+q(\theta)}\frac{E_t[\Psi_{t+1}]}{R_t+s^C+\left(1-s^C\right)s^L}$
- R_t strictly increasing in bargaining weight α_C

EXCESS RETURN: $R_t - R_t^0$

- R_t^0: competitive pricing in the credit market - creditor's surplus driven to 0 ($\alpha_C = 0$)

Bargaining and Equilibrium in the Labor Market

Each job is the joint interest of the firm and the creditor:

- Joint marginal value of a vacant job:

$$S_{lt} + B_{lt} \equiv F_{lt} = -\gamma + \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[q_t F_{gt+1} + (1 - q_t) F_{lt+1}\right] \quad (1)$$

- Joint marginal value of a filled job:

$$S_{gt} + B_{gt} \equiv F_{gt} = X_t - W_t + \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[(1 - s^L) F_{gt+1} + s^L F_{lt+1}\right] \quad (2)$$

- Equilibrium in the financial market determined $F_{lt} = K_t$

Bargaining and Equilibrium in the Labor Market

Job creation condition:

$$\frac{K_t + \gamma}{q(\theta_t)} = \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[F_{gt+1} + \left(\frac{1 - q(\theta_t)}{q(\theta_t)}\right) K_{t+1}\right]$$

- $\frac{K_t+\gamma}{q(\theta_t)}$: job creation costs
- F_{gt+1}: value a filled vacancy
- $\left(\frac{1-q(\theta_t)}{q(\theta_t)}\right) K_{t+1}$: present value of unfilled vacancy

Expanded JC condition: [Link]

Bargaining and Equilibrium in the Labor Market

Nash wage rule:

$$W_t = \alpha_L \left(X_t + \theta_t \left[\gamma + \left[\frac{r_t + s^C}{(1 + r_t)} \right] \frac{K_t}{(1 - s^C)} \right] \right) + (1 - \alpha_L) Z_t - \alpha_L \left[\frac{r_t + s^C}{1 + r_t} \right] K_t$$

Nash bargaining between firm and worker:

- Worker has bargaining weight $\alpha_L \in (0, 1)$
- Wage solves
$$W_t = argmax \left(\frac{H_{Nt} - H_{Ut}}{\lambda_t} \right)^{\alpha_L} \left(F_{gt} - F_{lt} \right)^{1 - \alpha_L}$$
- Wage satisfies sharing rule $\alpha_L \left(F_{gt} - K_t \right) = (1 - \alpha_L) \left(H_{Nt} - H_{Ut} \right) / \lambda_t$
- Limit $K_t \to 0 \ \forall t$: $W_t = \alpha_L \left(X_t + \theta_t \gamma \right) + (1 - \alpha_L) Z_t$

Outline

Estimation and analysis

1. Estimation by Simulated Method of Moments

2. Model moments and impulse responses

3. Non-linear Kalman filter: recovering the unobserved states

Simulated Method of Moments

$$\hat{\omega} = argmin \left(\mu - \frac{1}{S} \sum_{s=1}^{S} \mu_s(\omega) \right)' W^{-1} \left(\mu - \frac{1}{S} \sum_{s=1}^{S} \mu_s(\omega) \right)$$

- μ: vector of empirical moments of interest
- $\mu_s(\omega)$: vector of corresponding model models for a given vector of structural parameters ω
- S: number of model simulations of length T
- W: (optimal) weighting matrix, inverse of sample covariance matrix of moment condition

References: Duffie and Singleton (1993), Adda and Cooper (2003), Ruge-Murcia (2012)

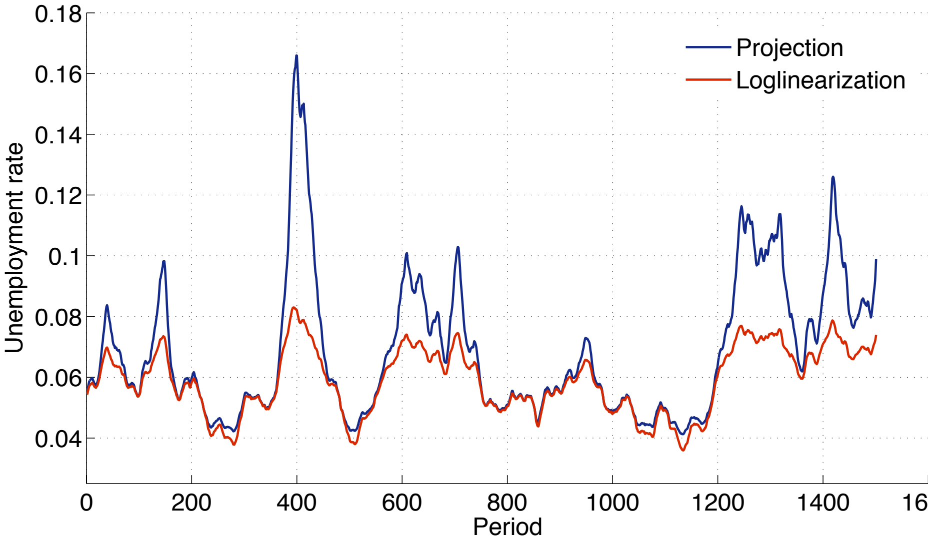
Model solution and moments

Policy function solved with projection over state space (X_t, κ_{Bt}, N_t)

- $\log(X_t)$ and $\log(\kappa_{Bt})$ discretized with 9 grid points each; cubic splines (20 basis functions) in N for each $\log(X)$ and $\log(\kappa_{Bt})$ -levels
- Model condenses to one functional equation, the job creation condition
- Our approach: solve for the conditional expectation $E_t[F_{gt+1}] \equiv \mathcal{F}(N_t, X_t, \kappa_{Bt})$ to satisfy the job creation condition
- Highly accurate method evaluated in Petrosky-Nadeau and Zhang (2013)

Average moments across 2000 simulations of length 474 (months)

Projection vs. Loglinearization



See Petrosky-Nadeau and Zhang (2016), Solving the DMP model Accurately

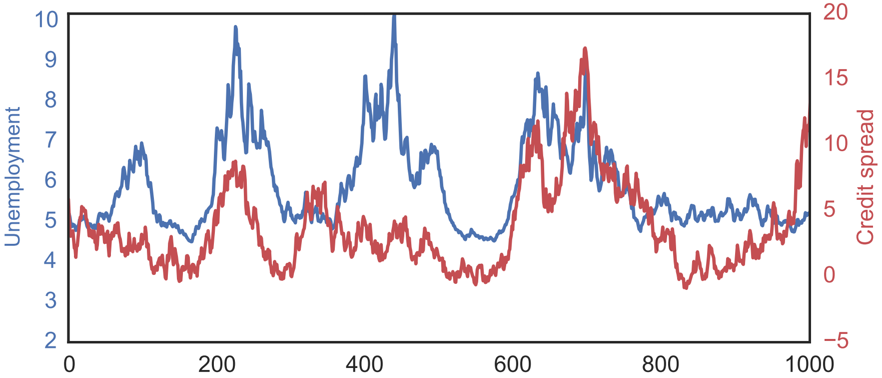
Estimation results - Data and Model Moments

		Data	Model
Mean unemployment	$mean(U)$	0.064	0.093
Unemployment volatility	σ_U	0.117	0.091
Mean vancancy rate	$mean(V_t)$	0.052	0.076
Vacancy rate volatility	σ_V	0.072	0.083
Vacancy-unem. correlation	$corr(U_t, V_t)$	-0.876	-0.379
Wage volatility	σ_V	0.010	0.045
Credit spread: mean	$mean(R_t)$	0.022	0.041
Credit spread: volatility	σ_R	0.12	0.078
Spread-unemp. correlation	$corr(U_t, R_t)$	0.448	0.192
Productivity: volatility	$std(X)$	0.008	0.009
Productivity: autocorrelation	$autocorr(X)$	0.739	0.732

Estimation results - Model parameters

	Parameter	Value	Std. Errors	Reference
Externally set:				
discount factor	β	.997	...	3 month U.S. T-bill
job-separation rate	s^L	0.032	...	JOLTS
credit separation rate	s^C	0.01/3	...	Firm exit rate
matching curvature	η_C	1.5	...	
search costs	κ_I	0.1	...	
Estimated paramaters:				
matching parameter	η_L	1.44	(...)	
worker bargaining weight	α_L	0.61	(...)	
creditor bargaining weight	α_C	0.38	(...)	
vacancy cost	γ	0.329	(...)	
non-employment value	z	0.806	(...)	
search costs	$\overline{\kappa}_B$	0.187	(...)	
persistence parameter	ρ_x	0.943	(...)	
persistence parameter	ρ_{κ_B}	0.717		
spillover parameter	ρ_{x,κ_B}	-0.147		
standard deviation	σ_x	0.009	(...)	
standard deviation	σ_{κ_B}	0.032	(...)	

Quantitative results - sample path



Sample model simulation: unemployment rate and credit spread

Quantitative results - moments

Table: Empirical and model moments

	U		Spread	
	data	model	data	model
Mean	0.064	0.093	0.022	0.041
S.d.	0.117	0.091	0.120	0.078
Skewness	0.573	0.489	1.679	0.113
Kurtosis	3.144	3.433	9.770	2.692

Quantitative results - theoretical IRFs to X shock

State dependent impulse responses:

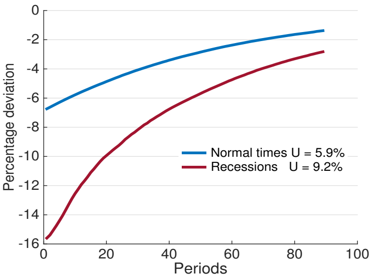
1. Labor market tightness: $\partial\theta_t/\partial\nu_{xt}$ increasing in $\mathcal{U}_t$

   $\rightarrow$ convexity of job filling rate $q(\theta_t)$

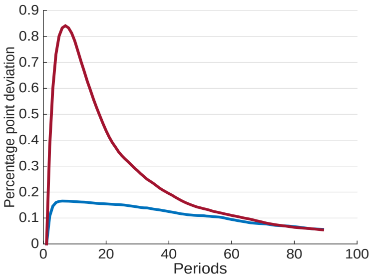
2. Unemployment: $\partial\mathcal{U}_t/\partial\theta_t$ increasing in $\mathcal{U}_t$

   $\rightarrow$ concavity and $\mathcal{U}$ $\mathcal{V}$ complementarity in $M_l(\mathcal{U}_t, \mathcal{V}_t)$

LABOR MARKET TIGHTNESS



UNEMPLOYMENT

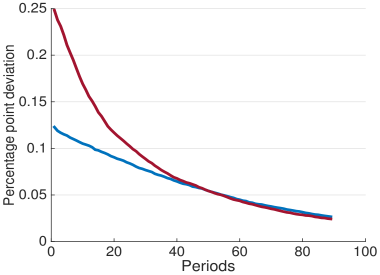


IMPULSE RESPONSES TO A PERCENTAGE DEVIATION NEGATIVE PRODUCTIVITY SHOCK

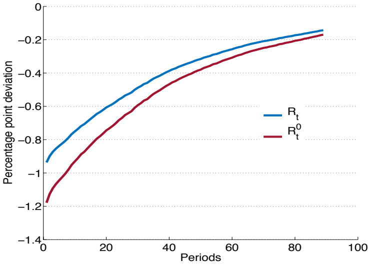
Quantitative results - theoretical IRFs to X shock

3. Credit spread $R_t - R_t^*$
   - Recall that $R_t = \frac{\mathbb{E}_t[\Psi_{t+1}]}{\gamma/q(\theta_t)} - \left(s^C + \left(1 - s^C\right) s^L\right)$
   - With bargaining power lenders receive more than the zero profit return to a project
   - Greater cushion in a recession

SPREAD $R_t - R_t^0$



R_t AND R_t^0



IMPULSE RESPONSES TO A PERCENTAGE DEVIATION NEGATIVE PRODUCTIVITY SHOCK

Quantitative results - theoretical IRFs to credit shock

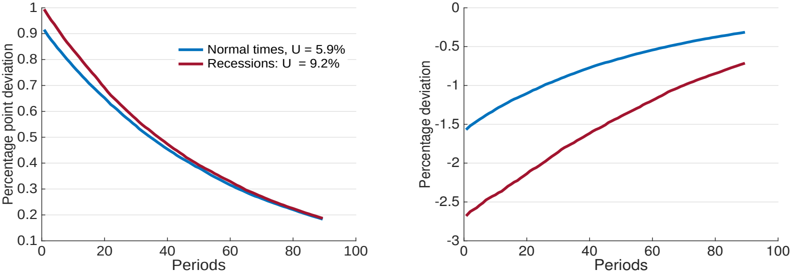
Shock to search costs κ_{Bt} for a 1 p.p. increase in credit spread:

- Two different initial U rates

- θ response 80% greater

SPREAD $R_t - R_t^*$

LABOR MARKET TIGHTNESS

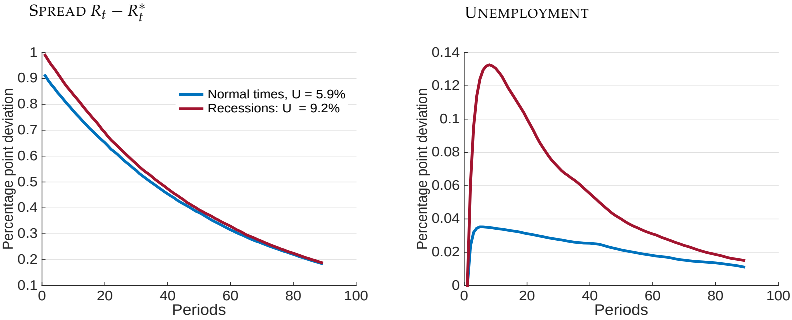


IMPULSE RESPONSES TO A FINANCIAL MARKET SHOCK

Quantitative results - theoretical IRFs to credit shock

Shock to search costs κ_{Bt} for a 1 p.p. increase in credit spread:

- Two different initial U rates
- *U* response 3.5 times larger



IMPULSE RESPONSES TO A FINANCIAL MARKET SHOCK

Particle filter

- Assess the conditional probability of date t observation of Y_t given a history of past realizations $Y^{t-1} = \{Y_j\}_{j=1}^{t-1}$:

$$L(Y_t \mid Y^{t-1})$$

- Sequence of conditional likelihoods:

$$L(Y) = \prod_{t=1}^{T} L(Y_t \mid Y^{t-1})$$

- Each assigned the likelihood of a candidate shock $\hat{\nu}_t$ by its assumed probability distribution:

$$L(Y_t \mid Y^{t-1}) = p_\nu(\hat{\nu}_t)$$

References: Fernandez-Villaverde and Rubio-Ramirez (2007), DeJong and Dave (2011)

Particle filter - no measurement error

- States and observables follow (dimension of ν_t matches the dimension of Y_t)

$$\begin{aligned} Z_t &= f(Z_{t-1}, \nu_t) \\ Y_t &= g(Z_t) \end{aligned}$$

- Conditional on the structural parameters ω, solve, in sequence:

$$Z_t = g^{-1}(Y_t) \text{ and } \nu_t = \nu(Y_t, Z_{t-1})$$

- For a given initial Z_0, use the recursion to obtain series

$$Z^T = \{Z_t\}_{t=1}^T \text{ and } \nu^T = \{\nu_t\}_{t=1}^T$$

Particle filter - no measurement error

- Construct the likelihood for Y^T conditional on Z_0:

$$L(Y^T \mid Z_0, \omega) = \prod_{t=1}^{T} p(\nu_t(Y^t, Z_0))$$

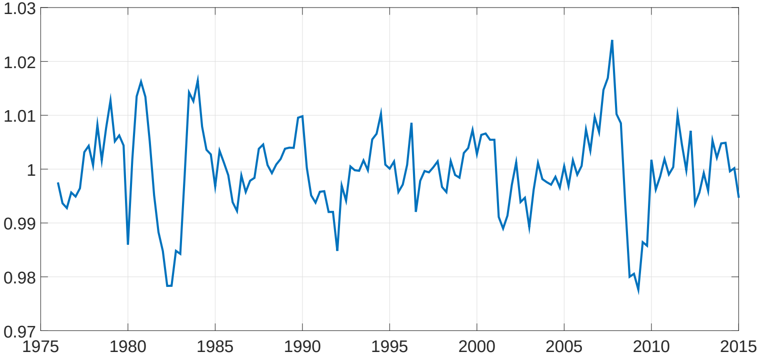
- Integrate over the model implied distribution for Z_0:

$$L(Y^T \mid \omega) = \prod_{t=1}^{T} \int p(\nu_t(Y^t, Z_0)) p(Z_0 \mid Y^t) dZ_0$$

$\Rightarrow$ Enter the particle filter

And turn to Fernandez-Villaverde and Rubio-Ramirez (2007), DeJong and Dave (2011) for details on implementation

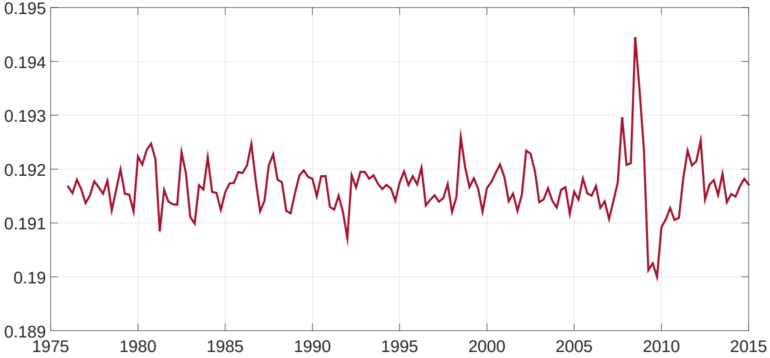
Model implied states, 1976-2015



MODEL IMPLIED PRODUCTIVITY

- Realizations symmetric around the mean

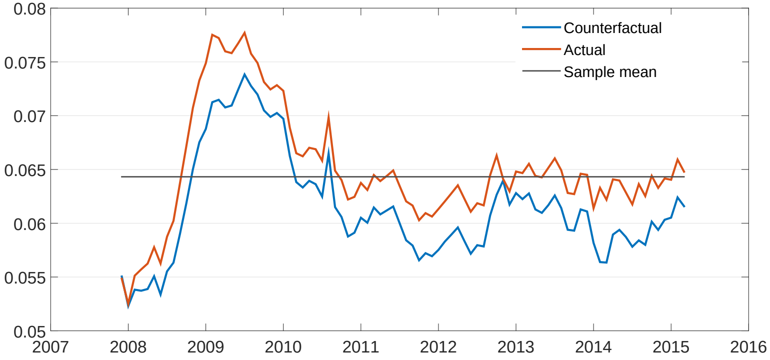
Model implied states, 1976-2015



Model implied credit search costs

- Skewness and kurtosis appear with the financial crisis

Counterfactual



Actual and counterfactual unemployment rate (detrended) fixing credit search cost from Dec. 2007 to historic mean

- Credit shocks have added, persistently, 0.5 p.p. to unemployment rate

Conclusion - work in progress

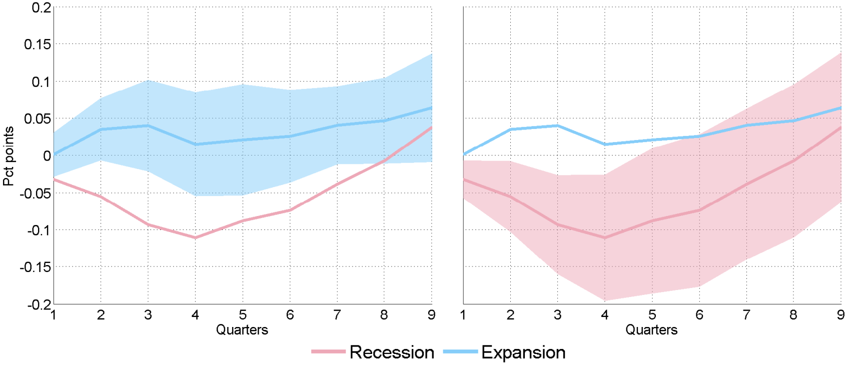
Summary:

- Asymmetry and state dependence in labor and credit market variables over the business cycle
- Arises in a macro model with search frictional credit and labor markets

To do (partial list):

1. Add higher order moments to estimation
2. Endogenize job destruction and credit destruction

State dependence - labor market tighntess



V-U ratio response to a 1 p.p. increase in the credit spread

Firm - marginal values

Additional project $\mathcal{N}_{ct}$ searching in the credit market:

$$S_{c,t} = -\kappa_I + p_t S_{l,t} + (1 - p_t)\mathbb{E}_t M_{t+1} S_{c,t+1}$$

Additional vacant position searching in the labor market:

$$S_{l,t} = \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[q_t S_{g,t+1} + (1 - q_t) S_{l,t+1}\right] + s^C \mathbb{E}_t M_{t+1} \left[S_{c,t+1}\right]$$

Additional filled position generating revenue:

$$S_{g,t} = X_t - W_t - \Psi_t + \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[\left(1 - s^L\right) S_{g,t+1} + s^L S_{l,t+1}\right] + s^C \mathbb{E}_t M_{t+1} \left[S_{c,t+1}\right]$$

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Financial institution - marginal values

Additional unit of effort $\mathcal{B}_{ct}$ searching in the credit market:

$$B_{c,t} = -\kappa_{Bt} + \bar{p}_t B_{l,t} + (1 - \bar{p}_t)\mathbb{E}_t M_{t+1} B_{c,t+1}$$

Additional vacant position searching in the labor market:

$$B_{l,t} = -\gamma + \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[q_t B_{g,t+1} + (1 - q_t) B_{l,t+1}\right] + s^C \mathbb{E}_t M_{t+1} B_{ct+1}$$

Additional filled position generating revenue:

$$B_{g,t} = \Psi_t + \left(1 - s^C\right) \mathbb{E}_t M_{t+1} \left[(1 - s^L) B_{g,t+1} + s^L B_{l,t+1}\right] + s^C \mathbb{E}_t M_{t+1} B_{c,t+1}$$

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Representative Household - marginal values

Additional unemployed worker $\mathcal{U}_t$:

$$\frac{H_{Ut}}{\lambda_t} = Z_t + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[f(\theta_t) \frac{H_{Nt+1}}{\lambda_{t+1}} + (1 - f(\theta_t)) \frac{H_{Ut+1}}{\lambda_{t+1}} \right]$$

Additional employed worker $\mathcal{N}_t$:

$$\frac{H_{Nt}}{\lambda_t} = W_t + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(1 - s^C\right) \left(1 - s^L\right) \frac{H_{Nt+1}}{\lambda_{t+1}} + \left(s^C + \left(1 - s^C\right) s^L\right) \frac{H_{Ut+1}}{\lambda_{t+1}} \right]$$

- λ_t: Lagrange multiplier on budget constraint
- $Z_t = b + l/\lambda_t$: flow utility when unemployed

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Bargaining and Equilibrium in the Labor Market

Job creation condition:

$$\frac{\Gamma_t}{q_t} = \mathbb{E}_t M_{t+1} \left[X_{t+1} - W_{t+1} + \left(1 - s^C\right) \left[\left(1 - s^L\right) \frac{\Gamma_{t+1}}{q_{t+1}} + s^L K_{t+1} \right] \right]$$

- $\Gamma_t = \frac{K_t + \gamma}{(1 - s^C)} - (1 - q_t)\, \mathbb{E}_t M_{t+1} K_{t+1}$

- Limit as $s^C = 0$ and $K_t \to 0 \forall t$:

$$\frac{\gamma}{q_t} = \mathbb{E}_t M_{t+1} \left[X_{t+1} - W_{t+1} + \left(1 - s^L\right) \frac{\gamma}{q_{t+1}} \right]$$

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