Abstract

I study unemployment insurance as a discretionary policy instrument in an environment with incomplete markets, search frictions, and nominal rigidities. An increase in generosity can be expansionary if the unemployed have a higher marginal propensity to consume or agents engage in precautionary saving. The resulting aggregate demand externality motivates optimally higher generosity if the economy is inefficiently slack. I calibrate the model to match U.S. patterns in wealth, income, and employment. In a labor market like that of 2008-09, temporarily extending benefit duration reduces the unemployment rate and raises utilitarian social welfare through these channels.

JEL codes: D52, D62, E21, E62, J64, J65

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1 Introduction

Economists have long viewed unemployment insurance (UI) as an important automatic stabilizer — but should it also serve as a discretionary tool in the stabilization of short-run fluctuations? Since the 1950s, policymakers in the United States have treated UI generosity as precisely such an instrument, routinely extending benefits in recessions. This practice was expanded in unprecedented and controversial fashion during the Great Recession, when benefit durations were raised almost four-fold at the depth of the downturn. While critics emphasized the costly supply-side effects of more generous UI, supporters pointed to potential stimulus benefits of transfers to the unemployed.\footnote{See Summers [2010], Congressional Budget Office [2012], and Blanchard et al. [2013] for examples of recent commentary emphasizing the potential stimulus from UI.}

The existing analysis of UI in the literature cannot speak fully to this debate because it has largely ignored these potential interactions between UI and aggregate demand. Most prior work has studied UI in partial equilibrium, while analyses in general equilibrium have focused on environments in steady-state or in the real business cycle tradition. This paper studies UI in a general equilibrium framework with macroeconomic shocks and nominal rigidities. In this setting, I analytically characterize the effects of UI on equilibrium employment and welfare, and quantitatively study these effects in simulations of UI policy.

I find that the interaction between UI and aggregate demand naturally motivates a role for higher generosity when the economy is slack. A contemporaneous increase in UI generosity can be expansionary if the unemployed have a higher marginal propensity to consume (MPC) than the employed. An expected future increase in UI generosity can be expansionary if agents have a precautionary saving motive in light of future income risk. Both channels generate an aggregate demand externality from transfers when the economy is inefficiently slack, implying optimal generosity which is higher than the classic formula from public finance would imply. Calibrating the model to match features of U.S. wealth, income, and employment suggests meaningful stimulus through these channels. In a macroeconomic environment close to that of the U.S. in 2008-09, extending benefits from 6 to 9 months for one year reduces the unemployment rate by an average of 0.04–0.07pp in the year of extended benefits and raises utilitarian social welfare.\footnote{Linearly extrapolating, this implies that extending benefits for one year from 26 to 99 weeks, their maximal duration during the Great Recession, reduces unemployment by 0.2–0.4pp through these channels.}

Several real and nominal frictions interact to set the stage for the paper’s novel results. First, search and matching frictions in the tradition of Diamond [1981], Mortensen [1982], and Pissarides [1984] give rise to involuntary unemployment, with unobservable worker search intensity in the matching process leading to a moral hazard cost of UI. Second, market in-
completeness with respect to unemployment risk, building on Bewley [1983], Huggett [1993], and Aiyagari [1994], generates consumption insurance gains from the public provision of UI. Third and most novel to this paper, nominal rigidities render the level of production partially demand-determined. Despite the richness of these frictions, I can obtain analytical insights regarding the effects of UI by first considering a stylized economy with three periods, and where the focus is on changes in UI in the second period.

In this environment, my first set of results demonstrate that nominal rigidities can reverse the conventional effects of UI on macroeconomic aggregates. A budget-balanced increase in transfers will raise the aggregate demand for consumption in the same period if the unemployed have a higher MPC than the employed, and will raise the aggregate demand for consumption in the prior period by reducing agents’ incentive to precautionary save. In the presence of nominal rigidities and monetary policy which does not respond to the change in transfers, these initial impulses can raise equilibrium tightness, employment, and output. These outcomes obtain despite the presence of moral hazard, as the reduction in unemployed workers’ search intensity is offset by a general equilibrium response in firms’ posted vacancies to meet the desired change in aggregate consumption.

In terms of social welfare, I find that the resulting aggregate demand externality from transfers can raise the optimal generosity of UI relative to the insurance-incentive benchmark from public finance. I first demonstrate that in a frictional labor market, macroeconomic inefficiency is usefully summarized by a wedge in surplus-sharing between workers and firms much like the conventional labor wedge captures inefficiency in a neoclassical setting. Absent such wedges, the Ramsey optimal generosity of UI balances the welfare gain from consumption insurance with the disincentive cost from moral hazard as in the classic partial equilibrium analysis. But in the presence of non-zero wedges signifying depressed demand, the social value of UI will exceed its private insurance value when UI can stimulate aggregate consumption through redistribution and precautionary saving as outlined above. Non-zero wedges and a resulting role for UI in macroeconomic stabilization arise naturally when monetary policy faces constraints, such as a zero lower bound.

To assess the strength of these channels, I then calibrate an enriched version of the model to the U.S. economy. Enriching the framework to an infinite horizon allows me to more credibly explore the dynamic interactions between incomplete markets and endogenous unemployment in shaping the effects of finite-duration UI policy. The model is calibrated to match U.S. data on the distribution of liquid wealth, the variance and persistence of shocks to income conditional on employment, the hazard rates out of unemployment by duration, and the (micro) disincentive elasticity to UI generosity. Analyzing the steady-state of the calibrated model, I find that a series of untargeted moments — wealth decumulation
among the unemployed, consumption declines among the unemployed, and the economy-wide average MPC — are consistent with available evidence.

Analyzing UI policy, I find that the implied pattern of MPCs and precautionary saving implies meaningfully positive effects of UI extensions on employment and welfare. Starting from steady-state, I consider an unanticipated shock to the economy’s average discount factor which, together with rigid prices and an assumed monetary policy rule, induces a binding zero lower bound and a path of long-term unemployment roughly consistent with that of the U.S. from July 2008 - June 2009. In this environment a one-year increase in UI duration from 6 months to 9 months reduces the unemployment rate by an average of 0.04–0.07pp in the year of extended benefits, with an associated contemporaneous output multiplier of 0.5–1.0. The extensions further raise utilitarian social welfare, with broadly shared welfare gains due to the demand externality from transfers. Consistent with the analytical formulas, the stimulus from UI is on the higher end of the simulated range when the steady-state calibration implies MPCs which rise more sharply with duration of unemployment or when agents have a higher degree of prudence. These results reverse those obtained under flexible prices, where disincentives dominate and generate a reduction in social welfare.

The analytical results of this paper bridge distinct literatures on optimal UI and second-best policies when monetary policy is constrained. It is especially related to the work of Landais et al. [2016a] from the first and Farhi and Werning [2016] from the second. Landais et al. [2016a] generalize the classic formulas of Baily [1978] and Chetty [2006] to account for the effects of UI on labor market tightness in general equilibrium. In their framework, UI can raise tightness and welfare through a “rat race effect” when jobs are rationed. Nominal rigidities render this search externality operative in my setting, but UI can also raise tightness and welfare through an aggregate demand externality in the class identified by Farhi and Werning [2016]. Conversely, my analysis demonstrates that a wedge between the private and social valuation of transfers arising from such a demand externality, identified by Farhi and Werning [2016] in their study of macroprudential policy, naturally applies to social insurance.

The quantitative analysis of this paper departs from most existing analyses of UI because of my focus on nominal rigidities; in doing so, my paper provides a new perspective on macroeconomic policy in incomplete markets New Keynesian economies. Krusell et al. [2010], Nakajima [2012], and Mitman and Rabinovich [2015] analyze UI in calibrated models with frictions in asset and labor markets, but with business cycle dynamics in the real business cycle tradition. My analysis shows that moving from flexible to sticky prices reverses the employment and welfare effects of UI. Part of the stimulus from UI arises from a positive feedback loop between lower precautionary saving, higher aggregate demand, and lower unemployment risk, as is also emphasized in Ravn and Sterk [2014], Challe et al. [2014], den
Haan et al. [2015], and Werning [2015]. Relative to these papers, as well as the analysis of automatic stabilizers in McKay and Reis [2016a,b], my analysis is distinguished by its focus on discretionary changes in the duration of UI benefits as the primary instrument of interest.

Finally, my results provide a theoretical counterpart to research using quasi-experimental variation to estimate the macroeconomic effects of UI, including the work of Hagedorn et al. [2016a], Hagedorn et al. [2016b], Coglianese [2015], di Maggio and Kermani [2016], and Chodorow-Reich and Karabarbounis [2017]. Researchers have estimated both expansionary and contractionary effects of UI in this literature. My results do not resolve this debate because they focus on understanding and quantifying a set of channels in the presence of nominal rigidities, with the appropriate model for and degree of these rigidities left outside the scope of my analysis. Rather, my contribution is to demonstrate that estimates of the expansionary effects of UI in this literature are consistent with a quantitative framework accounting for the redistribution and precautionary saving effects of UI.

The rest of the paper is structured as follows. I derive analytical results on the effects of UI in a three-period economy with incomplete markets, search frictions, and nominal rigidities in section 2. I enrich this framework in section 3 to assess the quantitative magnitudes of these channels in a more realistic dynamic setting. Finally, in section 4 I conclude.

2 Analytical insights in a three-period model

In this section I analytically study UI in a three-period general equilibrium model with nominal rigidities. Positively, I find that an increase in transfers to the unemployed can raise employment and output because of heterogeneity in MPCs by employment status and agents’ precautionary saving motive given future income risk. Normatively, I find that the resulting aggregate demand externality raises the optimal generosity of UI relative to the classic trade-off between insurance and incentives obtained in public finance.

2.1 Environment and equilibrium

Consider a closed economy with three periods: 0, 1, and 2. Period 1 features a frictional labor market, missing private markets to insure against unemployment risk, and government-provided UI. Period 0 is a neoclassical production economy with no such frictions, and period 2 is a simple endowment economy. I include both periods in the model because they give rise to a meaningful consumption-savings decision in the period before unemployment risk materializes, and in the period such risk is resolved, respectively. Any aggregate risk will be resolved at the beginning of period 0, such that an announced change to period 1 UI can be
interpreted as a discretionary response to the realization of the aggregate state.

2.1.1 Primitives and optimization

In the specification of primitives and agents’ optimization problems below, upper case variables will be nominal, with money serving (only) as the economy’s unit of account.\(^3\)

**Worker-consumers.** Measure one worker-consumers make consumption, savings, and labor supply decisions in periods 0 and 1. In both of these periods, each agent \(i\) consumes a CES aggregator of varieties

\[
    c^i_t = \left[ \int_0^1 (c^i_{tj})^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \quad t \in \{0, 1\}, \tag{1}
\]

while the consumption good in period 2 will simply be the same as the traded endowments. Given the standard solution to workers’ lower-stage optimization problem given CES preferences, we can define the price index

\[
    P_t = \left[ \int_0^1 P_{tj}^{1 - \varepsilon} dj \right]^{\frac{1}{1 - \varepsilon}}, \quad t \in \{0, 1\},
\]

and I focus on workers’ upper-stage problem here.

In period 0 agents are identical and the labor market is neoclassical. The representative agent receives flow utility \(u_0(c_0, h^s_0)\) given its choice of consumption \(c_0\) and hours to supply \(h^s_0\). Importantly, while the agent can trade a risk-free bond and shares in the economy’s firms at date 0, it is unable to trade claims on the risk of becoming unemployed (with probability \(\delta\)) in the frictional labor market of period 1. With discount factor \(\beta\), the agent thus faces

\[
    v_0 = \max_{c_0, h^s_0, z_1} u_0(c_0, h^s_0) + \beta \left[(1 - \delta)\tilde{v}_1^u(z_1) + \delta \tilde{v}_1^e(z_1)\right] \text{ s.t.} \tag{2}
\]

\[
    (RC)_1 : P_0c_0 + M_0P_1z_1 = W_0h^s_0 + \Pi'_0 + \int_0^1 \Pi'^R_0dj - T^R_0,
\]

where profits \(\Pi'_0\) and \(\Pi'^R_0\) and the lump-sum tax \(T^R_0\) are described below, \(M_0\) is the price of a risk-free bond paying one unit of the numeraire in period 1, and \(z_1\) denotes the combined position of the agent in the bond and equity markets (using the fact that, without aggregate risk between periods 0 and 1, arbitrage equalizes the return on the bond and equity claim).

At the start of period 1, some agents are matched with a job \((e)\), but others are unmatched \((u)\). Unmatched agents need to supply search effort \(s_1\) to find a job, with job-finding

\(^3\)Following Woodford [2003], I model the economy at the “cashless limit”.
probability per unit search effort $p(\theta_1)$ and non-pecuniary cost of search effort $\psi(s)$ which is increasing and convex ($\theta_1$ denotes labor market tightness, and is characterized further below).\(^4\) Hence, agents’ beginning of period value functions are given by

$$v^e_1(z_1) = v^e_1(z_1),$$
$$v^u_1(z_1) = \max_{s_1} (p(\theta_1)s_1)v^e_1(z_1) + (1 - p(\theta_1)s_1)v^u_1(z_1) - \psi(s_1).$$\(^3\)

Finally, in the middle of period 1, some agents will be employed ($e$) and others will be unemployed ($u$). With periods 1 and 2 consumption $\{c^i_1, c^i_2\}$ for agent $i \in \{e, u\}$, the agent receives utility $u^i(c^i_1, c^i_2)$, accommodating potential non-pecuniary costs of employment. The agent earns income $Y^i_1$ in period 1 which I characterize after specifying government policy, and earns an endowment $y^i_2$ in period 2 which can depend on the agent’s employment status in period 1 (capturing the persistent component of income differences between the employed and unemployed). Carrying wealth $z_1$ into the period, the agent thus faces

$$v^i_1(z_1) = \max_{c^i_1, c^i_2, z^i_2} u^i(c^i_1, c^i_2),$$
$$\text{subject to } (RC)^i_1 : P_1c^i_1 + M_1P_2z^i_2 \leq Y^i_1 + P_1z_1,$$
$$\text{and } (RC)^i_2 : P_2c^i_2 \leq P_2y^i_2 + P_2z^i_2,$$\(^4\)

where $z^i_2$ is the agent’s position in a risk-free bond paying one unit of the numeraire in period 2, and $M_1$ is the price of this bond in period 1.\(^5\)

**Producers.** Measure one producers of a homogenous intermediate good operate in periods 0 and 1. They are perfectly competitive in their product market and symmetric at each date, so we focus on the representative producer. Since it cannot affect its period 1 profits in period 0, we can focus on its period-by-period problem.\(^6\)

In period 0, the representative producer demands hours $h^d_0$ from workers at prevailing wage rate $W_0$ per hour. With a non-increasing-returns-to-scale production technology $f_0(\cdot)$ and price of intermediate goods $P^I_0$, it faces

$$\Pi^I_0 = \max_{h^d_0} P^I_0 f_0(h^d_0) - W_0h^d_0.$$\(^5\)

At the end of period 0, a fraction $\delta$ of workers exogenously separate from the firm, and in

\(^4\)Matched agents could choose to quit, though I will restrict attention to parameters such that the value of being employed exceeds that of being unemployed (equivalently, such that unmatched agents choose $s_1 > 0$).

\(^5\)In appendix B, I show that my results are robust to borrowing constraints at date 1.

\(^6\)This is no longer true in the infinite horizon environment studied in section 3, where incumbent employment is a state variable owing to frictional labor markets in each period.
period 1 the firm must post vacancies $\nu_1$ to re-hire (some of) these workers in the frictional labor market. A single vacancy requires that $k$ incumbent workers engage in recruiting rather than production, and has an expected yield $q(\theta_1)$. Suppose incumbents and new workers are paid the same wage $W_1$, discussed further below, and for simplicity assume hours per worker are fixed at 1 at this date. With intermediate good price $P^I_1$ and non-increasing-returns-to-scale technology $f_1(\cdot)$ (perhaps identical to $f_0(\cdot)$), the firm then faces

$$\Pi^I_1 = \max_{\nu_1} P^I_1 f_1 (1 - \delta - k\nu_1 + q(\theta_1)\nu_1) - W_1 (1 - \delta + q(\theta_1)\nu_1).$$  \hspace{1cm} (6)

**Retailers.** Measure one retailers operate in periods 0 and 1, purchasing the intermediate good and selling a differentiated variety to consumers. Retailers are monopolistically competitive in their product market, allowing me to accommodate nominal rigidity in prices.\(^7\) As is standard in the New Keynesian literature, I will assume retailers face a fixed ad-valorem tax $\tau^R_0 = \tau^R_1 = -\frac{1}{\varepsilon}$ on their purchase of intermediate goods, which the government balances with lump-sum taxes $T^R_0$ and $T^R_1$ on households in proportion to their holding of equity.\(^8\)

When retailers can choose prices each period, they face a sequence of static problems. Anticipating the residual demand for variety $j$, in period $t \in \{0, 1\}$ retailer $j$ faces

$$\Pi^R_{tj} = \max_{P_{tj}, y_{tj}, x_{tj}} P_{tj} y_{tj} - (1 + \tau^R_t)P^I_t x_{tj} \text{ s.t.}$$

$$(Tech)_{tj} : y_{tj} = x_{tj},$$

$$(Demand)_{tj} : y_{tj} = \left(\frac{P_{tj}}{P_t}\right)^{-\varepsilon} c_t,$$  \hspace{1cm} (7)

where $y_{tj}$ denotes the final good produced, $x_{tj}$ denotes the intermediate good purchased, $P_{tj}$ is the chosen price, and $c_t$ is aggregate consumption at date $t$.

**Policy.** Beyond $\{\tau^R_0, T^R_0, \tau^R_1, T^R_1\}$, which I assume are passively set, I focus on six other instruments available to policymakers. In the labor market in period 1, they can assess a lump-sum tax $t_1$ and ad-valorem payroll tax $\tau_1$ on employed workers and provide a lump-sum payment $b_1$ to unemployed workers. To conduct monetary policy, they can set the price of risk-free nominal bonds $M_0$ and $M_1$ (equivalently, set the nominal interest rates $i_0 \equiv \frac{1}{M_0} - 1$ and $i_1 \equiv \frac{1}{M_1} - 1$) and can choose the final period price level $P_2$.\(^9\)

\(^7\)My results are robust to nominal rigidity in wages instead, as discussed in appendix B.

\(^8\)None of my results are substantively affected by the existence of this retailer tax (really a subsidy). Indeed, because policymakers can also set the payroll tax $\tau_1$, the retailer tax $\tau^R_1$ does not change the set of implementable allocations and is thus redundant. I include it because a unitary gross mark-up in the flexible price and wage allocation simplifies the exposition of that model, discussed further in appendix B.

\(^9\)Policymakers’ control over $P_2$ is needed to ensure price level determinacy in this finite horizon environment. Implementation could be achieved by adjusting the (unmodeled) final period money supply, just as
Given policy, agents’ incomes in period 1 are thus

\[ Y^e_1 = (1 - \tau_1) W_1 - P_1 t_1, \]
\[ Y^u_1 = P_1 b_1. \]

**Wages.** Finally, I assume that the wage indeterminacy implied by labor market frictions in period 1 is resolved by the function

\[
\frac{W_1}{P_1} = (1 - \varphi) \bar{w}_1 + \varphi \left[ \frac{P_1'}{P_1} f'_1(\cdot) - \frac{1}{1 - \tau_1} \frac{1}{\phi} \frac{1}{\frac{\partial v_e^1}{\partial \nu^1_i}} (v^e_1 - v^u_1) \right]
\]

for \( \varphi \in [0, 1] \), \( \phi \in (0, 1) \), and some fixed \( \bar{w}_1 \). In appendix B I demonstrate that this function nests three leading cases of interest. When \( \varphi = 1 \) and \( \phi = 1 - \eta \), where \( \eta \) is the elasticity of job-finding with respect to tightness as described further below, this is the equilibrium wage resulting from competitive search. When \( \varphi = 1 \) and the production function is characterized by constant returns-to-scale, this is the equilibrium wage resulting from Nash bargaining with worker share \( \phi \). Finally, when \( \varphi = 0 \), this is consistent with a rigid real wage \( \bar{w}_1 \).\(^{10}\)

2.1.2 Market clearing and equilibrium

Asset market clearing in each period is given by

\[
z_1 = \frac{1}{P_1} \left( \Pi'_1 + \int_0^1 \Pi_{ij}^R dj - T^R_1 \right),
\]

\[
(1 - \delta + \delta p(\theta_1) s_1) z^e_2 + \delta (1 - p(\theta_1) s_1) z^u_2 = 0,
\]

where in the first case I have combined market clearing in bonds and equity. Since all agents have identical wealth entering period 1, and conditional on employment status have identical wealth entering period 2, I drop these as state variables in the notation which follows.

In the labor market, period 0 equilibrium in hours requires

\[
h^d_0 = h^s_0.
\]

In period 1, new hires result from an aggregate matching function

\[
m(\bar{s}_1, \bar{\nu}_1) = \bar{m} s_1^{1-\eta} \nu_1^n
\]

Implementation of a path of nominal interest rates could be achieved by adjusting the path of money supply.\(^10\)

\(^{10}\)In each case I assume for simplicity that incumbent and new workers are paid the same wage. Wage dispersion affected by UI would add another consideration to the positive and normative analysis here.
given aggregate search effort of initially unemployed workers $\tilde{s}_1$ and aggregate vacancies posted by firms $\tilde{\nu}_1$. Labor market tightness $\theta_1 \equiv \frac{\tilde{\nu}_1}{\tilde{s}_1}$ then determines the vacancy-filling probability perceived by firms

$$m(\tilde{s}_1, \tilde{\nu}_1) = \tilde{m}\theta_1^{n-1} \equiv q(\theta_1),$$

and the job-finding probability per unit search perceived by unemployed agents

$$m(\tilde{s}_1, \tilde{\nu}_1) = \tilde{m}\theta_1^q \equiv p(\theta_1).$$

Equilibrium in the period 1 labor market requires that tightness is consistent with individual behavior and the existence of a representative producer and unemployed agent:

$$\theta_1 = \frac{\nu_1}{\delta s_1}. \quad (11)$$

Intermediate goods market clearing in the first two periods is given by

$$\int_0^1 x_{0j} dj = f_0(h_0^d),$$

$$\int_0^1 x_{1j} dj = f_1(1 - \delta - k\nu_1 + q(\theta_1)\nu_1), \quad (12)$$

and final goods market clearing in each period is given by

$$c_{0j} = y_{0j} \ \forall j,$$

$$(1 - \delta + \delta p(\theta_1)s_1)c_{1j}^e + \delta (1 - p(\theta_1)s_1)c_{1j}^u = y_{1j} \ \forall j,$$  

$$(1 - \delta + \delta p(\theta_1)s_1)c_{2j}^e + \delta (1 - p(\theta_1)s_1)c_{2j}^u = (1 - \delta + \delta p(\theta_1)s_1)y_{2j}^e + \delta (1 - p(\theta_1)s_1)y_{2j}^u. \quad (13)$$

Finally, budget balance for the government requires

$$(1 - \delta + \delta p(\theta_1)s_1) [P_1 t_1 + \tau_1 W_1] = \delta (1 - p(\theta_1)s_1) P_1 b_1, \quad (14)$$

$$\tau_t^R \int_0^1 x_{1j} dj + T_t^R = 0, \ t \in \{0, 1\}. \quad (15)$$

We are now ready to define a flexible price and wage equilibrium:

**Definition 1.** A flexible price and wage equilibrium is an allocation and set of nominal prices and wages such that, given policy, workers solve their standard lower-stage problems given (1) and upper-stage problems (2)-(4); producers solve (5) and (6); retailers solve (7);
period 1 wages satisfy (8); markets clear according to (9)-(13); and the government’s budget is balanced according to (14) and (15).

I study UI in the flexible price and wage equilibrium in appendix B. I demonstrate that when the payroll tax can be used to induce efficient tightness in the labor market, optimal UI is characterized by a general equilibrium analog of the Baily [1978]-Chetty [2006] formula from public finance. This mirrors the benchmark result in Landais et al. [2016a].

Here, I move directly to my novel characterization of UI in a macroeconomic environment with nominal rigidities. Suppose that prior to period 0, all retailers have posted prices

$$P_{tj} = \bar{P}$$

(16)

for all $j$ and $t \in \{0, 1\}$. Then we have:

**Definition 2.** A fully sticky price equilibrium is an allocation and set of nominal prices and wages such that, given policy, retailers accommodate desired demand for their varieties provided they can earn non-negative profits at prices (16), and all other components of the equilibrium definition are as in Definition 1.

### 2.1.3 Characterizing equilibrium

Define real cash-on-hand for agents in period 1

$$y^e_1 \equiv \frac{Y^e_1}{P_1} + z_1,$$

$$y^u_1 \equiv \frac{Y^u_1}{P_1} + z_1,$$

given nominal incomes $Y^e_1, Y^u_1$ defined earlier. And define the inverse real interest rates

$$m_0 \equiv M_0 \frac{P_1}{\bar{P}_0} = M_0,$$

$$m_1 \equiv M_1 \frac{P_2}{P_1} = M_1 \frac{P_2}{\bar{P}},$$

given nominal bond prices $M_0, M_1$ and sticky prices from (16). UI generosity $b_1$ will affect macroeconomic outcomes through $y^u_1$. Because of sticky prices, monetary policy $\{M_0, M_1, P_2\}$ will directly change $m_0$ and $m_1$, the relative prices of future consumption.

Working backwards, (4) implies demand functions $c^i_t(m_1, y^i_t)$ and indirect utility functions $v^i_t(m_1, y^i_t)$ for $t \in \{1, 2\}$ and $i \in \{e, u\}$.

(3) then implies search effort $s_1(m_1, y^e_1, y^u_1, \theta_1)$.

These functions also depend on period 2 endowments $y^e_2$, but these are exogenous so I suppress them.
Goods market clearing in periods 1 and 2 requires zero excess supply

\[ x_1(s_1(m_1, y^u_1, y^u_1, \theta_1), \theta_1, c_1^e(m_1, y^e_1), c_1^u(m_1, y^u_1)) = 0, \]  
\[ x_2(s_1(m_1, y^u_1, y^u_1, \theta_1), \theta_1, c_2^e(m_1, y^e_1), c_2^u(m_1, y^u_1)) = 0, \]

where excess supply in each period is defined by

\[ x_1(s_1, \theta_1, c_1^e, c_1^u) \equiv f(1 - \delta + \delta(p(\theta_1)s_1 - k\theta_1s_1)) \]
\[ - (1 - \delta + \delta p(\theta_1)s_1)c_1^e - \delta(1 - p(\theta_1)s_1)c_1^u, \]
\[ x_2(s_1, \theta_1, c_2^e, c_2^u) \equiv (1 - \delta + \delta p(\theta_1)s_1)y^e_2 + \delta(1 - p(\theta_1)s_1)y^u_2 \]
\[ - (1 - \delta + \delta p(\theta_1)s_1)c_2^e - \delta(1 - p(\theta_1)s_1)c_2^u. \]

The Euler equation for the representative agent solving (2) is

\[ \frac{\partial u_0}{\partial c_0}(c_0, h_0) = \frac{\beta}{m_0} \left[ (1 - \delta + \delta p(\theta_1)s_1(m_1, y^e_1, y^u_1, \theta_1)) \frac{\partial v_1^e}{\partial y_1^e}(m_1, y^e_1) \right. \]
\[ + \left. \delta(1 - p(\theta_1)s_1(m_1, y^e_1, y^u_1, \theta_1)) \frac{\partial v_1^u}{\partial y_1^u}(m_1, y^u_1) \right], \]

and market clearing in period 0 requires

\[ c_0 = f_0(h_0). \]

Note that conditions (17) and (18) implicitly define \( y^e_1(m_1, y^u_1, \theta_1, m_1, y^u_1) \) and \( s_1(m_1, y^u_1) \). Conditions (19) and (20) then define \( c_0(m_0, m_1, y^u_1) \) and \( h_0(m_0, m_1, y^u_1) \). These conditions will prove useful in studying the positive and normative effects of UI, to which I now turn.

### 2.2 Positive impact of UI

I first characterize the positive effects of UI in the presence of nominal rigidities and fixed monetary policy. I find that an increase in transfers can be expansionary if the unemployed have a higher MPC out of income than the employed or if agents have a precautionary saving motive given future income risk. A reduction in worker search will be offset by the general equilibrium response of vacancy-creation to meet the desired demand for consumption.

To characterize the effects of UI, I will focus on exogenous changes in unemployed agents’ cash-on-hand \( y^u_1 \) rather than the underlying policy parameter \( b_1 \). These are related by

\[ y^u_1 = b_1 + \pi_1, \]
where \( \pi_1 \equiv \frac{1}{T_i}(\Pi_1^I + \int_0^1 \Pi_1^R dj - T_i^R) \) is the real value of firm dividends net of the tax financing the retailer subsidy. Focusing on the comparative statics of (17)-(20) with respect to \( \pi_i^a \) thus allows us to understand the equilibrium effects of changing \( b_1 \) independent of the specific assumptions made on wage determination and the resulting behavior of firm profits. The marginal effects of \( \pi_i^a \) will furthermore play a key role in the formula for optimal \( b_1 \) developed in the next subsection, owing to an equivalence of implementable allocations using \( \{b_1, t_1\} \) and using \( \{\pi_i^a, \pi_i^c\} \).

### 2.2.1 Contemporaneous effects of transfers

I begin by characterizing the effects on period 1 aggregates, holding fixed \( m_1 \) set by monetary policy. Differentiating (17) and (18) with respect to \( y_i^u \) yields the following useful result.

**Lemma 1.** In period 1, the responses of equilibrium tightness and search are related by

\[
\begin{align*}
\frac{\partial \theta_1(m_1, y_i^u)}{\partial y_i^u} &= \mu_{\theta_1}^{y_i^u} + \mu_{\theta_1}^{s_1} \frac{\partial s_1(m_1, y_i^u)}{\partial y_i^u}, \\
\frac{\partial s_1(m_1, y_i^u)}{\partial y_i^u} &= \mu_{s_1}^{y_i^u} + \mu_{s_1}^{\theta_1} \frac{\partial \theta_1(m_1, y_i^u)}{\partial y_i^u},
\end{align*}
\]

for coefficients \( \mu_{\theta_1}^{y_i^u}, \mu_{\theta_1}^{s_1}, \mu_{s_1}^{y_i^u}, \) and \( \mu_{s_1}^{\theta_1} \) in (22)-(25) below.

The direct effect on tightness captures the effect of redistribution on aggregate demand:

\[
\mu_{\theta_1}^{y_i^u} \equiv \frac{\text{Volume of transfers}}{f_1^*(\cdot)\delta(p'(\theta_1) - k)s_1 \left(1 - \delta p'(\theta_1)s_1\right)} \left(\frac{\partial c_1^e}{\partial y_i^u} - \delta p'(\theta_1)s_1\left(c_1^e - c_1^u\right) - \frac{\partial c_1^e}{\partial y_i^u}(y_i^e - y_i^u)\right)
\]

GE amplification

where I denote employment

\[ p_1^e \equiv 1 - \delta + \delta p(\theta_1)s_1 \]

to ease notation. Given an increase in transfers, there is an initial positive impulse to aggregate demand if the unemployed have a higher MPC out of income than the employed. This is scaled by the economy’s unemployment rate, which mechanically implies a larger volume of transfers. The resulting effect on tightness then depends on a feedback between tightness and aggregate demand for consumption summarized in the denominator. The first

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12 Nonetheless, for completeness, in appendix B I demonstrate how to obtain comparative statics in terms of \( b_1 \) using the results from the main text, (21), and alternative assumptions on wage determination.

13 Its proof, along with the proofs of all other analytical results in this section, is in appendix A.
term captures the typical Keynesian cross: provided higher tightness raises output (holding search fixed), the response of tightness is rising in the MPC of employed agents, who earn marginally higher income due to higher production. But incomplete markets mean that amplification also occurs on the extensive margin, captured by the second term. The higher is the relative level of consumption among the employed, the larger is the increase in tightness from an initial positive impulse to aggregate demand which raises employment. Owing to the economy’s resource constraint, the opposite is true for relative per-capita income.

Beyond this Keynesian channel, the search response to transfers will also affect equilibrium tightness according to

$$\mu_{s_1} \equiv \frac{p(\theta_1)}{p'(\theta_1)s_1} \frac{1}{\Delta} \left( 1 - \frac{\alpha_1}{\sigma_1} \right) \delta_k \theta_1 \left( \frac{1}{\Delta} - 1 \right) \frac{1}{\Delta} \delta p'(\theta_1)s_1 \left( c_1^e - c_1^u \right) \frac{\alpha_1}{\sigma_1} (y_1^e - y_1^u) \right). \tag{23}$$

Hence, vacancy-creation offsets the change in worker search to meet the demand for aggregate consumption, an application of Landais et al. [2016a,b]'s “rat race effect” when jobs are rationed. Absent recruiting costs $k$, the change in tightness would ensure that the response of search has no impact on employment. But because this changes recruiting costs, tightness must further adjust to meet the demand for aggregate consumption. For instance, if search falls and the feedback between tightness and aggregate demand is positive, tightness will overshoot to equilibrate goods markets in light of the efficiency loss from higher recruiting.

It only remains to characterize how equilibrium search is affected by transfers to the unemployed. The direct effect is characterized by

$$\mu_{s^1} \equiv \frac{1}{p_1} \left( f_1'(\cdot) \delta \left( p(\theta_1) - k \theta_1 \right) - \delta p(\theta_1)(y_1^e - y_1^u) \right) \frac{\partial s_1}{\partial y_1^u}, \tag{24}$$

and the response to the change in tightness is characterized by

$$\mu_{\theta_1} \equiv \left\{ \begin{array}{c} \text{Direct effect} \\ \frac{\partial s_1}{\partial \theta_1} + \frac{1}{p_1} \left( f_1'(\cdot) \delta \left( p(\theta_1) - k \theta_1 \right) s_1 - \delta p'(\theta_1)s_1(\delta p(\theta_1)(y_1^e - y_1^u)) \right) \frac{\partial s_1}{\partial y_1^u} \\ \text{Indirect effect} \\ 1 - \frac{1}{p_1} \left( f_1'(\cdot) \delta \left( p(\theta_1) - k \theta_1 \right) s_1 - \delta p(\theta_1)(y_1^e - y_1^u) \right) \frac{\partial s_1}{\partial y_1^u} \\ \text{GE amplification} \end{array} \right\}. \tag{25}$$
where all partial derivatives of search in these expressions correspond to the (micro) policy function \( s_1(m_1, y_e^i, y_u^i, \theta_1) \). The numerator of (24) captures the initial reduction in search arising from moral hazard. The numerator of (25) captures the response in search arising from a change in tightness, which directly changes search and indirectly changes it through the induced change in the income of the employed. Both effects are then amplified by the general equilibrium feedback between search and per-capita income of the employed. Since agents search harder when the returns are higher (\( \frac{\partial s_1}{\partial y_e^i} > 0 \)), the amplification of any impulse to agents’ search effort is rising in the effect of marginal changes in search on output, which generates additional income for the employed. Owing to the economy’s resource constraint, the opposite is again true for the difference in per-capita disposable income levels.

With these intermediate results in hand, we can readily characterize the equilibrium effects on tightness and search in closed form.

**Proposition 1.** In period 1, the responses of equilibrium tightness and search are given by

\[
\frac{\partial \theta_1(m_1, y_u)}{\partial y_u^i} = \frac{\mu_{\theta_1} y_u^i + \mu_{s_1} s_1 y_u^i}{1 - \mu_{s_1} \mu_{\theta_1}} ,
\]

\[
\frac{\partial s_1(m_1, y_u)}{\partial y_u^i} = \frac{\mu_{s_1} y_u^i + \mu_{\theta_1} s_1 y_u^i}{1 - \mu_{s_1} \mu_{\theta_1}} ,
\]

for coefficients \( \mu_{\theta_1}, \mu_{s_1}, \mu_{\theta_1}^{s_1}, \) and \( \mu_{s_1}^{\theta_1} \) given in (22)-(25).

In the normative analysis which follows, it will be useful to distinguish between the effects on tightness through the redistribution effect on aggregate demand and through the general equilibrium response to search:

\[
\frac{\partial \theta_1(m_1, y_u)}{\partial y_u^i} \bigg|_{AD} \equiv \frac{\mu_{\theta_1} y_u^i}{1 - \mu_{s_1} \mu_{\theta_1}} ,
\]

\[
\frac{\partial \theta_1(m_1, y_u)}{\partial y_u^i} \bigg|_{search} \equiv \frac{\mu_{s_1} \mu_{\theta_1}^{s_1}}{1 - \mu_{s_1} \mu_{\theta_1}} .
\]

These generate aggregate demand and search externalities, respectively, in the presence of macroeconomic inefficiency. Summarizing the net feedback loop between tightness and the demand for aggregate consumption

\[
\Delta_{\theta_1}^{AD} \equiv f_1'(\cdot) \delta (p'(\theta_1) - k) s_1 \left( 1 - \frac{\partial c_e^i}{\partial y_u^i} \right) - \delta p'(\theta_1) s_1 \left( c_e^i - c_u^i \right) - \frac{\partial c_e^i}{\partial y_u^i} (y_e^i - y_u^i) \left( 1 - \mu_{s_1} \mu_{\theta_1} \right) ,
\]

which accounts both for the direct effect of tightness (the denominator of (22)) and its
indirect effect via search effort, we have in particular that
\[
\left. \frac{\partial \theta_1(m_1, y_u)}{\partial y_1^u} \right|_{AD} = \frac{1}{\Delta_{\theta_1}} (1 - p_1^e) \left( \frac{\partial c_1^u}{\partial y_1^u} - \frac{\partial c_1^e}{\partial y_1^e} \right).
\]
(27)

That is, redistribution raises tightness when the unemployed have a higher MPC than the employed, and the resulting increase in aggregate demand raises tightness.

### 2.2.2 Effects of expected future transfers

I now turn to characterizing the effects of \(y_1^u\) on period 0 aggregates, holding fixed \(\{m_0, m_1\}\) set by monetary policy. I focus on equilibrium consumption \(c_0(m_0, m_1, y_1^u)\), which equals equilibrium output. Differentiating (19) and (20) and combining this with the effects of transfers in period 1 then yields the following result.

**Proposition 2.** In period 0, the response of equilibrium consumption (and thus output) is

\[
\frac{\partial c_0(m_0, m_1, y_1^u)}{\partial y_1^u} = \mu_{y_1^u} + \mu_{s_1} \frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u} + \mu_{\theta_1} \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}
\]

for \(\mu_{y_1^u}, \mu_{s_1}, \) and \(\mu_{\theta_1}\) in (28)-(30) below, and \(\frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u}\) and \(\frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}\) in Proposition 1.

The first component captures the insurance effect on aggregate demand:

\[
\mu_{y_1^u} \equiv \frac{m_0 \frac{\partial c_0(m_0, m_1, y_1^u)}{\partial m_0}}{p_1^e \frac{\partial y_1^u}{\partial y_1^u} + (1 - p_1^e) \frac{\partial v_1^u}{\partial y_1^u}} (1 - p_1^e) \left( \frac{\partial^2 v_1^e}{\partial y_1^u \partial y_1^u} - \frac{\partial^2 v_1^u}{\partial y_1^u \partial y_1^u} \right).
\]
(28)

We can better understand the drivers of this term by considering the benchmark case when utility functions are identical up to a constant so that

\[v_i^i(m_1, y_i^i) = v_1(m_1, y_1^i) + \chi^i\] for \(i \in \{e, u\}\).

Then a first-order Taylor approximation implies

\[
\mu_{y_1^u} = m_0 \left( \frac{\partial c_0(m_0, m_1, y_1^u)}{\partial m_0} \right) \left[ \begin{array}{c}
- \frac{\partial^2 v_1^e}{\partial y_1^u \partial y_1^u} \\
- \frac{\partial^2 v_1^e}{\partial y_1^u \partial y_1^u}
\end{array} \right] \left| \begin{array}{c}
y_u^i \\
y_e^i
\end{array} \right| (1 - p_1^e) (y_1^u - y_1^e) + o(||y_1^e - y_1^u||^2).
\]

Hence, expectations of greater future insurance will be expansionary by reducing agents’ need to precautionary save. This channel is increasing in the magnitude of income risk that agents face as well as the absolute prudence of indirect utility. It is further increasing in the semi-
elasticity of consumption to the (inverse) interest rate, capturing the extent to which agents’ consumption responds to news about the future. This insurance channel represents one way social insurance could potentially be even more stimulative than government spending.

The second component captures the change in period 0 consumption arising from the future search response to transfers

\[
\mu_{s_1}^c = \frac{m_0 \partial c_0(m_0, m_1, y_1^u)}{\partial m_0} \times \left( \frac{\partial v_1^c}{\partial y_1^u} - \frac{\partial^2 v_1^c}{\partial (y_1^u)^2} (y_1^e - y_1^u) \right) - \frac{\partial^2 v_1^c}{\partial (y_1^u)^2} \frac{f_1' (\cdot) \delta (p_1 \theta_1 - k \theta_1)}{f_1' (\cdot) \delta (p_1 \theta_1 - k \theta_1)} \right),
\]

(29)

while the third component captures the change in period 0 consumption arising from the future tightness response to transfers

\[
\mu_{\theta_1}^c = \frac{m_0 \partial c_0(m_0, m_1, y_1^u)}{\partial m_0} \times \left( \frac{\partial v_1^c}{\partial y_1^u} - \frac{\partial^2 v_1^c}{\partial (y_1^u)^2} (y_1^e - y_1^u) \right) - \frac{\partial^2 v_1^c}{\partial (y_1^u)^2} \frac{f_1' (\cdot) \delta (p_1 \theta_1 - k \theta_1)}{f_1' (\cdot) \delta (p_1 \theta_1 - k \theta_1)} \right). \]

(30)

In both cases, period 0 consumption responds to the change in the probability of being employed and the implied change in income when employed in period 1, which both change the representative agent’s expected income and her incentive to precautionary save. Since we showed in (23) how equilibrium tightness will offset the change in equilibrium search,

\[
\mu_{s_1}^c \frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u} + \mu_{\theta_1}^c \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}
\]

will ultimately reflect the redistribution effect on demand in period 1 and the change in recruiting costs due to the substitution of vacancies for search.

2.3 Normative role of UI

I now build on these positive results to characterize the optimal generosity of UI. In the presence of macroeconomic inefficiency, optimal UI departs from the classic insurance-incentive trade-off in public finance because of the search and aggregate demand externalities from transfers. Macroeconomic inefficiency is summarized by a wedge in surplus-sharing between workers and firms in a frictional labor market, much like the labor wedge summarizes in-
efficiency in a neoclassical setting. One reason for such wedges is if monetary policy is constrained in stabilizing the economy in response to shocks, as by a zero lower bound.

2.3.1 Planning problem and Ramsey optimal risk-sharing

I begin by studying the Ramsey optimal generosity of UI holding monetary policy fixed, as in the positive results. The problem of optimally choosing \{b_1, t_1\} is made more transparent by using the following equivalence of implementable allocations.

**Lemma 2.** Given \{m_0, m_1\} set by monetary policy, an allocation \{c_0, h_0, c^u_1, c^y_1, c^u_2, c^y_2, s_1, \theta_1\} forms part of a fully sticky price equilibrium if and only if there exists cash-on-hand \{y^c_1, y^u_1\} satisfying the economy-wide resource constraints

\[
\begin{align*}
& x_1(s_1, \theta_1, c^c_1, c^u_1) = 0, \\
& x_2(s_1, \theta_1, c^c_2, c^u_2) = 0,
\end{align*}
\]

given implementability constraints \[ c_0 = c_0(m_0, m_1, y^c_1, y^u_1, \theta_1), \quad h_0 = h_0(m_0, m_1, y^c_1, y^u_1, \theta_1), \quad c^c_1 = c^c_1(m_1, y^c_1), \quad c^u_1 = c^u_1(m_1, y^u_1), \quad c^c_2 = c^c_2(m_1, y^c_1), \quad c^u_2 = c^u_2(m_1, y^u_1), \quad s_1 = s_1(m_1, y^c_1, y^u_1, \theta_1). \]

Using a utilitarian metric corresponding to the utility of the period 0 representative agent, we can substitute these implementability constraints into (2) to obtain planning problem

\[
\max_{y^c_1, y^u_1, \theta_1} v_0(m_0, m_1, y^c_1, y^u_1, \theta_1) \text{ s.t.} \quad (RC)_1 : x_1(s_1(m_1, y^c_1, y^u_1, \theta_1), \theta_1, c^c_1(m_1, y^c_1), c^u_1(m_1, y^u_1)) = 0, \quad (RC)_2 : x_2(s_1(m_1, y^c_1, y^u_1, \theta_1), \theta_1, c^c_2(m_1, y^c_1), c^u_2(m_1, y^u_1)) = 0.
\]

Since these constraints define \( y^c_1(m_1, y^u_1) \) and \( \theta_1(m_1, y^u_1) \), we can equivalently study

\[
\max_{y^u_1} v_0(m_0, m_1, y^u_1) \equiv v_0(m_0, m_1, y^c_1(m_1, y^u_1), y^u_1, \theta_1(m_1, y^u_1)).
\]

Before characterizing the optimality conditions of (32), it will prove useful to define two wedges summarizing macroeconomic inefficiency in periods 0 and 1.\(^{14}\) In period 0, inefficiency can be usefully summarized by the labor wedge

\[
\tau^h_0 \equiv \frac{\partial u_0}{\partial c_0} + \frac{1}{f_0'(h_0)} \frac{\partial u_0}{\partial h_0},
\]

\(^{14}\) These wedges capture inefficiency because both are zero at the constrained efficient allocation (chosen by a planner who directly sets consumption, hours, and tightness subject to agents’ optimal choice of search), recognizing in (34) that \( m_1 = -\frac{\partial u_0}{\partial c_0} = \frac{\partial u}{\partial x_1} \) by agents’ optimal intertemporal allocation of consumption.
as studied by Chari et al. [2007], Shimer [2009], and others. In period 1, labor market frictions imply that an analogous tightness wedge

\[
\tau_1^\theta = \left( 1 - \frac{1}{\beta_1} \left( \frac{\partial v^e}{\partial y_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial y_1}{\partial y_1} \right) \times \left[ \delta p'(\theta_1) s_1 \frac{1}{\partial v^e} (v^e - v^e_1) + \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) + 1 - \frac{\partial x_1}{\partial s_1} \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} \right]
\]

(34)

can summarize inefficiency in the level of production.

To better understand the intuition behind the tightness wedge, we can use the definitions of excess supply in (17) and (18) to relate \(\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\) and \(\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\), implying

\[
\tau_1^\theta = \left( 1 - \frac{1}{\beta_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial y_1}{\partial y_1} \right) \delta \eta f'_1(\cdot) k s_1 \times \left[ \frac{1}{\delta f'_1(v^e - v^e_1)} \left( \frac{1}{\eta} \right) + \frac{1}{\delta f'_1(v^e - v^e_1)} \left( \eta + 1 - \frac{\partial x_1}{\partial \theta_1} \frac{\partial x_2}{\partial \theta_1} \right) \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \right].
\]

Deviation in relative worker surplus from Hosios [1990]

Fiscal externality from search

This has two implications. First, to achieve a zero wedge, the division of surplus between workers and firms must depart from Hosios [1990] if there is a fiscal externality associated with marginal changes in search.\(^{15,16}\) Second, the wedge is increasing in the surplus share of workers relative to firms, so that a positive wedge reflects inefficiently low tightness.\(^{17}\)

Armed with these wedges, we can assess the marginal social welfare effect of changing \(y_1^u\) starting from an arbitrary allocation, obtaining the following key result:

\(^{15}\)I interpret \(\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\) as a fiscal externality because it is the general equilibrium analog to the effect of a change in search on the government’s budget constraint — the standard fiscal externality in public finance.

\(^{16}\)At the constrained efficient allocation (as defined in footnote 14), the planner’s optimal allocation of consumption across agents implies

\[
\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \propto \frac{\partial v^e}{\partial y_1} - \frac{\partial v^e_1}{\partial y_1}.
\]

Hence, to achieve \(\tau_1^\theta = 0\) as part of the constrained efficient allocation when moral hazard and risk aversion necessitate \(\frac{\partial v^e}{\partial y_1} - \frac{\partial v^e_1}{\partial y_1} > 0\), workers’ surplus share must be less than that in Hosios [1990]. When agents are risk-neutral, the Hosios [1990] condition remains efficient. These results, which to my knowledge are novel, build on earlier observations in Lehmann and van der Linden [2007] and Landais et al. [2016a] to generalize Hosios [1990] under moral hazard and incomplete markets.

\(^{17}\)This statement presumes that \(1 - \frac{1}{\beta_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial x_1}{\partial s_1} > 0\). While I have been unable to prove this starting from an arbitrary allocation, it is indeed holds at the Ramsey optimum with zero wedges characterized in Proposition 3, and thus by continuity holds at the optimum with small wedges as well.
Proposition 3. Starting from any allocation,

\[
\frac{\partial v_0(m_0, m_1, y_1^u)}{\partial y_1^u} = \beta \left( 1 - p_1^u \right) \left( \frac{\partial v_1^u}{\partial y_1^u} - \frac{\partial v_1^e}{\partial y_1^e} \right) \frac{1 - \frac{1}{1 - p_1^u} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial x_1}{\partial s_1}}{1 - \frac{1}{\beta} \left( \frac{\partial c_1^u}{\partial y_1^u} + m_1 \frac{\partial c_1^e}{\partial y_1^e} \right) \frac{\partial c_1^u}{\partial y_1^u}} \\
+ \theta_1 \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \bigg|_{search} + \theta_1 \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \bigg|_{AD} + \frac{1}{\beta} \frac{\partial c_0(m_0, m_1, y_1^u)}{\partial y_1^u} \right) \tag{35}
\]

for \( \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \bigg|_{search} \), \( \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \bigg|_{AD} \), and \( \frac{\partial c_0(m_0, m_1, y_1^u)}{\partial y_1^u} \) characterized in Propositions 1 and 2. At the Ramsey optimum, the expression in brackets must equal zero.

This result isolates three externalities associated with marginal changes in transfers to the unemployed starting from an arbitrary allocation. It further defines Ramsey optimal risk-sharing in the economy, where (35) equals zero.

First, a fiscal externality in the public finance tradition reflects the impact of transfers on agents’ search responses and thus economy-wide resources available for consumption. If \( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} > 0 \), lower search reduces the net present value of these resources in periods 1 and 2, which is socially costly. Since transfers to the unemployed reduce incentives to search (that is, \( \frac{\partial s_1}{\partial y_1^u} < 0 \) and \( \frac{\partial s_1}{\partial y_1^e} > 0 \)), this externality drives incomplete insurance at the optimum.

Second, a search externality reflects the impact of transfers on agents’ search and thus tightness, affecting the returns to search for other agents in the economy. If tightness is inefficiently low as summarized by a positive wedge, this externality raises the social value of transfers to the unemployed provided that agents’ search responses induce a general equilibrium increase in tightness. This externality is the focus of Landais et al. [2016a,b].

Third, and novel to my analysis, an aggregate demand externality reflects the impact of transfers on the demand for aggregate consumption and thus the level of economic activity. Consider an economy featuring deficient demand in period 1, which will lead to a positive (negative) tightness wedge \( \tau_1^D \) if the relationship between tightness and demand \( \Delta_y^{AD} \) is positive (negative). In either case, since (27) implies

\[
\tau_1^D \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \bigg|_{AD} = \tau_1^D \frac{1}{\Delta_y^{AD}} (1 - p_1^u) \left( \frac{\partial c_1^u}{\partial y_1^u} - \frac{\partial c_1^e}{\partial y_1^e} \right),
\]

the social value of transfers is rising in the difference in MPCs between the unemployed and employed, as this raises demand and mitigates the macroeconomic inefficiency. Similarly, consider an economy featuring deficient demand in period 0 as summarized by a positive
labor wedge. Then the social value of transfers is rising in the degree to which it stimulates aggregate consumption before employment uncertainty is resolved, driven by the insurance effect on aggregate demand in (28) and the general equilibrium feedback from effects in period 1. These results extend those of Farhi and Werning [2016], who focus on macroprudential policy, to the setting of optimal social insurance.\textsuperscript{18}

2.3.2 Implementation and a generalized Baily-Chetty formula

Characterizing the implementation of Ramsey optimal risk-sharing in Proposition 3 leads to a generalization of the optimal UI formula from public finance which accounts for the search and aggregate demand externalities from transfers. A key step is to recognize that

\[
\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} = \delta p(\theta_1) \left[ f'_1(\cdot) \left( 1 - \frac{k}{q(\theta_1)} \right) + m_1(y_u^e - y_u^e) - \left[ (c_1^e + m_1 c_2^e) - (c_1^u + m_1 c_2^u) \right] \right]_{\text{e-u diff in lifetime marginal product}}
\]

\[
_{\text{e-u diff in lifetime consumption}}
\]

The expression in brackets can be interpreted as a measure of transfers: if the difference between employed and unemployed consumption is less than their difference in marginal product, this must be induced by a wealth transfer between these agents in equilibrium. The following lemma clarifies how such a transfer is implemented under fully sticky prices.

**Lemma 3.** In a fully sticky price equilibrium,

\[
\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} = \frac{1}{P_1^e} b_1 + (\mu - 1) w_1,
\]

where \(\mu_1 \equiv \frac{\mu}{P_1}\) is the gross retailer mark-up and \(w_1 \equiv \frac{W_1}{P}\) is the real wage.

Lemma 3 isolates the two sources of transfers between agents in equilibrium: UI policy and mark-up variation under sticky prices. The direction of the latter depends on agents’ holding of firm equity. In the present environment all agents enter period 1 with identical assets, but only the employed earn labor income; as a result, an increase in mark-ups amounts to a reallocation of wealth from labor to profit income, and thus a transfer to the unemployed.

Let the payroll tax \(\tau_1\) be used to eliminate the equilibrium mark-up, allowing us to focus attention on redistribution through UI alone.\textsuperscript{19} We then can generalize the Baily [1978]-Chetty [2006] formula for optimal UI, the final main analytical result of the paper:

\textsuperscript{18}In appendix B I provide an equivalent characterization of the Ramsey optimum in which the appropriate notion of agents’ social marginal utilities of income are equated. This provides an easier way to see how my results build on those in Farhi and Werning [2016].

\textsuperscript{19}This assumption is defensible because if some retailers could adjust their prices, using the tax to eliminate mark-ups would eliminate costly price dispersion. It is innocuous under fully sticky prices because the mark-up implementing the Ramsey optimum is indeterminate, provided the real wage in (8) is not fully
Proposition 4. Assuming the payroll tax $\tau_1$ is used to eliminate any mark-ups ($\mu_1 = 1$), the optimal UI benefit $b_1$ implements Ramsey optimal risk-sharing in Proposition 3 by satisfying

\[
\begin{align*}
\text{Incentives} & \left( \frac{1}{p_1^1} \right) \varepsilon^{1-p_1^1} b_1 - \frac{1}{1-p_1^1} \frac{1}{\delta^{1}} \tau \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} |\text{search} & = \frac{\partial u^u}{\delta^{1}} \frac{\partial u_1^h}{\delta^{1}} + \frac{1}{1-p_1^1} \frac{1}{\delta^{1}} \left( \frac{1}{3} \tau_0 \frac{dc_0(m_0, m_1, y_1^u)}{dy_1^u} + \tau_1 \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} |\text{AD} \right) + o(\|\tau_0^h, \tau_1^\theta \|^2), \\
\text{Search externality} & + \frac{1}{1-p_1^1} \frac{1}{\delta^{1}} \frac{\partial u^u}{\delta^{1}} \frac{\partial u_1^h}{\delta^{1}} |\text{AD externality} \right),
\end{align*}
\]

where $\varepsilon^{1-p_1^1}$ is the micro elasticity of unemployment in period 1 with respect to UI

\[
\varepsilon^{1-p_1^1} = - \left( \frac{\delta p(\theta_1)}{1-p_1^1} \right) \frac{\partial s_1}{\delta y_1^u} b_1 s_1 > 0,
\]

and $\frac{\partial u^u}{\delta^{1}}$ is the harmonic mean of agents’ marginal utilities of consumption in period 1.

Absent macroeconomic inefficiency, optimal UI trades off the private gains from consumption insurance against the (micro) disincentive effect, reflecting Ramsey optimal risk-sharing accounting for the fiscal externality alone. But in the presence of macroeconomic inefficiency, optimal UI also accounts for the search and aggregate demand externalities of transfers.\(^{20}\) If demand is inefficiently low, the latter in particular means that the social value of UI exceeds its private consumption insurance value if the unemployed have a higher MPC than the employed or the risk of unemployment leads agents to precautionary save.

### 2.3.3 Sources of macroeconomic inefficiency

The preceding analysis has been silent on the sources of non-zero wedges $\tau_0^h$ and $\tau_1^\theta$ which motivate optimal UI away from the Baily-Chetty level. Such inefficiency can in fact be motivated by constraints on monetary policy in a richer stabilization problem.

Appendix B provides a formal analysis of jointly optimal monetary policy and UI, which generalizes planning problem (31) to include $\{m_0, m_1\}$ as additional controls. Absent constraints on its use, monetary policy achieves $\tau_0^h = \tau_1^\theta = 0$ and the formula for optimal UI in Proposition 4 then collapses to that in public finance. Consistent with the monetary economics literature, monetary policy is the appropriate instrument to undo the effects rigid ($\varphi < 1$). At the limit of a fully rigid real wage, a similar indeterminacy would apply if policymakers could assess the payroll tax on producers rather than workers.

\(^{20}\)If competitive intermediaries offered privately-provided UI, they would be expected to account for fiscal externalities in the manner described in Prescott and Townsend [1984] but would not internalize the search and aggregate demand externalities of UI. Hence, another way of interpreting the results described here is that in the presence of macroeconomic inefficiency, these externalities justify public intervention in UI.
of nominal rigidity; reminiscent of the Diamond and Mirrlees [1971] result on production efficiency, UI is then left to solve the partial equilibrium problem.

When monetary policy is constrained, however, UI plays a second-best role in macroeconomic stabilization. I focus on one particularly relevant set of constraints in recent years (also studied in the quantitative analysis of the next section), a zero lower bound on the nominal interest rates \( \{i_0, i_1\} \). When it binds only in period 1, the Ramsey planner must accept a non-zero tightness wedge \( \tau^\theta_1 \) taking the same sign as the relationship between tightness and demand \( \Delta^AD_1 \). When it binds in period 0, the Ramsey planner must accept a positive labor wedge \( \tau^h_0 \) and will also accept additional distortions in period 1 (\( \tau^\theta_1 \)) to lessen those in period 0. Either way, the resulting inefficiencies and externalities from transfers motivate departures from the Baily-Chetty level of UI summarized in Proposition 4.

These insights should extend to other settings where monetary policy is unable to fully stabilize the economy on its own. For instance, in a small open economy with sticky prices or wages, a fixed exchange rate would motivate a role for UI in stabilization through very similar channels. A more distinct setting of interest would be one with partially sticky prices or wages and “cost-push” shocks. The aggregate demand externality of UI would carry over, but the effects on inflation might introduce new and interesting trade-offs as well.

3 Quantitative insights in the infinite horizon

In a richer infinite horizon environment, I now quantitatively evaluate the effects of UI characterized in the analytical results. I discipline the model in steady-state to match salient patterns in wealth, income, and employment in U.S. data. I then evaluate the effect of extending UI benefits from 6 to 9 months over a one year period when prices are sticky and labor market slackness is close to that of the U.S. during the Great Recession. The implied pattern of MPCs and precautionary saving generate substantially expansionary effects of UI through aggregate demand: the unemployment rate falls by an average of 0.04–0.07pp in the year of extended benefits, and utilitarian social welfare rises.

3.1 Environment and equilibrium

To quantitatively evaluate the effects of UI, I first extend the environment from section 2 to a richer infinite horizon setting. By repeating an infinite sequence of search, matching, production, consumption, and separation, confined to period 1 of the three-period model, we

\[21\] I combine this with an upper bound on \( P_2 \), implying an upper bound on expected inflation which can be generated despite flexible prices in the final period. In practice, such constraints on expected inflation may arise from unmodeled costs of inflation, political economy constraints, or anchored expectations.
can account for the interactions between incomplete markets and endogenous unemployment in a long-lived recession. By introducing heterogeneity beyond unemployment and enriching the specification of policy and matching, we can generate patterns in wealth, income, and employment which are necessary for a more credible quantitative analysis of UI.

I first discuss these features before revisiting agent optimization and equilibrium. The increased complexity forces us to study the model numerically in the remainder of the paper. But in deciding which features to add, choosing calibration targets, and interpreting the simulation results, we will be guided by the analytical insights of the three-period model.

### 3.1.1 Heterogeneity beyond unemployment

To have confidence in the model’s MPCs and precautionary saving elasticities, key moments in the analytical formulas, I will attempt to match patterns in U.S. wealth and income. To that end, I introduce two sources of heterogeneity beyond employment: shocks to workers’ labor productivity and heterogeneous discount factors. To further refine the wealth distribution implied by these features, I account for credit constraints and non-own-labor income.

**Shocks to labor productivity.** Labor productivity shocks capture the sources of income volatility that workers face even conditional on employment. I follow much of the literature on incomplete asset markets in assuming a persistent-transitory process for productivity, summarized by the pair \((a^P_t, a^T_t)\). When a worker is employed in period \(t\), these components evolve as

\[
\begin{align*}
\log a^P_t & = \log \bar{a}_t + \eta^P_t, \\
\eta^P_t & = \rho^P \eta^P_{t-1} + \varepsilon^P_t, \quad \varepsilon^P_t \sim N(0, (\sigma^P)^2), \\
\log a^T_t & \sim N(0, (\sigma^T)^2),
\end{align*}
\]

where \(\bar{a}_t\) controls the economy-wide average productivity and follows an exogenous process. When a worker is unemployed in period \(t\), the transitory component of productivity is irrelevant (as described below), and the persistent component of productivity remains unchanged:

\[
\begin{align*}
\log a^P_t & = \log \bar{a}_t + \eta^P_t, \\
\eta^P_t & = \eta^P_{t-1}.
\end{align*}
\]

In contrast to the literature focused only on incomplete asset markets, labor market frictions complicate the mapping between productivity and workers’ resulting wages when employed.\(^{22}\) To eliminate these complexities while preserving the link between productivity
and income shocks, I make two additional assumptions. First, I assume that producers’
technology \( f(\cdot) \) is constant returns-to-scale. Second, I assume real wages are determined by

\[
\begin{align*}
   w_t(a^P_t, a^T_t) &= a^P_t + a^T_t - (\bar{a}_t - \bar{w}_t) 
\end{align*}
\]

where \( \bar{w}_t \) follows an exogenous process and must be less than \( \bar{a}_t \) for bilateral efficiency.\(^{23}\)

**Heterogeneous discount factors.** As argued by Carroll et al. [2015] and Krueger et al. [2016a], discount factor heterogeneity can help in matching the empirical distribution of wealth, perhaps because it captures the heterogeneous motives for saving along the cross-section which would arise from life-cycle considerations in a richer model. I assume that \( \nu^{\beta} \) indexes this heterogeneity, with a fraction one-third of worker-consumers having \( \nu^{\beta} = -\Delta^{\beta} \), one-third having \( \nu^{\beta} = 0 \), and one-third having \( \nu^{\beta} = \Delta^{\beta} \) for some positive dispersion parameter \( \Delta^{\beta} \). For an agent with any particular \( \nu^{\beta} \), its period \( t \) discount factor is then

\[
\beta_t(\nu^{\beta}) = \bar{\beta}_t + \nu^{\beta} 
\]

where \( \bar{\beta}_t \) is the economy-wide average discount factor and follows an exogenous process.

**Borrowing constraint and endowment.** Lastly, I include a borrowing constraint and exogenous endowment to capture additional determinants of agents’ savings decisions in practice. To reflect credit constraints in the economy, I assume that worker-consumers cannot short equity and cannot take a position below \( z_t \) in the riskless bond. To reflect the implicit insurance within households and families, I assume that all agents are endowed with

\[
\omega_t = \omega^e p^e_t 
\]

units of each variety regardless of their employment status, where this varies with the economy-wide employment rate \( p^e_t \) to roughly capture fluctuations in spousal income.

### 3.1.2 Richer specification of policy and matching

Beyond matching patterns in wealth and income, it is also important that the model feature a realistic specification of UI policy and realistic patterns in unemployment. To that end, I account for both incomplete eligibility/take-up and duration-dependence.

**Incomplete eligibility and take-up.** In practice, not all newly unemployed workers are eligible for benefits, and many who are still do not take it up (Blank and Card [1991]). To capture this pattern in the data, I assume that only with probability \( \zeta \) does a newly unemployed worker begin receiving benefits, a state denoted with indicator \( 1_{UI} \).

\(^{23}\)In appendix D, I further discuss and evaluate bilateral efficiency in this environment.
Duration-dependent UI. The generosity of UI varies with unemployment duration in the U.S. I thus assume that, coupled with the above indicator for eligibility and take-up, the transfer to an unemployed agent with persistent productivity $a_t^P$ and duration $d_t$ is

$$b_t(a_t^P, d_t, 1_{UI}) = \begin{cases} rr_t w_t(a_t^P, 0) & \text{if } 1_{UI} = 1, d_t < \bar{d}_t, \\ b^{SA} & \text{if } 1_{UI} = 0 \text{ or } d_t \geq \bar{d}_t \end{cases},$$

(42)

where the policy parameters of the UI program in period $t$ are replacement rate $rr_t$ and duration $\bar{d}_t$, and agents not receiving UI simply receive fixed social assistance $b^{SA}$.

Duration-dependent matching. Finally, duration-dependence in matching allows the model to better reflect empirical hazard rates out of unemployment through “structural” duration-dependence in job-finding rates, consistent with the evidence of Ghayad [2013], Kroft et al. [2013], Eriksson and Rooth [2014] and others. I assume that an unemployed agent with duration $d_t$ faces job-finding probability per unit effort

$$p_t(\theta_t; d_t) = (\bar{m}_t(0))^{1-\eta} \frac{\bar{m}_t(d_t)}{\bar{m}_t(0)} \theta_t^{\eta},$$

(43)

where labor market tightness $\theta_t$ is characterized in further detail below, and

$$\bar{m}_t(d) = \begin{cases} \bar{m}_t(0)(1 - \lambda_0 + \lambda_0 \exp(\lambda_1 d)) & \text{for } d < 8, \\ \bar{m}_t(7) & \text{for } d \geq 8. \end{cases}$$

(44)

Here, $\bar{m}_t(0)$ controls the overall level of match efficiency in the economy in period $t$, while $\{\lambda_0, \lambda_1\}$ control the relative efficiencies by duration. I assume furthermore that match efficiencies are flat after an unemployed agent has been unemployed for 8 months or more.

An alternative literature has argued that dynamic selection among heterogeneous job-seekers better explains observed duration-dependence in job-finding rates (Ahn and Hamilton [2015], Alvarez et al. [2015]). I describe the required changes to the model to accommodate such heterogeneity, and discuss the sensitivity of my results to this approach, in section 3.3.

3.1.3 Optimization and equilibrium revisited

Armed with these enriched primitives, we can now generalize the optimization problems and definition of equilibrium from section 2.

---

24The replacement rate is applied to a wage reflecting the agent’s persistent productivity, consistent with U.S. practice of computing benefits using a base period of earnings prior to job loss.

25This is computationally convenient because it limits the state space. But it is also consistent with the flatter empirical hazards out of unemployment after 8 months reported in Figure 7(A) of Kroft et al. [2016].
Worker-consumers. These agents are now heterogeneous in more ways than employment and wealth. It proves notationally convenient to collect these new state variables into

\[
\zeta^e_t \equiv (\beta_t, a^P_t, a^T_t), \\
\zeta^u_t \equiv (\beta_t, a^P_t, d_t, 1_{UI}),
\]

for employed and unemployed agents, respectively, with associated transition probabilities \( \Gamma(t(\zeta^e_t|\zeta^u_t), \Gamma(t(\zeta^u_{t+1}|\zeta^e_t), \Gamma(t(\zeta^u_{t+1}|\zeta^u_t), \text{and } \Gamma(t(\zeta^u_{t+1}|\zeta^u_t) \text{ which follow from (37)-(40). As in the three-period model, we can continue to consolidate agents’ holding of bonds and firm equity into a single state variable (z_t) owing to the absence of aggregate risk.}^{26}

Then agents’ beginning-of-period value functions generalize (3) to account for the new state variables and duration-dependence in job-finding:

\[
\tilde{v}^e_t(z_t; \zeta^e_t) = v^e_t(z_t; \zeta^e_t), \\
\tilde{v}^u_t(z_t; \zeta^u_t) = \max_{s_t} (p(\theta_t; \zeta^u_t)s_t) \int_{\zeta^e_t} v^e_t(z_t; \zeta^e_t) \Gamma(t(\zeta^e_t|\zeta^u_t)d\zeta^e_t \\
+ (1-p(\theta_t; \zeta^u_t)s_t)v^u_t(z_t; \zeta^u_t) - \psi(s_t).
\]

And agents’ middle-of-period value functions generalize (4) to account for the new state variables, borrowing constraint, and future risk of separation: the employed face

\[
v^e_t(z_t; \zeta^e_t) = \max_{c^e_t, z_{t+1}} u(c^e_t) + \beta_t \left[ (1 - \delta_t) \int_{\zeta^e_{t+1}} \tilde{v}^e_{t+1}(z^e_{t+1}; \zeta^e_{t+1}) \Gamma(t(\zeta^e_{t+1}|\zeta^e_t)d\zeta^e_{t+1} \\
+ \delta_t \int_{\zeta^u_{t+1}} \tilde{v}^u_{t+1}(z^u_{t+1}; \zeta^u_{t+1}) \Gamma(t(\zeta^u_{t+1}|\zeta^u_t)d\zeta^u_{t+1} \right] \text{s.t. (4e')}
\]

\[
(4e')_t : P_t c^e_t + M_t P_{t+1} z^e_{t+1} \leq Y^e_t(\zeta^e_t) + P_t z_t,
\]

and the unemployed face

\[
v^u_t(z_t; \zeta^u_t) = \max_{c^u_t, z_{t+1}} u(c^u_t) + \beta_t \int_{\zeta^u_{t+1}} \tilde{v}^u_{t+1}(z^u_{t+1}; \zeta^u_{t+1}) \Gamma(t(\zeta^u_{t+1}|\zeta^u_t)d\zeta^u_{t+1} \text{s.t. (4u')}
\]

\[
(4u')_t : P_t c^u_t + M_t P_{t+1} z^u_{t+1} \leq Y^u_t(\zeta^u_t) + P_t z_t,
\]

\[
(BC)^e_t : z^e_{t+1} \geq z_t,
\]

\[
(BC)^u_t : z^u_{t+1} \geq z_t,
\]

---

26The distinction between holdings of bonds and firm equity is important, however, when evaluating the equilibrium after an unanticipated macroeconomic shock. I discuss this further in section 3.3.
where $c_i^t$ is a CES aggregator as in (1), flow utility is now assumed time-separable and identical across agents, and $\delta_t$ denotes the separation rate at end of period $t$.

**Producers.** Intermediate good producers face a richer problem in view of the frictional labor market each period. Focusing on the representative producer, let $n_t$ denote its stock of incumbent workers at the beginning of period $t$, $\varphi_i^n(\zeta^e_t)$ denote the distribution of state variables among these incumbents, and $\tilde{\varphi}_i^u(\zeta^u_t)$ denote the distribution of state variables among the pool of unemployed workers. For simplicity, assume that recruiting can only be done by workers with (economy-wide average) productivity $\bar{a}_t$. Then (6) generalizes to

$$J_t(n_t, \varphi_i^n(\zeta^e_t)) = \max_{\nu_t, n_{t+1}, \varphi_{t+1}^n(\zeta^e_{t+1})} \int_{\zeta^e_t} \left( P_t \omega_t(\zeta^e_t) - P_t w_t(\zeta^e_t) \right) \times \left[ n_t \varphi_i^n(\zeta^e_t) + q(\theta_t) \nu_t \int_{\zeta^e_t} \Gamma_t(\zeta^e | \zeta^u_t) \tilde{\varphi}_i^u(\zeta^u_t) d\zeta^u_t \right] d\zeta^e_t \quad (6')$$

subject to the evolution of the stock of incumbents

$$n_{t+1} = (1 - \delta_t)(n_t + q(\theta_t)\nu_t)$$

and the evolution of $\varphi_{t+1}^n(\zeta^e_{t+1})$ consistent with $\varphi_i^n(\zeta^e_t)$, $\tilde{\varphi}_i^u(\zeta^u_t)$, and Bayes’ Rule. The producer discounts future profits at the bond price $M_t$ since this is the nominal stochastic discount factor of the (unconstrained) owners of firm equity and there is no aggregate risk.

**Retailers.** If retailers can update prices each period they continue to face a sequence of static problems identical to (7). For simplicity, I continue to assume a passive tax $\tau^R = -\frac{1}{\varepsilon}$ on retailers to undo the effects of monopolistic competition on the flexible price mark-up, financed by a lump-sum tax $T_t^R$ on households in proportion to their holding of equity.

**Policy.** For simplicity I eliminate the payroll tax $\tau$ from the analysis, so the government’s marginal source of funds will be the lump-sum tax $t_t$.\textsuperscript{27} Agents’ incomes are thus

$$Y_t^e(\zeta^e_t) = P_t w_t(\zeta^e_t) + \int_0^1 P_t \omega_t d\zeta - P_t t_t,$$

$$Y_t^u(\zeta^u_t) = P_t b_t(\zeta^u_t) + \int_0^1 P_t \omega_t d\zeta$$

given the wage function (39), endowment (41), and government transfers (42). Monetary

\textsuperscript{27}As noted by Chetty [2006], the taxes which finance the UI program in the U.S. are inframarginal for most workers, and are thus best captured in $t$ rather than $\tau$ (note $t$ is still distortionary because of incomplete markets). Since in my calibration I use estimates of the income process already using post-tax data, there is little additional reason to explicitly model payroll or income taxes in this framework.
policy can be specified with a nominal interest rate rule in the usual way (see, e.g., Woodford [2003]). Finally, I allow the government to take a time-invariant position in the bond market \( z^g \), which will later be used to target the observed average level of household liquid wealth.

**Equilibrium.** Let \( \bar{p}_t^e \), \( \varphi_t^e(z_t; \zeta_t^e) \), and \( \bar{\varphi}_t^e(z_t; \zeta_t^e) \) denote the employment rate and distributions over state variables among employed and unemployed workers at the beginning of period \( t \). Let \( p_t^e \), \( \varphi_t^u(z_t; \zeta_t^u) \), and \( \bar{\varphi}_t^u(z_t; \zeta_t^u) \) denote the analogs in the middle of period \( t \).

Then asset market clearing becomes

\[
 p_t^e \int_{\zeta_t} \int_{s_t} z_t^{e}(z_t; \zeta_t^{e}) \varphi_t^{e}(z_t; \zeta_t^{e}) dz_t d\zeta_t^{e} + (1 - p_t^e) \int_{\zeta_t} \int_{s_t} z_t^{u}(z_t; \zeta_t^{u}) \varphi_t^{u}(z_t; \zeta_t^{u}) dz_t d\zeta_t^{u} = -z^g + \frac{1}{M_t P_{t+1}} Q_t, \quad (9')
\]

where \( Q_t \), the end-of-period-\( t \) price of a claim to economy-wide profits net of the tax financing the retailer subsidy, solves

\[
 Q_t = M_t \left[ \left( \int_j P_{t+1} y_{t+1,j} dj - P_{t+1} \int_{\zeta_t^{e}} w_{t+1}(\zeta_t^{e}) \varphi_{t+1}(\zeta_t^{e}) d\zeta_t^{e} \right) + Q_{t+1} \right].
\]

Given duration-dependent job-finding, equilibrium in the labor market now requires

\[
 \theta_t = \frac{\nu_t}{s_t}, \quad (11')
\]

where

\[
 s_t \equiv (1 - \lambda_t^{e}) \int_{\zeta_t} \frac{\bar{m}(d(\zeta_t^{u}))}{\bar{m}(0)} \int_{z_t} s_t(z_t; \zeta_t^{u}) \bar{\varphi}_t^{u}(z_t; \zeta_t^{u}) dz_t d\zeta_t^{u}.
\]

summarizes aggregate (weighted) search of initially unemployed workers.\(^{29}\)

Intermediate goods market clearing becomes

\[
 \int_{0}^{1} x_{tj} dj = \int_{\zeta_t} a_t(\zeta_t^{e}) \left[ n_t \varphi_t^{u}(\zeta_t^{u}) + q(\theta_t) \nu_t \int_{\zeta_t^{u}} \Gamma_t(\zeta_t^{e}|\zeta_t^{u}) \bar{\varphi}_t^{u}(\zeta_t^{u}) d\zeta_t^{u} \right] d\zeta_t^{e} - \bar{a}_t k \nu_t, \quad (12')
\]

\(^{28}\)Hence, \( \bar{\varphi}_t^{u}(\zeta_t^{u}) \) in (6') is the marginal distribution of \( \bar{\varphi}_t^{u}(z_t; \zeta_t^{u}) \) over the non-wealth state variables.

\(^{29}\)Hence, the aggregate number of matches corresponding to (43) is

\[
 m_t(\bar{s}_t, \nu_t) = (\bar{m}_t(0))^{1-\eta} s_t^{-\eta} \nu_t^\eta,
\]

and the vacancy-filling probability facing firms is given by

\[
 q_t(\theta_t) = (\bar{m}_t(0))^{1-\eta} \theta_t^{-\eta}. \quad (9')
\]

When \( \bar{m}_t(d) = \bar{m}_t \) for all \( d \), this matching process collapses to that in the three-period model.
and final goods market clearing for each variety \( j \) becomes

\[
p_t \int_{\zeta_t} \int_{z_t} c_{ij}^e(z_t; \zeta_t^e) \varphi_t^e(z_t; \zeta_t^e) dz_t d\zeta_t^e + (1 - p_t^e) \int_{\zeta_t^u} \int_{z_t} c_{ij}^u(z_t; \zeta_t^u) \varphi_t^u(z_t; \zeta_t^u) dz_t d\zeta_t^u = y_{tj} + \omega_t. \tag{13'}
\]

Finally, budget balance for the government becomes

\[
p_t^P P_{t+i} + (P_t - M_t P_{t+1}) z^g = (1 - p_t^e) \int_{\zeta_t^u} \int_{z_t} P_t b_t(\zeta_t^u) \varphi_t^u(z_t; \zeta_t^u) dz_t d\zeta_t^u, \tag{14'}
\]

\[
\tau^R \int_0^1 x_{tj} dj + T^R_t = 0. \tag{15'}
\]

Policy is summarized by \( \{rr_t, \bar{d}_t, t_t, \bar{z}_t^g\} \), the monetary policy rule, and the subsidy towards retailers, while aggregate fundamentals are \( \{\bar{a}_t, \bar{w}_t, \bar{b}_t, \bar{m}_t, \bar{m}_t(0), \bar{d}_t\} \). Conditional on policy and aggregates, we can redefine a flexible price equilibrium in this enriched environment:

**Definition 3.** A flexible price equilibrium is a sequence of value and policy functions, tightness, employment, nominal prices, and probability measures such that, given policy and macroeconomic aggregates, workers solve their standard lower-stage problem given (1) and upper-stage problems (3'-4u'); producers solve (6'); retailers solve (7); markets clear according to (9'-13'); the government’s budget is balanced according to (14') and (15'); and the probability measures are consistent with \( \tilde{p}_0^e = n_0 \) and \( \int_{z_0} \tilde{p}_0^e(z_0; \zeta_0^e) = p_0^e(\zeta_0^e) \) in period 0 and the above policies and stochastic elements of the model for all future periods.

### 3.2 Calibration and properties of the stationary RCE

I begin by studying the stationary RCE in which policy and aggregates are constant. I calibrate the model at a monthly frequency to match the distribution of wealth, patterns of income, hazards out of unemployment, and disincentive effects of UI estimated in U.S. data. Untargeted moments reveal that wealth and consumption fall upon job loss while MPCs rise sharply with duration — consistent with available evidence, and important drivers of the policy response in the next section.

#### 3.2.1 Calibrating the stationary RCE

I first specify functional forms and certain parameters beyond those already assumed in (37)-(44). For worker-consumers, I assume CRRA flow utility from consumption

\[
u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}
\]
as well as isoelastic disutility of searching when unemployed

\[ \psi(s) = s^{\xi+1}. \]

Relative risk aversion is set to \( \sigma = 1 \), allowing us to normalize average productivity to \( \bar{a} = 1 \) since the environment is consistent with balanced growth. The elasticity of job-finding with respect to tightness is set to \( \eta = 0.7 \) consistent with Petrongolo and Pissarides [2001], and the separation rate is set to \( \delta = 0.034 \) as calculated by Shimer [2010] at a monthly frequency. The persistent-transitory process for worker productivity is set to \( \rho = 0.997, \sigma^P = 0.057, \) and \( \sigma^T = 0.228 \), based on Krueger et al. [2016a]’s estimates adjusted to a monthly frequency.\footnote{Using annual data from the Panel Study of Income Dynamics, these authors estimate a persistent-transitory process for post-tax earnings conditional on employment of \( \rho_P = 0.9695, (\sigma_P^P)^2 = 0.0384, (\sigma_P^T)^2 = 0.0522 \). They then translate these estimates to a quarterly frequency; using their same approach, I adjust these to a monthly frequency using \( \rho = (\rho_P)^{1/12}, (\sigma^P)^2 = (1 - (\rho_P)^2) \frac{(\sigma_P^P)^2}{1 - (\rho_P)^2}, \sigma^T = \sigma_P^T \).} Finally, the assumed parameters of the UI program are a replacement rate of \( rr = 50\% \) for the first \( \bar{d} = 6 \) months of unemployment, consistent with regular benefits in the U.S.\footnote{This replacement rate is roughly consistent with that for the average worker reported by the Department of Labor’s Employment and Training Administration, after accounting for additional savings in taxes.}

I calibrate the remaining parameters to match salient patterns in wealth, income, and unemployment in U.S. data prior to the Great Recession.\footnote{The computational algorithm used to solve the stationary RCE is discussed in appendix D.} The targeted moments and simulated values are summarized in Table 1. The table also indicates the value of the model parameter which is primarily varied in order to target the given moment.

The first set of parameters target key features of wealth and income. Using data on liquid wealth from the 2004 Survey of Consumer Finances (SCF), I find that median wealth equaled 0.3 times average monthly household income, and that the 25th and 75th percentiles equaled 0.0 times and 2.0 times income, respectively.\footnote{My definition of liquid wealth is transaction account balances (checking, saving, money market, and call accounts) plus directly held bonds less credit card balances, like that used in Kaplan et al. [2016].} I use the discount factor \( \beta \) to target median wealth, the borrowing constraint \( \hat{z} \) to target the 25th percentile, and the dispersion in discount factors \( \Delta^\beta \) to target the interquartile range (IQR). To achieve this wealth distribution at a targeted 2\% annualized real interest rate, I use the government’s position in the bond market \( z^g \). Finally, to match Rothstein and Valletta [2014]'s evidence that household income falls by roughly 25\% in the month after job loss and an additional 15\% in the month after UI benefit exhaustion, I use the endowment \( \omega \) and social assistance \( b^{SA} \), respectively.

The second set of parameters target key features of unemployment and job search. Based on monthly data over 1995-2007 from the Bureau of Labor Statistics, I calculate a 5.0\% average unemployment rate with 14\% of the unemployed having duration between 15-26 weeks and 17\% having duration greater than 26 weeks. I use the average wage \( \bar{w} \) to target
### Table 1: targeted moments and calibration results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Achieved</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wealth and income</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real interest rate (ann.)</td>
<td>2%</td>
<td>2.0%</td>
<td>$z^g/\bar{a}$</td>
<td>-1.33</td>
</tr>
<tr>
<td>Median wealth / monthly HH income</td>
<td>0.3</td>
<td>0.3</td>
<td>$\beta$</td>
<td>0.9969</td>
</tr>
<tr>
<td>25ptile wealth / monthly HH income</td>
<td>0.0</td>
<td>-0.1</td>
<td>$\tilde{z}/\bar{a}$</td>
<td>-1.1</td>
</tr>
<tr>
<td>IQR wealth / monthly HH income</td>
<td>2.0</td>
<td>1.4</td>
<td>$\Delta^g$</td>
<td>0.0011</td>
</tr>
<tr>
<td>HH income with UI / pre job loss</td>
<td>0.75</td>
<td>0.75</td>
<td>$\omega/\bar{a}$</td>
<td>1.07</td>
</tr>
<tr>
<td>HH income without UI / pre job loss</td>
<td>0.60</td>
<td>0.61</td>
<td>$b^{SA}/\bar{a}$</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>Unemployment and job search</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>5%</td>
<td>5.0%</td>
<td>$\bar{\bar{u}}/\bar{a}$</td>
<td>0.996</td>
</tr>
<tr>
<td>Fraction w/ duration 4-6 mos</td>
<td>0.14</td>
<td>0.18</td>
<td>$\lambda_0$</td>
<td>1.1</td>
</tr>
<tr>
<td>Fraction w/ duration &gt; 6 mos</td>
<td>0.17</td>
<td>0.20</td>
<td>$\lambda_1$</td>
<td>-0.14</td>
</tr>
<tr>
<td>Share receiving UI</td>
<td>0.39</td>
<td>0.41</td>
<td>$\zeta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Duration elasticity to benefit duration</td>
<td>0.1</td>
<td>0.13</td>
<td>$\xi$</td>
<td>9</td>
</tr>
<tr>
<td>Conventional market tightness</td>
<td>0.634</td>
<td>0.638</td>
<td>$\bar{m}(0)$</td>
<td>0.213</td>
</tr>
<tr>
<td>Fraction of monthly wage to hire worker</td>
<td>0.108</td>
<td>0.112</td>
<td>$k/\bar{a}$</td>
<td>0.069</td>
</tr>
</tbody>
</table>

the 5% unemployment rate, and use the duration-dependence of match efficiency \(\{\lambda_0, \lambda_1\}\) to target fractions 0.14 and 0.17 of unemployed with duration between 4-6 months and greater than 6 months, respectively. I use the probability of UI receipt conditional on job loss \(\zeta\) to target the 39% of unemployed receiving benefits computed by Chodorow-Reich and Karabarbounis [2016] over 1961Q1-2008Q2. Since much of my analysis will focus on policy affecting UI duration, I use the elasticity of workers’ disutility from search \(\xi\) to target a micro-elasticity of unemployment duration to potential duration of benefits of 0.1, within the range surveyed by Schmieder and von Wachter [2016]. Finally, I use the level of match efficiency \(\bar{m}(0)\) to target conventional labor market tightness of 0.634 reported by Hagedorn and Manovskii [2008],\(^{34}\) and the cost \(k\) to target the 10.8% of a recruiter’s monthly wage used in hiring one worker reported by Silva and Toledo [2009].

### 3.2.2 Properties of the stationary RCE

An important takeaway of the calibrated economy is that unemployment, especially long-term unemployment, results in large consumption drops and high sensitivities of consumption to cash-on-hand. This is consistent with available evidence and is the underlying reason for the stimulative effects of UI extensions found in the next section.

The consumption drop and high MPCs associated with unemployment in the model follow from agents’ consumption policy functions and dynamic evolution of wealth. Figure 1 shows

---

\(^{34}\)Conventional tightness refers to vacancies divided by the measure of unemployed agents. It is thus distinct from (unobservable) tightness \(\theta\), which includes search effort and match efficiencies in the denominator.
Figure 1: consumption policy functions and marginal wealth distributions in stationary RCE

Table 2: model-generated quarterly MPC by duration of unemployment

<table>
<thead>
<tr>
<th>Group</th>
<th>Quarterly MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>0.14</td>
</tr>
<tr>
<td>Short-term unemployed ($d \in {0, 1, 2}$)</td>
<td>0.36</td>
</tr>
<tr>
<td>Medium-term unemployed ($d \in {3, 4, 5}$)</td>
<td>0.55</td>
</tr>
<tr>
<td>Long-term unemployed ($d \geq 6$)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The consumption policies and the marginal distributions over wealth for employed agents and unemployed agents in their first month of unemployment ($d = 0$), in their fourth month ($d = 3$), and just after the expiration of benefits ($d = 6$), averaging over all other state variables using the relevant marginal distributions. The policy functions make clear that conditional on wealth, the income losses associated with unemployment lead to lower consumption, amplified at low wealth by precautionary forces. The marginal distributions are consistent with agents decumulating assets through an unemployment spell, as unemployment is a temporary shock. Together, these forces imply consumption losses which worsen, and MPCs which generally rise, through an unemployment spell. Table 2 quantifies the rise in quarterly MPCs out of an unexpected rebate of $500 by duration of unemployment. To borrow a phrase from Akerlof [1978], this suggests that the long-term unemployed are an extremely promising “tag” for high MPCs.

We can have confidence in these features of the model because of the consistency between untargeted moments and available evidence on wealth and consumption. Table 3

---

35 When averaging, I use the same weights at each level of wealth to avoid discontinuities in the graphs.
36 Note that the wealth distribution of the initially unemployed is almost identical to that of the employed.
37 To compute these MPCs, I translate this rebate into model scale using average monthly household income from the 2004 SCF and then simulate agents over the three months including and after receipt. As in Kaplan and Violante [2014], I focus on a $500 rebate to be consistent with the evidence on tax rebates.
Moment Model Estimate

Wealth of unemployed vs. employed

(Median unemployed - Median employed) / monthly HH income 

-0.4 -0.3

Consumption losses from unemployment

Unemployed vs. 12 months prior

-16% -6.8%

Unemployed after UI exhaustion vs. pre job loss

-21% -22%

Quarterly MPC

Economy-wide average

0.15 0.12-0.3

Table 3: untargeted moments and comparison to empirical estimates

summarizes several of these untargeted moments. The model comes close to matching the 30% lower median wealth of unemployed agents relative to employed agents as a share of average monthly income reported in the 2004 SCF.\textsuperscript{38} The model’s consumption losses from unemployment are higher than Gruber [1997]’s estimate that unemployed workers have 6.8% lower consumption than one year earlier, but match almost exactly Ganong and Noel [2017]’s estimate that the consumption in the month after UI exhaustion is 22% lower than just prior to job loss. Finally, the model’s economy-wide quarterly MPC is 15%, consistent with the range of estimates in Parker et al. [2013] and surveyed in Carroll et al. [2015].\textsuperscript{39}

3.3 Transitional dynamics in response to UI policy

I now simulate the effect of UI policy after a shock to fundamentals generates slackness in the labor market like that experienced by the U.S. during the Great Recession. In contrast to the results obtained under flexible prices, nominal rigidities mean that UI extensions lead to a meaningful increase in employment and utilitarian social welfare through their effect on aggregate demand. These effects are more pronounced when MPCs rise more sharply by duration of unemployment or agents have a higher degree of prudence, consistent with the redistribution and precautionary saving channels characterized in the analytical results.

3.3.1 Baseline effects on employment and welfare

To simulate a slack labor market and the effects of UI extensions, I focus on shocks which are unanticipated but then characterized by perfect foresight. This simplifies the computation and seems a reasonable approximation to the beginning of the Great Recession in particular.

I assume these shocks occur in the context of a stark form of nominal rigidity: prior to

\textsuperscript{38}I use variables X6670-X6677 of the full public dataset to identify respondents’ employment status.

\textsuperscript{39}While researchers have estimated MPCs across characteristics such as income and age, I am not aware of estimates of MPC heterogeneity by duration of unemployment. Given its importance in my analytical formulas and quantitative results, measurement of this moment in future work would be extremely valuable.
period 0, all retailers have posted prices as in (16) for all $t \geq 0$. This eliminates any stimulus from UI which might arise from inflation expectations when nominal interest rates are fixed (as at the zero lower bound), allowing me to maintain focus on the effects of redistribution and precautionary saving alone.\textsuperscript{40} Given this assumption on prices, we have:

**Definition 4.** A fully sticky price equilibrium is a sequence of value and policy functions, tightness, employment, nominal prices, and probability measures such that, given policy and macroeconomic aggregates, retailers accommodate desired demand for their varieties provided they can earn non-negative profits at prices (16), and all other components of the equilibrium definition are as in Definition 3.

To capture a slack labor market relevant for the evaluation of changes in UI policy, I choose a shock to fundamentals at $t = 0$ which, together with the assumed stickiness of prices and monetary policy rule described below, can deliver a path for long-term unemployment close to the U.S. experience between July 2008 and June 2009. I focus on an unanticipated shock $\epsilon_0^\beta$ with persistence $\rho^\beta$ to the economy-wide average discount factor

$$\ddot{\beta}_t = \beta + \epsilon_t^\beta,$$

$$\epsilon_t^\beta = \rho^\beta \epsilon_{t-1}^\beta,$$

while I assume that monetary policy follows

$$i_t = \begin{cases} r_t^n & \text{when } r_t^n \geq 0, \\ 0 & \text{when } r_t^n < 0, \end{cases}$$

where the nominal interest rate $i_t \equiv \frac{1}{M_t} - 1$ and the natural rate of interest $r_t^n$ is the real interest rate under flexible prices. All other macroeconomic parameters $\{\bar{a}_t, \bar{w}_t, \bar{z}_t, \bar{m}_t(0), \delta_t\}$ remain at their steady-state levels.\textsuperscript{41} To revalue agents’ wealth given the initial change in the value of firm equity, I assume portfolio shares at the beginning of $t = 0$ which are consistent with the 2004 SCF.\textsuperscript{42} The resulting sticky price equilibrium, depicted in Figure 2, features

\textsuperscript{40}The mechanism through inflation expectations has been characterized in general in Eggertsson [2010] and Eggertsson and Krugman [2012], and studied in the UI context by Christiano et al. [2015].

\textsuperscript{41}A zero wage elasticity to UI is consistent with available micro evidence (e.g., Card et al. [2007], Lalove [2007], and van Ours and Vodopivec [2008]). In recent work, Hagedorn et al. [2016a] have estimated a positive macro wage elasticity to a change in benefit duration. I study the effects of such an elasticity in appendix C, and summarize my analysis later in this section.

\textsuperscript{42}The only component of liquid wealth (as defined in footnote 33) which involves exposure to corporate profits is directly held bonds. Hence, using the 2004 SCF, I first compute the average fraction of liquid wealth made up by directly held bonds for each 5% quantile of the wealth distribution. I find that this measure becomes positive only in the 10th quantile (that is, households with wealth between the 45th and 50th percentiles) and tends to rise after that, reaching 1.2% in the 16th quantile and 2.3% in the 19th
Figure 2: equilibrium without UI shock

a binding zero lower bound and long-term unemployment which closely tracks the data.\textsuperscript{43,44}

In this context, I find that an increase in UI duration in the presence of nominal rigidities reverses the positive and normative conclusions obtained in the standard flexible price analysis. I focus in particular on an extension of benefits from 6 to 9 months announced at \( t = 0 \) and lasting through \( t = 11 \). As is evident from Figure 3 and Table 4, the unemployment rate falls by an average of 0.07pp in the year of extended benefits, and utilitarian social welfare rises when the benefits are announced, both in contrast to the flexible price case. Expressing the welfare effects as equivalent changes to consumption at all dates and states, Figure 4 demonstrates that the welfare gains under sticky prices are especially high for unemployed agents and asset-poor but income-rich employed agents. Evidently, for this latter set of agents, the aggregate demand externality from more generous UI considerably outweighs the welfare cost from higher taxes when employed.\textsuperscript{45}

\textsuperscript{43}The shock parameters chosen to achieve these outcomes are \( \epsilon^0 = 0.0051 \) and \( \rho^0 = 0.9659 \) (consistent with a half-life of 20 months). The computational algorithm used to characterize the transitional dynamics builds on the approach of Guerrieri and Lorenzoni \cite{GuerrieriLorenzoni2017} and is discussed further in appendix D.

\textsuperscript{44}Under sticky prices, the initial collapse in aggregate demand resulting from perfect foresight and a binding zero lower bound means that the greatest reduction in hiring occurs in the initial period of the simulation, explaining why the long-term unemployment rate peaks 6 months after the shock. Indeed, I find that the collapse in demand is such that firms do not wish to post any vacancies in the initial period of the simulation. To obtain an interior equilibrium, I allow them to costlessly (and randomly) lay off workers.

\textsuperscript{45}A comprehensive analysis of agents’ change in welfare across each dimension of the idiosyncratic state space is presented in appendix C. In the present simulation under sticky prices, extending UI benefits in fact leads to a Pareto improvement. This is not, however, the case under all of the alternative calibrations.
Figure 3: effects of UI shock on unemployment

<table>
<thead>
<tr>
<th></th>
<th>Flex</th>
<th>Sticky + fixed MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>%Δ utilitarian social welfare</td>
<td>-0.001%</td>
<td>+0.005%</td>
</tr>
<tr>
<td>Equivalent %Δ lifetime consumption</td>
<td>-0.001%</td>
<td>+0.003%</td>
</tr>
</tbody>
</table>

Table 4: effects of UI shock on social welfare

3.3.2 Sensitivity and roles of redistribution and precautionary saving

Assessing the sensitivity of the policy responses to alternative calibrations allows us to better understand the roles of redistribution and precautionary saving, as characterized in section 2, in driving these equilibrium effects under sticky prices.

Table 5 compares three such alternative calibrations of the steady-state to the baseline.\(^{46}\) First, I study a version of the model featuring dynamic selection rather than structural duration dependence: job-finding probabilities per unit effort are constant through an unemployment spell, but there are two types of agents having different match efficiencies.\(^{47}\) Relative to the baseline, the resulting stationary RCE features a considerably flatter profile of MPCs by duration of unemployment, consistent with the long-term unemployed having lower permanent income than in the baseline model. Second, I study a version of the model featuring a greater role for precautionary saving: the assumed coefficient of relative risk aversion is \(\sigma = 4\). Relative to the baseline with \(\sigma = 1\), this implies a higher coefficient of relative prudence under CRRA utility, but also flattens the profile of MPCs by duration of

\(^{46}\)Further details for each calibration are provided in appendix C.

\(^{47}\)Concretely, I set \(\lambda_0 = \lambda_1 = 0\) and instead introduce a new source of heterogeneity among agents: a fraction \(\mu\) have match efficiency \(\bar{m}^1\) whenever they are unemployed, and a fraction \(1 - \mu\) have match efficiency \(\bar{m}^2\) whenever they are unemployed. Workers know their own match efficiencies. In the steady-state calibration, \(\{\mu, \bar{m}^2\} \) replace \(\{\lambda_0, \lambda_1\} \) in matching the share of medium-term and long-term unemployed.
unemployment. Third and finally, I study a version of the model featuring a greater search response to UI: the targeted micro elasticity of unemployment duration to potential duration of benefits is 0.4. This exceeds the baseline target of 0.1, and is at the high end of estimates for the U.S. summarized in Schmieder and von Wachter [2016].

Table 6 compares the macroeconomic effects of extending UI duration from 6 to 9 months for one year in a similarly slack labor market starting from the alternative steady-states.\textsuperscript{48} In each case I provide the employment and welfare effects as well the output multiplier

\[
\text{output multiplier} = \frac{\sum_{t=0}^{11} \Delta \text{output}_t}{\sum_{t=0}^{11} \Delta \text{UI payments}_t},
\]

where the denominator is computed using the initial distribution of agents across the state space.\textsuperscript{49,50} This serves as a convenient metric across which to compare experiments. To further summarize the decomposition between the initial and final impulses to aggregate consumption, I define the partial equilibrium output multiplier with the change in \(c_t^{PE}\) in

\textsuperscript{48}For this sensitivity analysis, I first re-calibrate the discount factor shock to achieve a similar path of long-term unemployment as that in Figure 2. In the dynamic selection case, this implies \(\epsilon^\beta = 0.0053\) and \(\rho^\beta = 0.9690\) (consistent with a half-life of 22 months); in the \(\sigma = 4\) case, this implies \(e^\beta = 0.0087\) and \(\rho^\beta = 0.9798\) (consistent with a half-life of 34 months); and in the case of a higher target for the disincentive effect, this implies \(\epsilon^\beta = 0.0047\) and \(\rho^\beta = 0.9690\) (again consistent with a half-life of 22 months).

\textsuperscript{49}The denominator thus ignores the possibility that UI lowers the long-term unemployment rate during the year of extended benefits — as it does across all of the sticky price simulations presented in this paper.

\textsuperscript{50}An alternative measure of interest replaces the numerator by the discounted sum of output effects in all future periods. Owing to my assumption of fully sticky prices, this measure tends to be slightly lower than the output multiplier reported here (though still positive): the reduction in permanent income among the employed lowers their demand, which mildly depresses equilibrium output after the UI extensions have ended. In simulations available on request, I find that this effect is eliminated if prices become flexible in month 12, and is thus likely to be minimized in the (empirically realistic) case of partial price stickiness.
Table 5: alternative calibrations of the steady-state

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Dynamic selection</th>
<th>$\sigma = 4$</th>
<th>Higher disincentive effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly MPC LT unemp. - emp.</td>
<td>0.69</td>
<td>0.37</td>
<td>0.58</td>
<td>0.67</td>
</tr>
<tr>
<td>Coeff. of relative prudence</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Duration elast. to UI duration</td>
<td>0.13</td>
<td>0.12</td>
<td>0.15</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 6: sensitivity of effects of UI shock under sticky prices

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Dynamic selection</th>
<th>$\sigma = 4$</th>
<th>Higher disincentive effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>PE output multiplier</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Output multiplier</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Avg change in unemp. rate</td>
<td>-0.07pp</td>
<td>-0.04pp</td>
<td>-0.07pp</td>
<td>-0.07pp</td>
</tr>
<tr>
<td>%Δ util. SW, lifetime cons. equiv.</td>
<td>+0.003%</td>
<td>+0.002%</td>
<td>+0.001%</td>
<td>+0.004%</td>
</tr>
</tbody>
</table>

Comparing the effects of UI across these calibrations confirms that the redistribution and precautionary saving responses to UI generosity, rather than the conventional effect on labor supply, dominate the equilibrium response. Relative to the baseline calibration, the flatter profile of MPCs by duration of unemployment under dynamic selection mitigates the redistribution effect and lowers the stimulus from UI. While a similar effect would be at work in the $\sigma = 4$ calibration, the offsetting amplification from greater prudence and thus precautionary savings leaves the stimulus from UI virtually unchanged from the baseline case. And while the calibration with a higher disincentive effect of UI is associated with considerable mitigation of the partial equilibrium effects of UI extensions, this effect is undone by the general equilibrium response of vacancies to leave the overall stimulus from UI unchanged — consistent with the insights of section 2. Accounting for the range of these effects, I conclude that in a slack labor market like that of the U.S. during the first year of the Great Recession, a 3-month extension of UI for one year reduces the unemployment rate by an average of 0.04pp-0.07pp in the year of extended benefits and raises utilitarian social welfare.

In appendix C, I describe several additional policy experiments illustrating the quantita-

---

$^{51}$I compute the required change in taxes on the employed in the month of extended benefits by balancing the government budget without regard to the behavioral responses to UI policy. Hence, $c_{t}^{PE}$ should be not be interpreted as reflecting the path of consumption in standard public finance models which, despite working in partial equilibrium, account for the fiscal externality.
tive importance of the redistribution and precautionary saving channels. First, I decompose the one-year extension of UI into month-by-month extensions which isolate each channel. I find that both are important and that they interact and amplify each other in general equilibrium. Second, I compare the effects of UI extensions in a slack labor market with those starting from steady-state. I find that the stimulus and welfare gains from UI are amplified when the labor market is slack, consistent with section 2’s result that such slackness increases the wedge between the social and private value of UI. Third, I assess how the effects of UI extensions change when equilibrium wages rise in response. In stark contrast to the flexible price case, the stimulus and welfare gains from UI only further increase: the temporary increase in wages reallocates income away from dividends (disproportionately earned by the low-MPC asset-rich) and towards labor income (disproportionately earned by the high-MPC asset-poor), further raising aggregate demand just like the redistribution effect of UI. Finally, I compare UI extensions to an alternative policy of raising the replacement rate. I find higher multipliers associated with extensions, consistent with the long-term unemployed being an especially good “tag” in stabilization because they have very high MPCs and because long-term unemployment is an important risk against which agents save.

3.3.3 Relation to debates over UI policy during Great Recession

The 3-month extension of UI simulated in the above experiments is (deliberately) consistent with the 13-week extension signed into law on June 30, 2008 in the Emergency Unemployment Compensation Act of 2008. However, subsequent extensions together with the Extended Benefits program meant that UI duration eventually rose to 99 weeks in some states at the depth of the Great Recession. Linearly extrapolating, the present framework under sticky prices and fixed monetary policy implies that an extension from 26 to 99 weeks for one year would reduce the unemployment rate by 0.2–0.4pp that year.

These magnitudes mean that the redistribution and precautionary saving effects of UI can rationalize recent estimates of its macroeconomic stimulus. In particular, the reduction in unemployment is consistent with the confidence interval estimated by Chodorow-Reich and Karabarbounis [2017] in their analysis of extensions during the Great Recession. The output multipliers are further consistent with the confidence interval estimated by di Maggio and Kermani [2016] in their analysis of UI stabilizing individual states in response to shocks.

This being said, because I simplify several real-world features to focus on quantification of the redistribution and precautionary saving effects of UI, my approach cannot resolve the

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52 These authors extrapolate from their 95% confidence interval in Table 4 to conclude that extensions from 26 to 99 weeks reduced the unemployment rate by at most 0.5pp and increased it by at most 0.3pp.

53 Substituting these authors’ 95% confidence interval from Table 8(B) into their formula for the earnings multiplier in section 6, their estimates imply that UI has a (local) multiplier between 0.8 and 3.0.
empirical debate over its overall effects. First, I study nominal rigidities in the polar case of fully rigid prices. As recently explored by Christiano et al. [2015], the effect of UI on inflation would also play a role when prices are partially sticky, either mitigating or amplifying the results here depending on the response of monetary policy and the resulting behavior of real interest rates. Second, my analysis ignores capital investment and accumulation, motivated again by my focus on the effects of UI when real interest rates are fixed. If real interest rates rise in response to UI, greater generosity could crowd out capital accumulation and render it more contractionary, consistent with the recent findings of Krueger et al. [2016a,b]. Finally, my analysis ignores the typical financing of UI extensions through debt rather than taxes, as was the case during the Great Recession. Because the model is non-Ricardian, this would be expected to render UI more expansionary than the simulations here.

4 Conclusion

This paper contributes to our understanding of the role that the UI system, a key part of the social safety net in advanced economies, can play in macroeconomic stabilization.

In a general equilibrium environment combining incomplete markets, search frictions, and nominal rigidities, I find that the interaction between UI and aggregate demand naturally motivates higher generosity when the economy is slack. An increase in transfers to the unemployed can be expansionary if the unemployed have a higher MPC than the employed or if agents have a precautionary saving motive, motivating optimally more generous UI than the public finance benchmark when the economy is slack. In a calibration matching features of the U.S. economy in 2008-09, extending benefits by 3 months for one year reduces the unemployment rate by an average of 0.04–0.07pp in the year of extended benefits and raises utilitarian social welfare through these channels.

The mechanisms emphasized in this paper should apply more broadly to other social insurance and cash transfer programs. This suggests that a small set of behavioral responses, including MPCs and precautionary savings elasticities, will play a key role in determining the relative effects of these programs in macroeconomic stabilization — a useful perspective, in view of the variety and importance of these programs in actual government budgets (Oh and Reis [2012]). I leave the comparison of these programs, and the comparison of UI and these programs to more standard government purchases, to future research.
References


Appendix For Online Publication

A Proofs

In this section I provide proofs of the results outlined in section 2 of the main text.

A.1 Lemma 1

Proof. By (17) and (18), $y^c_t(m_1, y^u_t)$, $\theta_t(m_1, y^u_t)$, and $s_t(m_1, y^u_t)$ are implicitly defined by the three-equation system

\begin{align}
  x_t(s_t(m_1, y^u_t), \theta_t(m_1, y^u_t), c_t^1(m_1, y^u_t), c_t^2(m_1, y^u_t)) &= 0, \\
  x_t(s_t(m_1, y^u_t), \theta_t(m_1, y^u_t), c_t^2(m_1, y^u_t), c_t^3(m_1, y^u_t)) &= 0, \\
  s_t(m_1, y^u_t) &= s_t(m_1, y^c_t(m_1, m_1, y^u_t), y^u_t, \theta_t(m_1, y^u_t)).
\end{align}

Differentiating (A.1) and (A.2) with respect to $y^u_t$, we obtain

\begin{equation}
  \frac{\partial x_t}{\partial s_t} \frac{\partial s_t(m_1, y^u_t)}{\partial y^u_t} + \frac{\partial x_t}{\partial \theta_t} \frac{\partial \theta_t(m_1, y^u_t)}{\partial y^u_t} - p_{t1} \frac{\partial c_t^1}{\partial y^u_t} \frac{\partial y^1_t(m_1, y^u_t)}{\partial y^u_t} - (1 - p_{t1}) \frac{\partial c_t^u}{\partial y^u_t} = 0
\end{equation}

for $t \in \{1, 2\}$, where I have used $\frac{\partial x_t}{\partial s_t} = -p_{t1}$ and $\frac{\partial x_t}{\partial \theta_t} = -(1 - p_{t1})$ given employment $p_t$ as defined in the main text. Adding up (A.4) across $t \in \{1, 2\}$ and using the identity for Marshallian demand that $\frac{\partial c_t^1}{\partial y^u_t} + m_1 \frac{\partial c_t^2}{\partial y^u_t} = 1$, we obtain

\begin{equation}
  \frac{\partial y^1_t(m_1, y^u_t)}{\partial y^u_t} = \frac{1}{p_{t1}} \left[ - (1 - p_{t1}) + \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1(m_1, y^u_t)}{\partial y^u_t} + \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) \frac{\partial \theta_1(m_1, y^u_t)}{\partial y^u_t} \right].
\end{equation}

Substituting (A.5) into (A.4) at $t = 1$ and collecting terms, we obtain

\begin{equation}
  \frac{\partial \theta_1(m_1, y^u_t)}{\partial y^u_t} = \frac{(1 - p_{t1}) \left( \frac{\partial c_t^1}{\partial y^u_t} - \frac{\partial c_t^2}{\partial y^u_t} \right)}{\frac{\partial x_1}{\partial s_1} - \frac{\partial x_2}{\partial s_1} \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right)} - \frac{\partial x_1}{\partial \theta_1} - \frac{\partial x_2}{\partial \theta_1} \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) \frac{\partial s_1(m_1, y^u_t)}{\partial y^u_t}.
\end{equation}

This defines $\mu_{y^1_t}^{\theta_1}$ and $\mu_{s_1}^{\theta_1}$, where expanding out $\frac{\partial x_1}{\partial \theta_1}$, $\frac{\partial x_2}{\partial \theta_1}$, and $\frac{\partial x_2}{\partial s_1}$ yields the expressions
in (22) and (23). And substituting (A.5) into (A.3) and collecting terms, we obtain

\[
\frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u} = \frac{\frac{\partial s_1}{\partial y_1^u} - \frac{1-p^e_1}{p_1} \frac{\partial s_1}{\partial y_1^u}}{1 - \frac{1}{p_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right)} + \frac{\frac{\partial s_1}{\partial y_1^u} + \frac{1}{p_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right)}{1 - \frac{1}{p_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right)} \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}. \tag{A.7}
\]

This defines \(\mu^{s_1}_{y_1^u}\) and \(\mu^{s_1}_{\theta_1}\) in (24) and (25) after expanding out \(\frac{\partial x_1}{\partial y_1^u}, \frac{\partial x_2}{\partial y_1^u}, \frac{\partial x_1}{\partial s_1}, \) and \(\frac{\partial x_2}{\partial s_1}\). \(\square\)

### A.2 Proposition 1

**Proof.** The result is an immediate consequence of solving the pair of equations (A.6) and (A.7) for \(\frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}\) and \(\frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u}\). \(\square\)

### A.3 Proposition 2

**Proof.** By (19) and (20) and given \(y_1^e(m_1, y_1^u), \theta_1(m_1, y_1^u)\), and \(s_1(m_1, y_1^u)\) described in the proof of Lemma 1, \(c_0(m_0, m_1, y_1^u)\) is implicitly defined by

\[
\frac{\partial u_0}{\partial c_0} (c_0(m_0, m_1, y_1^u), f_0^{-1}(c_0(m_0, m_1, y_1^u))) = \frac{\beta}{m_0} \left[ (1 - \delta + \delta p(\theta_1(m_1, y_1^u))s_1(m_1, y_1^u)) \frac{\partial v_1^e}{\partial y_1^u} (m_1, y_1^u)) + \delta (1 - p(\theta_1(m_1, y_1^u))s_1(m_1, y_1^u)) \frac{\partial v_1^u}{\partial y_1^u} (m_1, y_1^u)) \right]. \tag{A.8}
\]

Differentiating by \(y_1^u\), we obtain

\[
\frac{\partial c_0(m_0, m_1, y_1^u)}{\partial y_1^u} = -\frac{d}{dc_0} u_0, c_0, f_0^{-1}(c_0) \times \left[ - (1 - p^e_1) \frac{\partial^2 v_1^u}{\partial (y_1^u)^2} - p^e_1 \frac{\partial^2 v_1^e}{\partial (y_1^u)^2} \frac{\partial y_1^u}{\partial y_1^u} + \delta p(\theta_1) \left( \frac{\partial v_1^u}{\partial y_1^u} - \frac{\partial v_1^e}{\partial y_1^u} \right) \frac{\partial s_1(m_1, y_1^u)}{\partial y_1^u} \right] + \delta p'(\theta_1)s_1 \left( \frac{\partial v_1^u}{\partial y_1^u} - \frac{\partial v_1^e}{\partial y_1^u} \right) \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \right].
\]
where I write \( u_{0,c}(c_0, f_0^{-1}(c_0)) \equiv \frac{\partial m_0}{\partial c_0} (c_0, f_0^{-1}(c_0)) \) for brevity. Plugging in (A.5) and collecting terms, we then obtain

\[
\frac{\partial c_0(m_0, m_1, y^u_1)}{\partial y^u_1} = \frac{\beta m_0}{1 - \frac{d}{dc_0} u_{0,c}(c_0, f_0^{-1}(c_0))} \times \\
(1 - p_1^e) \left( \frac{\partial^2 v_1^e}{\partial (y^u_1)^2} - \frac{\partial^2 v_1^u}{\partial (y^u_1)^2} \right) + \\
\left( \delta p(\theta_1) \left( \frac{\partial v_1^u}{\partial y^u_1} - \frac{\partial v_1^e}{\partial y^u_1} \right) - \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial^2 v_1^e}{\partial (y^u_1)^2} \right) \frac{\partial s_1(m_1, y^u_1)}{\partial y^u_1} + \\
\left( \delta p(\theta_1)^{s_1} \left( \frac{\partial v_1^u}{\partial y^u_1} - \frac{\partial v_1^e}{\partial y^u_1} \right) - \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) \frac{\partial^2 v_1^e}{\partial (y^u_1)^2} \right) \frac{\partial \theta_1(m_1, y^u_1)}{\partial y^u_1} 
\]

Finally, noting by (A.8) that

\[
\frac{\beta m_0}{1 - \frac{d}{dc_0} u_{0,c}(c_0, f_0^{-1}(c_0))} = \frac{m_0}{p_1^e \frac{\partial v_1^e}{\partial y^u_1}} + (1 - p_1^e) \frac{\partial v_1^u}{\partial y^u_1},
\]

this defines \( \mu^c_{y^u_1}, \mu^c_{s_1}, \) and \( \mu^c_{\theta_1} \) in (28)-(30) after expanding out \( \frac{\partial x_1}{\partial y^u_1}, \frac{\partial x_2}{\partial y^u_1}, \frac{\partial x_1}{\partial s_1}, \) and \( \frac{\partial x_2}{\partial s_1} \).

\[\square\]

**A.4 Lemma 2**

*Proof.* The text in section 2.1.3 proves \( \Rightarrow \), so here I focus on \( \Leftarrow \). The goal is to show that an allocation \( \{c_0, c_1', c_1'', c_2, c_2', s_1, \theta_1\} \) satisfying the given resource and implementability constraints at \( \{y^u_1, y^u_2\} \) and \( \{m_0, m_1\} \) forms part of an equilibrium according to Definition 2. I will demonstrate that appropriate prices, wages, and policies ensure that first-order conditions, resource constraints, and market clearing conditions are satisfied (and will assume that agents’ first-order conditions are sufficient to characterize optimality).

I start with monetary policy, which implements \( \{m_0, m_1\} \) as in the claim. Indeed, given \( P_0 = P_1 = \bar{P} \), the Fisher equation pins down the necessary \( i_0 \) and set of \( \{i_1, P_2\} \).

I now turn to prices, wages, and the payroll tax \( \tau_1 \). Producers’ first-order condition with respect to \( \nu_1 \) pins down \( \frac{W_1}{P_1} = W_2 \frac{P_1}{P_1} \). The specific mix of real wage \( \frac{W_1}{P} \) and markup \( \frac{P}{P_1} \) is indeterminate, however, because of policymakers’ ability to vary the payroll tax \( \tau_1 \).\footnote{As noted in footnote 19 in the main text, this argument of course does not apply at the limit of a fully rigid real wage in (8), where the markup would then be determinate. But it becomes indeterminate if the tax could be assessed on producers rather than workers in that case.}

Conditional on picking any \( \tau_1 \) (subject only to \( \frac{P}{P_1} \geq 1 \), as described further below), the wage equation (8) then pins down \( \frac{W_1}{P} \) and \( \frac{P}{P_1} \), and thus \( W_1 \) and \( P_1 \) given sticky prices, respectively.
Finally, I consider UI policy. The given allocation coupled with above real wage pin down firm dividends (net of the tax financing the retailer subsidy) \( \pi_1 \). Together with the given cash-on-hand of unemployed agents \( y_u^1 \), the resource constraint of unemployed agents then pins down UI \( b_1 \). The resource constraint of employed agents pins down lump-sum taxes \( t_1 \).

Since the implementability constraints are assumed satisfied at the given allocation, it follows that worker-consumer first-order conditions and resource constraints are satisfied. Retailers are indeed acting optimally provided that \( \frac{\bar{P}_1}{P_1} \geq 1 \), which can be ensured with the payroll tax \( \tau_1 \). It only remains to check market clearing.

Market clearing in both intermediate and final goods is implied by the resource constraints which are assumed satisfied at the allocation. Labor market clearing in period 0 is implied by \( c_0 = c_0(m_0, m_1, y_e^1, y_u^1, \theta_1) \) and \( h_0 = h_0(m_0, m_1, y_e^1, y_u^1, \theta_1) \). \( s_1 \) and \( \theta_1 \) imply \( \nu_1 \equiv \theta_1(\delta s_1) \) satisfying consistency with individual optimization in the period 1 labor market. Asset market clearing and the government budget constraint are then satisfied by Walras’ Law.  

### A.5 Proposition 3

**Proof.** First note that

\[
\frac{\partial v_0(m_0, m_1, y_u^1)}{\partial y_u^1} = \frac{\partial}{\partial y_u^1} \left[ u_0(c_0(m_0, m_1, y_u^1), h_0(m_0, m_1, y_u^1)) + \beta(1 - \delta + \delta p(\theta_1(m_1, y_u^1))s_1(m_1, y_u^1))v_e^1(m_1, y_e^1(m_1, y_u^1)) + \delta (1 - p(\theta_1(m_1, y_u^1))s_1(m_1, y_u^1))v_u^1(m_1, y_u^1) - \delta \psi(s_1(m_1, y_u^1))) \right]
\]

\[
= \frac{\partial u_0}{\partial c_0} \frac{\partial c_0(m_0, m_1, y_u^1)}{\partial y_u^1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0(m_0, m_1, y_u^1)}{\partial y_u^1} + \beta \left[ (1 - p_1) \frac{\partial v_1^1}{\partial y_u^1} + p_1 \frac{\partial v_1^e}{\partial y_u^1} \frac{\partial y_u^1}{\partial y_u^1} + \delta p'(\theta_1) s_1(v_e^1 - v_u^1) \frac{\partial \theta_1(m_1, y_u^1)}{\partial y_u^1} \right]
\]

(A.9)

where the Envelope Theorem implies that the marginal social welfare effect of changing search is zero. Using (20) we obtain

\[
\frac{\partial h_0(m_0, m_1, y_u^1)}{\partial y_u^1} = \frac{1}{f_0'(h_0)} \frac{\partial c_0(m_0, m_1, y_u^1)}{\partial y_u^1}.
\]
And combining (A.5) with (A.7), we obtain

\[
\frac{\partial y_1^u(m_1, y_1^u)}{\partial y_1^u} = -\left(1 - \frac{p_1^e}{p_1^c}\right) \left(1 - \frac{1}{1 - p_1^e} \left(\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\right) \frac{\partial s_1}{\partial y_1^u}\right) + \frac{1}{p_1^e} \left[\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\right] \frac{\partial s_1}{\partial y_1^u} \partial \theta_1(m_1, y_1^u).
\]

Plugging these last two results into (A.9) and collecting terms, we have

\[
\frac{\partial u_0(m_0, m_1, y_1^u)}{\partial y_1^u} = \left[\frac{\partial u_0}{\partial c_0} + \frac{1}{f_0'(h_0)} \frac{\partial u_0}{\partial h_0}\right] \frac{\partial c_0(m_0, m_1, y_1^u)}{\partial y_1^u} + \beta \left(1 - p_1^e\right) \left(\frac{\partial v_1^e}{\partial y_1^u} - \frac{\partial v_1^c}{\partial y_1^u}\right) \left(1 - \frac{1}{1 - p_1^e} \left(\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\right) \frac{\partial s_1}{\partial y_1^u}\right) + \left[\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1}\right] \frac{\partial s_1}{\partial y_1^u} \partial \theta_1(m_1, y_1^u).
\]

The first expression in brackets is simply the labor wedge (33). Using

\[
\frac{\partial s_1}{\partial \theta_1} = p'(\theta_1)(v_1^e - v_1^u) \frac{\partial s_1}{\partial y_1^u},
\]

implied by workers’ optimal search \(s_1(m_1, y_1^e, y_1^u, \theta_1)\), it is straightforward to see that the second expression in brackets is the tightness wedge (34). Factoring out \(\beta\) and splitting \(\frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u}\) into the components corresponding to aggregate demand and search described in Proposition 1 then yields the claimed result.

**A.6 Lemma 3**

**Proof.** By differentiating \(x_1(\cdot)\) and \(x_2(\cdot)\), it is straightforward to show

\[
\frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} = \delta p(\theta_1) \left[f_1'(\cdot) \left(1 - \frac{k}{q(\theta_1)}\right) + m_1(y_2^e - y_2^u) - [(c_1^e + m_1 c_2^e) - (c_1^u + m_1 c_2^u)]\right]
\]
as stated in the main text. Now
\[ f'_1(\cdot) \left( 1 - \frac{k}{q(\theta_1)} \right) = \frac{W_1}{P'_1} = \mu_1 w_1, \]
where the first equality uses producer optimization in vacancy-posting and the second uses
the definitions of the gross retailer mark-up \( \mu_1 \equiv \frac{P_1}{P'_1} \) and real wage \( w_1 \equiv \frac{w_1}{P_1} \). Thus,
\[ f'_1(\cdot) \left( 1 - \frac{k}{q(\theta_1)} \right) + \frac{m_1(y^e_2 - y^u_2)}{\sqrt{1 - p^e_1 \frac{m_1}{m^e_0}}} \left[ \frac{1}{\beta} \frac{\partial c_0(m_0, m_1, y^u_1)}{\partial y^u_1} + \tau_1 \frac{\partial \theta_1(m_1, y^u_1)}{\partial y^u_1} \right] \left( 1 - \frac{1}{p'_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial s_1} \right) = 0. \] (A.12)

We further have that
\[
\begin{align*}
& w_1 + m_1(y^e_2 - y^u_2) - [(c^e_1 + m_1 c^e_2) - (c^u_1 + m_1 c^u_2)] \\
& = w_1 + m_1(y^e_2 - y^u_2) - [(c^e_1 + m_1 c^e_2) - (c^u_1 + m_1 c^u_2)], \\
& = w_1 - (y^e_1 - y^u_1), \\
& = \tau_1 w_1 + t_1 + b_1, \\
& = \frac{1 - p^e_1}{p^e_1} b_1 + b_1, \\
& = \frac{1}{p^e_1} b_1, \\
& = 1 - p^e_1 (p^e_1 b_1 + b_1), \\
& = 1 - p^e_1 b_1,
\end{align*}
\] (A.11)

where the first equality uses agents’ intertemporal resource constraints starting in period 1,
the third equality uses the definition of agents’ cash-on-hand in period 1, and the fourth
equality uses the government budget constraint in period 1. Combining (A.10) and (A.11)
yields the claimed result.

A.7 Proposition 4

Proof. Setting marginal social welfare in Proposition 3 to zero, we have
\[
\frac{\partial v^u_1}{\partial y^u_1} + \frac{1}{1 - p^e_1} \left[ \frac{1}{\beta} \frac{\partial c_0(m_0, m_1, y^u_1)}{\partial y^u_1} + \tau_1 \frac{\partial \theta_1(m_1, y^u_1)}{\partial y^u_1} \right] \left( 1 - \frac{1}{p'_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial s_1} \right) = 0.
\] (A.12)
Thus (A.14) can be written

\[
\frac{\partial v_1^n}{\partial y_1^n} \frac{\partial s_1}{\partial y_1^n} = - \frac{\partial v_1^e}{\partial y_1^e} \frac{\partial s_1}{\partial y_1^e},
\]

(A.13)

implied by workers’ optimal search, (A.12) becomes

\[
- \frac{1}{p_1^e(1 - p_1^e)} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n} + m_1 \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial y_1^n} = \frac{\partial v_1^e}{\partial y_1^e} - \frac{\partial v_1^e}{\partial y_1^e} + \frac{1}{1 - p_1^e} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n},
\]

Then note that

\[
1 \frac{1}{1 - p_1^e} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n} = \frac{1}{1 - p_1^e} \frac{1}{p_1^e} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n} + \frac{1}{p_1^e(1 - p_1^e)} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n},
\]

where the first equality uses (A.13), the second is a straightforward algebraic manipulation, and the last uses the harmonic mean of marginal utilities of income in period 1:

\[
\frac{\partial v_1}{\partial y_1} = \frac{p_1^e}{\frac{\partial x_1}{\partial s_1} + 1 - p_1^e}.
\]

Thus (A.14) can be written

\[
- \frac{1}{p_1^e(1 - p_1^e)} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^n} = \frac{\partial v_1^e}{\partial y_1^e} - \frac{\partial v_1^e}{\partial y_1^e} + \frac{1}{1 - p_1^e} \frac{1}{p_1^e} \left[ \frac{1}{\beta} \gamma_0 \frac{\partial c_0(m_0, m_1, y_1^n)}{\partial y_1^n} + \gamma_1 \frac{\partial \theta_1(m_1, y_1^n)}{\partial y_1^n} \right] + \frac{1}{1 - p_1^e} \frac{1}{p_1^e} \left[ \frac{1}{\beta} \gamma_0 \frac{\partial c_0(m_0, m_1, y_1^n)}{\partial y_1^n} + \gamma_1 \frac{\partial \theta_1(m_1, y_1^n)}{\partial y_1^n} \right].
\]

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Finally, implementation of this Ramsey optimality condition uses Lemma 3, which implies that on the left-hand side
\[
- \frac{1}{p_1^c (1 - p_1^c)} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^u} = - \frac{1}{p_1^c (1 - p_1^c)} \left( \delta p(\theta_1) \left( \frac{1}{p_1^c} b_1 + (\mu_1 - 1) w_1 \right) \right) \frac{\partial s_1}{\partial y_1^u},
\]

\[
= \left( \frac{1}{p_1^c} \right)^2 \varepsilon b_1^{1 - p_1^c} \left( 1 + (\mu_1 - 1) w_1 \right),
\]

where the second equality uses the micro elasticity of unemployment in period 1 with respect to UI characterized in (36). It follows that the implementation of Ramsey optimal risk-sharing requires

\[
\left( \frac{1}{p_1^c} \right)^2 \varepsilon b_1^{1 - p_1^c} \left( 1 + (\mu_1 - 1) w_1 \right) = \frac{\partial v_1^e}{\partial y_1^u} - \frac{\partial v_1^u}{\partial y_1^u} + \frac{1}{1 - p_1^c} \frac{1}{\sigma_{11}} \left[ \frac{1}{\beta} \tau_0^h \frac{\partial c_{01}(m_0, m_1, y_1^u)}{\partial y_1^u} + \tau_1^h \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \right] + \frac{1}{1 - p_1^c} \frac{1}{\sigma_{11}} \left[ \frac{1}{\beta} \tau_0^h \frac{\partial c_{11}(m_0, m_1, y_1^u)}{\partial y_1^u} + \tau_1^h \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \right].
\]

Taking a Taylor approximation to further simplify the right-hand side for small \( \tau_0^h \) and \( \tau_1^h \), assuming that the payroll tax \( \tau_1 \) implements \( \mu_1 = 1 \), replacing \( \frac{\partial v_i}{\partial y_i} \) with \( \frac{\partial u_i}{\partial c_i} \) for \( i \in \{ e, u \} \) using the Envelope Theorem, and splitting \( \frac{\partial \theta_1(m_1, y_1^u)}{\partial y_1^u} \) into the components corresponding to aggregate demand and search described in Proposition 1 yields the claimed result. \( \Box \)

\section{Additional analytical results}

In this section I describe supplementary results accompanying those in section 2 of the main text. I first microfound the wage function in (8) in the cases of competitive search, constant returns-to-scale and Nash bargaining, and rigid real wages. I describe how to relate the positive effects of changes in cash-on-hand \( y_1^u \) to the underlying policy parameter \( b_1 \). I then provide additional normative results: an alternative characterization of the Ramsey optimum using agents’ social marginal utilities of income; jointly optimal monetary and UI policy; and optimal policy in the flexible price and wage benchmark. I finally discuss the robustness of my positive and normative results to the presence of borrowing constraints and sticky wages rather than prices. Proofs of results provided in this appendix follow.

\subsection{Microfoundations of wage function}

As noted in the main text, if there are incumbents in period 1 \( (\delta < 1) \), I assume for simplicity that incumbents and new hires earn the same wage. Under competitive search or Nash
bargaining, this can be achieved if incumbents earn the economy-wide average wage, equalling
the single posted or bargained wage since firms and unemployed workers are symmetric.

B.1.1 Competitive search

In this case I follow Acemoglu and Shimer [1999]. All producers anticipate a tightness sched-
ule \( \theta(\hat{W}_1) \) indexed over submarkets indexed by their prevailing wage. Given the objective
function in (6) and the assumption that a single firm’s posted wage has a negligible impact
on the economy-wide average earned by incumbents, the optimal posted wage satisfies

\[
P^f f'_1(x) - W_1 = \frac{1}{\eta - 1} \frac{\theta_1(W_1)}{\theta'_1(W_1)}.
\]

The schedule \( \theta_1(\hat{W}_1) \) keeps unemployed workers indifferent across submarkets. To charac-
terize it, I index an employed worker’s ex-post problem in (4) with its wage

\[
v^e_1(z_1; \hat{W}_1) = \max_{c_1^e, c_2^e, z_2^e} u^e(c_1^e, c_2^e) \text{ s.t.} \]

\[
(RC)_1^e : P_1 c_1^e + M_1 P_2 z_2^e \leq Y_1^e(\hat{W}_1) + P_1 z_1,
\]

\[
(RC)_2^e : P_2 c_2^e \leq Y_2^e + P_2 z_2^e,
\]

and then index the ex-ante utility of the worker with the submarket to which it applies

\[
\tilde{v}^u_1(z_1; \hat{W}_1) = \max_{s_1} (p(\theta_1)s_1)v^e_1(z_1; \hat{W}_1) + (1 - p(\theta_1)s_1)v^u_1(z_1) - \psi(s_1).
\]

Suppressing the dependence of both value functions on assets \( z_1 \) since there is a representative
unemployed worker at the start of period 1, worker indifference across submarkets requires that

\[
(\tilde{v}^u_1(\hat{W}_1) - \tilde{v}^{u*}_1(\hat{W}_1))(\theta(\hat{W}_1))^{-1} \leq 0, \text{ with equality if } \theta(\hat{W})^{-1} > 0,
\]

where

\[
\tilde{v}^{u*}_1 \equiv \tilde{v}^u_1(W_1).
\]

In words, this means that in any off-path submarket which would attract a positive measure
of workers, their expected utility by directing search to that submarket must equal that
obtained in equilibrium. Assuming that the equilibrium wage is at an interior point in the
\( \theta(\hat{W}_1) \) schedule, local worker indifference and the Envelope theorem imply

\[
\eta \frac{P_1}{1 - \tau_1} \frac{1}{\theta''_1} (v^e_1 - v^u_1) = - \frac{\theta(W_1)}{\theta'_1(W_1)}
\]
Combining firm optimality and worker indifference at the equilibrium wage, it is characterized by

$$\frac{P_1 f'_1(\cdot)}{P_1} - \frac{W_1}{P_1} = \frac{1}{1 - \tau_1} \frac{\eta}{1 - \eta} \frac{1}{\frac{\partial v_{1e}}{\partial y_{1e}}} (v_{1e} - v_{1u}).$$

This is consistent with the wage function (8) for $\varphi = 1$ and $\phi = 1 - \eta$.

**B.1.2 Constant returns-to-scale and Nash bargaining**

In this case I determine wages according to Nash bargaining with worker share $\phi$, assuming the production function $f_1(\cdot)$ is constant returns-to-scale:

$$f_1(p(\theta_1)s_1 - k\theta_1s_1) = a_1(p(\theta_1)s_1 - k\theta_1s_1).$$

Having matched with a worker, the surplus to the representative producer from employing that worker at wage $\hat{W}_1$ is

$$S_{1f}(\hat{W}_1) = P_1a_1 - \hat{W}_1.$$

Having matched with a firm, the surplus to the representative worker from accepting the job at wage $\hat{W}_1$ is

$$s_{1w}(\hat{W}_1) = v_{1e}(\hat{W}_1) - v_{1u}$$

where $v_{1e}(\hat{W}_1)$ is as defined in the competitive search case above, and I again suppress the dependence of these value functions on the agent’s level of assets because of the existence of a representative worker at the start of period 1. The equilibrium wage then solves

$$W_1 = \arg \max_{\hat{W}_1} s_{1w}(\hat{W}_1)^{\phi} S_{1f}(\hat{W}_1)^{1-\phi},$$

implying

$$\frac{P_1 f'_1(\cdot)}{P_1} - \frac{W_1}{P_1} = \frac{1}{1 - \tau_1} \frac{1 - \phi}{\phi} \frac{1}{\frac{\partial v_{1e}}{\partial y_{1e}}} (v_{1e} - v_{1u}).$$

With $f_1(\cdot) = a_1$, this is consistent with the wage function (8) for $\varphi = 1$ and worker bargaining weight $\phi$.

**B.1.3 Rigid real wages**

In this case the equilibrium wage simply solves

$$\frac{W_1}{P_1} = w_1.$$
for some fixed $\bar{w}_1$.\(^2\) This is also consistent with the wage function (8) for $\varphi = 0$. In the usual way, labor market frictions render such a rigid real wage bilaterally efficient across a range of realizations of macroeconomic shocks.

### B.2 Mapping positive results between cash-on-hand and UI

Here I relate the comparative statics of changing unemployed agents’ cash-on-hand $y^u_1$, studied in the main text, to underlying changes in the policy parameter $b_1$. Since

\[
\begin{align*}
\theta_1(m_1, b_1) &= \theta_1(m_1, y^u_1(m_1, b_1)), \\
s_1(m_1, b_1) &= s_1(m_1, y^u_1(m_1, b_1)), \\
c_0(m_0, m_1, b_1) &= c_0(m_0, m_1, y^u_1(m_1, b_1)),
\end{align*}
\]

where $\theta_1(m_1, y^u_1)$, $s_1(m_1, y^u_1)$, and $c_0(m_0, m_1, y^u_1)$ are characterized by the equilibrium conditions (17)-(20), the Chain Rule implies that the comparative statics of interest follow directly from those in the main text and $\frac{\partial y^u_1(m_1, b_1)}{\partial b_1}$. Hence, I focus here on characterizing $\frac{\partial y^u_1(m_1, b_1)}{\partial b_1}$.

I first note that by $y^u_1(m_1, y^u_1)$, $\theta_1(m_1, y^u_1)$, and $s_1(m_1, y^u_1)$ defined by (17) and (18), we can fully characterize the equilibrium real wage $w_1(m_1, y^u_1)$. In particular, combining producers’ optimality condition with respect to vacancies $\nu_1$

\[
\frac{P^f_1}{P^f_1} f^*_1(\cdot) \left( 1 - \frac{k}{q(\theta_1)} \right) = w_1
\]

with the wage function (8), $w_1(m_1, y^u_1)$ is implicitly defined by

\[
w_1(m_1, y^u_1) = (1 - \varphi) \bar{w}_1 + \varphi \left[ \frac{w_1(m_1, y^u_1)}{1 - \frac{k}{q(\theta_1(m_1, y^u_1))}} - \frac{1}{1 - \tau_1} \frac{1 - \phi}{\phi} \frac{1}{\frac{\partial y^u_1}{\partial b_1}(m_1, y^u_1)} \left( v^e_1(m_1, y^u_1(m_1, y^u_1)) - v^u_1(m_1, y^u_1) \right) \right]. \tag{A.15}
\]

Given the wage $w_1(m_1, y^u_1)$ along with tightness $\theta_1(m_1, y^u_1)$ and search $s_1(m_1, y^u_1)$, equilibrium firm profits (net of the tax financing the retailer subsidy) is then

\[
\pi_1(m_1, y^u_1) = f(1 - \delta + \delta(p(\theta_1(m_1, y^u_1))s_1(m_1, y^u_1) - k\theta_1(m_1, y^u_1)s_1(m_1, y^u_1)))) \\
- w_1(m_1, y^u_1)(1 - \delta + \delta(p(\theta_1(m_1, y^u_1))s_1(m_1, y^u_1)), \tag{A.16}
\]

simply aggregate output net of the aggregate wage bill.

\(^2\)The case with a rigid nominal wage is studied later in this appendix.
Finally, (21) then implicitly defines $y_u^i(m_1, b_1)$:

$$y_u^i(m_1, b_1) = b_1 + \pi_1(m_1, y_u^i(m_1, b_1)).$$

It follows immediately that the marginal effect on unemployed agents’ cash-on-hand from a change in UI is

$$\frac{\partial y_u^i(m_1, b_1)}{\partial b_1} = \frac{1}{1 - \frac{\partial \pi_1(m_1, y_u^i)}{\partial y_u^i}}.$$

By (A.16), $\frac{\partial \pi_1(m_1, y_u^i)}{\partial y_u^i}$ depends on the response of real wages $\frac{\partial w_1(m_1, y_u^i)}{\partial y_u^i}$ and the results in Proposition 1. By (A.15), $\frac{\partial w_1(m_1, y_u^i)}{\partial y_u^i}$ in turn depends on the assumptions on wage determination $\{\varphi, \phi\}$, the results in Proposition 1, and $\frac{\partial y^e_1(m_1, y_u^i)}{\partial y_u^i}$ as characterized in (A.5).

### B.3 Additional normative results

Here I provide additional normative results excluded from the main text for brevity. First I provide an alternative characterization of the Ramsey optimum in section 2.3.1 using agents’ social marginal utilities of income. I then formally study the joint monetary and UI policy problem, the results of which are summarized in section 2.3.3. I finally characterize optimal policy in the flexible price and wage benchmark.

#### B.3.1 Ramsey optimal risk-sharing using social marginal utilities of income

Proposition 3 characterizes the Ramsey optimum in terms of the positive effects of transfers studied in section 2.2. The following set of results provides an alternative (but equivalent) perspective on the optimum as equating agents’ social marginal utilities of income, building on the approach of Farhi and Werning [2016] in their study of macroprudential policy.

Consider planning problem (31) and define the multipliers on the economy’s period 1 and 2 resource constraints $\lambda_{RC1}$ and $\lambda_{RC2}$, respectively. It will be useful to define the relative price wedge between periods 1 and 2

$$\tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} m_1$$

alongside the labor wedge $\tau_0^h$ and tightness wedge $\tau_1^\theta$ defined earlier. Intuitively, a positive relative price wedge will correspond to depressed demand in period 1 because it means that the equilibrium real interest rate between periods 1 and 2 ($\frac{1}{m_1}$) renders consumption in period 1 “too expensive” relative to the social marginal rate of transformation across dates.
Then we can manipulate the first-order conditions of (31) to obtain:

**Lemma B.1.** At the Ramsey optimum,

\[
1 - \frac{\frac{1}{\beta h} \frac{\partial c}{\partial y_i}}{\frac{1}{p_i} \frac{\partial c}{\partial y_i}} + \frac{\frac{1}{\beta} \frac{\partial v_i}{\partial y_i}}{\frac{1}{p_i} \frac{\partial v_i}{\partial y_i}} - \frac{\frac{1}{\beta} \frac{\partial c}{\partial y_i}}{\frac{1}{p_i} \frac{\partial c}{\partial y_i}} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_i} \] \quad (A.18)

for each \( i \in \{e, u\} \). It follows that the left-hand side is equated across agents at the optimum.

This generalizes the definition of the social marginal utility of income obtained in Farhi and Werning [2016] to the present environment with labor market frictions and moral hazard. Even when \( \tau_0^h = \tau_{1,2} = 0 \), a fiscal externality motivates departures from complete risk-sharing in this economy, and when demand is depressed in period 1 (\( \tau_{1,2} > 0 \)), it changes the social cost of (dis)incentivizing labor supply via a search externality. But as in Farhi and Werning [2016], when demand is depressed in either period (\( \tau_0^h > 0 \) or \( \tau_{1,2} > 0 \)), an aggregate demand externality means that the social value of transfers to a given agent is rising in the degree to which it stimulates consumption that period.

While the relative price wedge involves unobservable Lagrange multipliers, at the Ramsey optimum it is in fact closely related to the tightness wedge defined in the main text.

**Lemma B.2.** At the Ramsey optimum,

\[
\frac{1}{\beta} \frac{\lambda_{RC}^2}{m_1} \tau_{1,2} = \frac{1}{\beta} \frac{\lambda_{RC}^0}{\Delta_{\theta_1}^c} + \frac{\lambda_{RC}^0}{\Delta_{\theta_1}^c} \frac{1}{\Delta_{\theta_1}^c} \quad (A.19)
\]

for

\[
\Delta_{\theta_1}^{AD} \equiv \left( \frac{\partial x_1}{\partial \theta_1} + \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} \right) - \left( \frac{\partial c}{\partial y_i} - \frac{\partial c}{\partial y_i} \right) \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_i} \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) \frac{\partial s_1}{\partial \theta_1} \right) \quad (A.19)
\]

\[3\]The proofs of this result and the results which follow are provided in the final part of this section.
identical to its definition in (26) and

$$
\Delta_{\theta_1}^{c_0} = \frac{\partial c_0}{\partial \theta_1} + \left( \frac{1}{1 - \frac{1}{p_1} (\frac{\partial c_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1})} \right) \left[ \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) + \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial \theta_1} \right].
$$

We can interpret (A.18) by first noting the relationship between the coefficients $\Delta_{\theta_1}^{AD}$ and $\Delta_{\theta_1}^{c_0}$ and the positive effects of transfers studied in section 2.2 of the main text. Recall from the analysis therein that $\Delta_{\theta_1}^{AD}$ summarizes the net feedback between tightness and the demand for aggregate consumption, accounting both for the direct effect of tightness and its effect via search effort. And $\Delta_{\theta_1}^{c_0}$ summarizes the net effect on period 0 consumption from a change in tightness, accounting both for the direct effect of tightness and its effect on search effort which then affects period 0 consumption. That is,

$$
\Delta_{\theta_1}^{c_0} = \mu_{\theta_1}^{c_0} + \mu_{s_1}^{c_0} \mu_{\theta_1}^{s_1}
$$

for $\mu_{\theta_1}^{s_1}$ defined in (25), $\mu_{s_1}^{c_0}$ defined in (29), and $\mu_{\theta_1}^{c_0}$ defined in (30).

We can then understand (A.18) as follows. Suppose the optimum is free of distortions in period 0 ($\tau_{h_0}^0 = 0$). Then if demand is depressed in period 1 ($\tau_{2} > 0$), it will mean inefficiently low tightness ($\tau_{\theta_1} > 0$) if and only if low demand leads to low tightness ($\Delta_{\theta_1}^{AD} > 0$). Now suppose the economy is depressed in period 0 ($\tau_{h_0}^0 > 0$). Then the planner can eliminate intertemporal distortions from period 1 onwards ($\tau_{1,2} = 0$) by making tightness inefficiently high ($\tau_{\theta_1} < 0$) if and only if high tightness stimulates period 0 consumption ($\Delta_{\theta_1}^{c_0} > 0$).

Finally, combining Lemmas B.1 and B.2, we obtain the following representation of agents’ social marginal utilities of income and Ramsey optimal risk-sharing.

**Proposition B.1.** At the Ramsey optimum,

$$
\frac{\partial v_i}{\partial y_i} + \frac{1}{\beta} \tau_{h_0} \left[ \frac{1}{p_1} \frac{\partial c_1}{\partial y_i} + \frac{\Delta_{\theta_1}^{c_0}}{\Delta_{\theta_1}^{AD}} \left( \frac{\partial c_1}{\partial y_i} - \frac{1}{p_1} \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial y_i} \right) \right] + \tau_{1} \left[ \frac{1}{\Delta_{\theta_1}^{AD}} \left( \frac{\partial c_1}{\partial y_i} - \frac{1}{p_1} \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial y_i} \right) \right] = \frac{1}{\beta} \frac{\lambda_{RC2}}{m_1}
$$

for each $i \in \{e, u\}$, where $\Delta_{\theta_1}^{AD}$ and $\Delta_{\theta_1}^{c_0}$ are defined in (26) and (A.20), respectively. It follows that the left-hand side is equated across agents at the optimum.

Based on the role of $\Delta_{\theta_1}^{AD}$ and $\Delta_{\theta_1}^{c_0}$ in the positive effects of transfers, it should not be
surprising that the characterizations of the optimum in Propositions 3 and B.1 are equivalent.

### B.3.2 Jointly optimal monetary and UI policy

Section 2.3.3 of the main text described how the macroeconomic inefficiency motivating optimal UI away from the Baily-Chetty level could in turn be motivated by constraints on monetary policy in a generalized planning problem. I formally study that problem here.

Recall that monetary policy is described by the choice of \( \{i_0 = \frac{1}{M_0} - 1, i_1 = \frac{1}{M_1} - 1, P_2\} \).

Consider the following constraints on policy associated with a zero lower bound:

\[
\begin{align*}
    i_0 &\geq 0, \\
    i_1 &\geq 0, \\
    P_2 &\leq \bar{P}_2.
\end{align*}
\] (A.21)

The first two constraints require that the nominal interest rate is no lower than the zero rate of return on cash.\(^4\) The third constraint implies an upper bound on the amount of expected inflation which can be generated despite flexible prices in the final period, which may arise from unmodeled costs of inflation, political economy constraints, or anchored expectations.

The problem of jointly choosing monetary and UI policy is then equivalent to solving a generalization of planning problem (31) where \( \{m_0, m_1\} \) are additional controls and

\[
\begin{align*}
    (ZLB)_0 &: m_0 \leq \frac{\bar{P}}{\bar{P}} = 1, \\
    (ZLB)_1 &: m_1 \leq \frac{\bar{P}_2}{\bar{P}},
\end{align*}
\] (A.22)

are additional constraints. Intuitively, in periods 0 and 1, (A.21) implies lower bounds on real interest rates, or equivalently upper bounds on the relative prices of future consumption.

In this case the economics of the optimum is made clearer by solving (31) using the tools of price theory as described earlier in this appendix, rather than solving (32) (though of course the results would be equivalent). Doing so, two conditions characterize the optimum alongside risk-sharing in Proposition B.1:

**Proposition B.2.** Let \( \lambda_{ZLB0} \) and \( \lambda_{ZLB1} \) be the multipliers satisfying complementary slack-

\(^4\)As argued by Eggertsson and Woodford [2004], this constraint holds even in an economy at the “cashless limit” such as the one under present study provided that agents have the option of holding currency.
ness with constraints (A.22). Then the Ramsey optimum features

\[
\frac{1}{\beta} \tau_0^h \zeta_0^h = \lambda_{ZLB0},
\]

\[
\frac{1}{\beta} \tau_0^h (\zeta_0^1 + \Delta_{\theta_1}^{AD} \zeta_1^1) + \tau_1^\theta \frac{1}{\Delta_{\theta_1}^{AD}} \zeta_1^1 = \frac{1}{\beta} \lambda_{ZLB1},
\]

for \( \Delta_{\theta_1}^c \) and \( \Delta_{\theta_1}^{AD} \) from (26) and (A.20), and coefficients \( \zeta_0^0, \zeta_0^1, \zeta_1^1 \) given by

\[
\zeta_0^0 = \frac{\partial c_0}{\partial m_0},
\]

\[
\zeta_0^1 = \frac{\partial c_0}{\partial m_1} + (c_e^u - y_e^u) \frac{\partial c_0}{\partial y_e^u} + (c_u^e - y_u^e) \frac{\partial c_0}{\partial y_u^u},
\]

\[
\zeta_1^1 = p^e_1 \frac{\partial c_{e,h}}{\partial m_1} + (1 - p^e_1) \frac{\partial c_{u,h}}{\partial m_1},
\]

where the “h” superscript in \( \zeta_1^1 \) signifies compensated demand, and all are weakly positive under standard conditions on preferences.

As described in the main text, slack constraints (\( \lambda_{ZLB0} = \lambda_{ZLB1} = 0 \)) imply that optimal monetary policy has achieved \( \tau_0^h = \tau_1^\theta = 0 \), leaving optimal UI at the Baily-Chetty level as described in Proposition 4. But binding constraints on monetary policy generically imply non-zero wedges \( \tau_0^h \) and \( \tau_1^\theta \), giving UI a second-best role in macroeconomic stabilization. In particular, when the zero lower bound binds in period 1 only (\( \lambda_{ZLB1} > 0 \)), the economy will feature a non-zero tightness wedge \( \tau_1^\theta \), the sign of which depends on whether tightness rises or falls given the deficiency in demand, summarized by \( \Delta_{\theta_1}^{AD} \). When the zero lower bound binds in period 0 (\( \lambda_{ZLB0} > 0 \)), the economy will feature a positive labor wedge \( \tau_0^h \) and optimal policy calls for additional distortions in period 1 to lessen those in period 0.

### B.3.3 Optimal UI in the flexible price and wage benchmark

While the main text focused on my novel characterization of optimal UI under nominal rigidities, here I characterize optimal policy in the flexible price and wage benchmark. The main insight is that when policymakers can choose the payroll tax \( \tau_1 \) alongside UI policy \( \{b_1, t_1\} \), the Ramsey optimum implements the Baily-Chetty formula, an application of the recent insights of Landais et al. [2016].

In light of the generalized planning problem with monetary policy and UI described in the prior subsection, this suggests a useful perspective on my results under nominal rigidities: monetary policy can take the place of the payroll tax to induce production efficiency. When it does so, UI continues to solve the Baily-Chetty formula and the natural allocation is
achieved; but when monetary policy is constrained, the inefficiency impinges upon optimal UI, and conversely UI has a second-best role to play in macroeconomic stabilization.

The analysis of the optimal \( \{b_1, t_1, \tau_1\} \) is facilitated by the following equivalence of implementable allocations under flexible prices and wages:

**Lemma B.3.** An allocation \( \{c_0, h_0, c_1^e, c_1^u, c_2^e, c_2^u, s_1, \theta_1\} \) forms part of a flexible price and wage equilibrium if and only if there exists cash-on-hand \( \{y_1^e, y_1^u\} \) satisfying the economy-wide resource constraints

\[
\begin{align*}
  x_1(s_1, \theta_1, c_1^e, c_1^u) &= 0, \\
  x_2(s_1, \theta_1, c_2^e, c_2^u) &= 0,
\end{align*}
\]

given implementability constraints \( c_0 = c_0(m_0, m_1, y_1^e, y_1^u, \theta_1) \), \( h_0 = h_0(m_0, m_1, y_1^e, y_1^u, \theta_1) \), \( c_1^e = c_1^e(m_1, y_1^e) \), \( c_1^u = c_1^u(m_1, y_1^u) \), \( c_2^e = c_2^e(m_1, y_1^e) \), \( c_2^u = c_2^u(m_1, y_1^u) \), and \( s_1 = s_1(m_1, y_1^e, y_1^u, \theta_1) \).

The proof of this claim is analogous to that of Lemma 2, except for the use of the payroll tax \( \tau_1 \) to ensure (8) is satisfied at the given allocation.

It follows that the Ramsey planning problem is

\[
\begin{align*}
\max_{m_0, m_1, y_1^e, y_1^u, \theta_1} & \quad v_0(m_1, m_1, y_1^e, y_1^u, \theta_1) \\
\text{s.t.} & \quad (RC)_1: x_1(s_1(m_1, y_1^e, y_1^u, \theta_1), \theta_1, c_1^e(m_1, y_1^e), c_1^u(m_1, y_1^u)) = 0, \\
& \quad (RC)_2: x_2(s_1(m_1, y_1^e, y_1^u, \theta_1), \theta_1, c_2^e(m_1, y_1^e), c_2^u(m_1, y_1^u)) = 0,
\end{align*}
\]

identical to the planning problem of jointly choosing monetary policy and UI under fully sticky prices and slack constraints in the prior subsection. Combining Proposition B.2 with Proposition B.1, we can immediately characterize the Ramsey optimum.

**Proposition B.3.** At the Ramsey optimum,

\[
\begin{align*}
1 - \frac{1}{p_1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^u} &= \frac{1}{p_1 - \frac{1}{1 - p_1}} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_1^e},
\end{align*}
\]

and

\[
\tau_0^h = \tau_1^\theta = 0
\]

for labor and tightness wedges defined in (33) and (34), respectively.

Finally, using Lemma 3 and the fact that \( \mu = 1 \) in the flexible price and wage allocation, implementation implies that optimal UI satisfies the classic Baily-Chetty formula:
Proposition B.4. The Ramsey optimum is implemented in part by UI benefits $b_1$ satisfying a general equilibrium version of the Baily-Chetty formula

$$
\left(\frac{1}{P_1^e}\right)^2 \varepsilon_{b_1}^{-1} = \frac{\partial \mu^u}{\partial c_1} - \frac{\partial \mu^e}{\partial c_1}.
$$

where $\varepsilon_{b_1}^{-1}$, the micro elasticity of unemployment in period 1 with respect to an increase in UI is given by (36).

B.4 Robustness of positive and normative results

In this subsection I demonstrate that the positive and normative results regarding UI under sticky prices are robust to the presence of borrowing constraints and sticky wages rather than prices. Together with the robustness to alternative wage determination mechanisms embedded in (8), this implies that the paper’s main results are robust to the details of search frictions, market incompleteness, and nominal rigidity in the environment under study.

B.4.1 Borrowing constraints

If agents face borrowing constraints in period 1, (4) generalizes to

$$
v_i^t(z_1) = \max_{c_1^i, c_2^i, z_2^i} u^t(c_1^i, c_2^i) \text{ s.t.}
$$

$$(RC)_1^i : P_1 c_1^i + M_1 P_2 z_2^i \leq Y_1^i + P_1 z_1^i,$$

$$(RC)_2^i : P_2 c_2^i \leq P_2 y_2^i + P_2 z_2^i,$$

$$(BC)_1^i : z_2^i \geq z_1^i.$$

Note that this can capture employment-specific borrowing constraints, such as hand-to-mouth behavior among (only) the unemployed if $z_1^i$ is sufficiently high. Analogous to the way in which (4) implied indirect utility $v_i^t(m_1, y_1^i)$ and Marshallian demand $c_1^i(m_1, y_1^i)$, (4') implies indirect utility $v_i^t(m_1, y_1^i, y_2^i)$ and generalized Marshallian demand $c_1^i(m_1, y_1^i, y_2^i)$, since the solution to (4') will be the same as the solution to

$$
v_i^t(m_1, y_1^i, y_2^i) = \max_{c_1^i, c_2^i} u^t(c_1^i, c_2^i) \text{ s.t.}
$$

$$(RC)_1^i : c_1^i + m_1 c_2^i \leq y_1^i + m_1 y_2^i,$$

$$(BC)_1^i : c_2^i \geq y_2^i + z_1^i.$$
It follows that all of the positive and normative results of section 2 remain unchanged, except that \( v_i(m_1, y_1, y_2) \) and \( c_i(m_1, y_1, y_2) \) replace \( v_i^1(m_1, y_1^1) \) and \( c_i^1(m_1, y_1^1) \) everywhere. The MPC of an agent up against her borrowing constraint will simply be \( \frac{\partial c_i^1}{\partial y_1^i} = 1 \). In the proofs of the supplemental normative results provided earlier in this appendix, the results are unchanged because generalized versions of key price theory identities continue to hold, including Roy’s

\[
\frac{\partial v_i^1}{\partial m_1} = -(c_2^2 - y_2^2) \frac{\partial v_i^1}{\partial y_1^i},
\]

Slutsky’s

\[
\frac{\partial c_i^1}{\partial m_1} = \frac{\partial c_i^{1,h}}{\partial m_1} - (c_2^2 - y_2^2) \frac{\partial c_i^1}{\partial y_1^i},
\]

and the identities among demand functions

\[
\frac{\partial c_i^1}{\partial y_1^i} + m_1 \frac{\partial c_2^1}{\partial y_1^i} = 1,
\]

\[
\frac{\partial c_i^{1,h}}{\partial y_1^i} + m_1 \frac{\partial c_2^{1,h}}{\partial y_1^i} = 0,
\]

for generalized compensated demand solving

\[
e_i(m_1, u, y_2^i) = \min_{c_1^{i,h},c_2^{i,h}} c_1^{i,h} + m_1 c_2^{i,h} \text{ s.t.}
\]

\[
(U)^i : u_i(c_1^{i,h}, c_2^{i,h}) \geq u,
\]

\[
(BC)^1_i : c_2^{i,h} \geq y_2^{i,h} + z_1^{i,h}.
\]

**B.4.2 Sticky wages rather than prices**

Now suppose wages are sticky at \( W_0 = W_1 = \bar{W} \), but prices are flexible. For standard Keynesian reasons, this means that the nature of firms’ labor demand takes on greater importance. In period 0, producers’ first-order condition for hours in (5) combined with retailers’ optimal pricing in (7) implies the labor demand relation

\[
P_0 = \frac{W}{f_0'(h_0)}.
\]

In period 1, producers’ first-order condition for vacancies in (6) combined with retailers’ optimal pricing in (7) analogously implies the labor demand relation

\[
P_1 = \frac{W}{f_1'(1 - \delta + \delta(p(\theta_1)s_1 - k\theta_1 s_1)) \left(1 - \frac{k}{q(\theta_1)}\right)}.
\]
Alongside the equilibrium conditions (17)-(20), these labor demand relations imply that real interest rates are now endogenous even if monetary policy \( \{M_0, M_1, P_2\} \) is held fixed, since

\[
\begin{align*}
m_0 &\equiv M_0 \frac{P_1}{P_0} = M_0 \frac{f'_0(h_0)}{f'_1(1 - \delta + \delta(p(\theta_1)s_1 - k\theta_1 s_1)) \left(1 - \frac{k}{q(\theta_1)}\right)}, \quad (A.24) \\
m_1 &\equiv M_1 \frac{P_2}{P_1} = M_1 \frac{P_2}{W} f'_1(1 - \delta + \delta(p(\theta_1)s_1 - k\theta_1 s_1)) \left(1 - \frac{k}{q(\theta_1)}\right). \quad (A.25)
\end{align*}
\]

From a positive perspective, this introduces a contractionary effect of greater UI generosity via the increase in firms’ marginal cost from greater recruiting. From a normative perspective, the change in real interest rates induced by a change in UI generosity will affect welfare in the presence of macroeconomic inefficiency summarized by \( \tau^h_0 \neq 0 \) or \( \tau^a_1 \neq 0 \). But aside from these new forces, the effects of UI on employment and social welfare operating through heterogeneity in MPCs and precautionary savings remain robust.

**B.5 Proofs**

Here I provide proofs of the supplemental results provided in this appendix.

**B.5.1 Lemma B.1**

**Proof.** For each \( i \in \{e, u\} \), the first-order condition of planning problem (31) with respect to \( y_i^1 \) is

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial y_i^1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial y_i^1} + \beta p^i_1 \frac{\partial v^i_1}{\partial y_i^1} + \lambda_{RC1} \left( \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial y_i^1} - p^i_1 \frac{\partial c_1}{\partial y_i^1} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial y_i^1} - p^i_1 \frac{\partial c_2}{\partial y_i^1} \right) = 0 \quad (A.26)
\]

given \( p^u_1 \equiv 1 - p^e_1 = \delta(1 - p(\theta_1)s_1) \). The Envelope theorem is again used to ignore the change in the planner’s objective from the search effort response to changes in the distribution of income.

To simplify this condition, first note that since

\[
\frac{\partial c_0}{\partial y_i^1} = f'_0(h_0) \frac{\partial h_0}{\partial y_i^1},
\]

by (20), we have that

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial y_i^1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial y_i^1} = \tau^h_0 \frac{\partial c_0}{\partial y_i^1}.
\]
given the labor wedge defined in (33). Then, defining the relative price wedge as in (A.17)

$$\tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} m_1$$

and recalling that

$$\frac{\partial c_i}{\partial y_i} + m_1 \frac{\partial c_i}{\partial y_i} = 1$$

using the standard identity for Marshallian demand functions, we have that

$$\lambda_{RC1} \left( \frac{\partial x_1}{\partial s_1} - p_i \frac{\partial c_i}{\partial y_i} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial s_1} - p_i \frac{\partial c_i}{\partial y_i} \right)$$

$$= \frac{\lambda_{RC2}}{m_1} \left[ (1 - \tau_{1,2}) \left( \frac{\partial x_1}{\partial s_1} - p_i \frac{\partial c_i}{\partial y_i} \right) + m_1 \left( \frac{\partial x_2}{\partial s_1} - p_i \frac{\partial c_i}{\partial y_i} \right) \right],$$

$$= - \frac{\lambda_{RC2}}{m_1} \left[ p_i - \tau_{1,2} p_i \frac{\partial c_i}{\partial y_i} - \left( 1 - \tau_{1,2} \right) \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial y_i} \right].$$

Taken together, these results imply that (A.26) becomes

$$\tau_0^h \frac{1}{p_i} \frac{\partial c_0}{\partial y_i} + \beta \frac{\partial v^i}{\partial y_i} - \frac{\lambda_{RC2}}{m_1} \left[ 1 - \tau_{1,2} \frac{\partial c_i}{\partial y_i} - \frac{1}{p_i} \left( \left( 1 - \tau_{1,2} \right) \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_i} \right] = 0. \quad (A.27)$$

This implies that the social marginal utility of income for agent $i \in \{e, u\}$ can be written

$$\frac{\tau_0^h \frac{1}{p_i} \frac{\partial c_0}{\partial y_i} + \beta \frac{\partial v^i}{\partial y_i} - \frac{\lambda_{RC2}}{m_1} \left[ \frac{1}{p_i} \left( \left( 1 - \tau_{1,2} \right) \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_i} \right]}{1 - \tau_{1,2} \frac{\partial c_i}{\partial y_i} - \frac{1}{p_i} \left( \left( 1 - \tau_{1,2} \right) \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial y_i}} = \frac{\lambda_{RC2}}{m_1},$$

a generalization of that obtained in Farhi and Werning [2016] to the present environment with search frictions and moral hazard.

\[\Box\]

**B.5.2 Lemma B.2**

**Proof.** The first-order condition of planning problem (31) with respect to $\theta_1$ is

$$\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial \theta_1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial \theta_1} + \beta \delta p'(\theta_1) s_1 (v^e_1 - v^u_1) +$$

$$\lambda_{RC1} \left( \frac{\partial x_1}{\partial \theta_1} + \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial \theta_1} + \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} \right) = 0. \quad (A.28)$$

which again makes use of the Envelope theorem to ignore the change in the planner’s objective from the search effort response to changes in tightness. Analogous results to those in the
proof of Lemma B.1 imply
\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial \theta_1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial \theta_1} = \tau^h_0 \frac{\partial c_0}{\partial \theta_1}
\]
and
\[
\lambda_{RC1} \left( \frac{\partial x_1}{\partial \theta_1} + \frac{\partial x_1 \partial s_1}{\partial \theta_1} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial \theta_1} + \frac{\partial x_2 \partial s_1}{\partial \theta_1} \right) = \lambda_{RC2} \left( \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} + \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial \theta_1} \right) - \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \left( \frac{\partial x_1}{\partial \theta_1} + \frac{\partial x_1 \partial s_1}{\partial \theta_1} \right).
\]
Hence, (A.28) becomes
\[
\tau^h_0 \frac{\partial c_0}{\partial \theta_1} + \beta \delta \theta^0(\theta_1) s_1 (v^e_1 - v^u_1) + \lambda_{RC2} \left( \frac{\partial x_1}{\partial \theta_1} + m_1 \frac{\partial x_2}{\partial \theta_1} \right) - \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \left( \frac{\partial x_1}{\partial \theta_1} + \frac{\partial x_1 \partial s_1}{\partial \theta_1} \right) = 0. \tag{A.29}
\]
Now, since (A.27) for \( i = e \) implies
\[
\frac{\lambda_{RC2}}{m_1} = \frac{1}{1 - \frac{1}{p^1} \left( \frac{\partial x_1}{\partial s_1} + m_1 \frac{\partial x_2}{\partial s_1} \right) \frac{\partial s_1}{\partial \theta_1} \frac{\partial s_1}{\partial \theta_1} \left( \beta \frac{\partial v^e_1}{\partial y^e_1} + \tau^h_0 \frac{1}{p^1} \frac{\partial c_0}{\partial y_1} + \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \left( \frac{\partial c^e_1}{\partial y^e_1} - \frac{1}{p^1} \frac{\partial x_1 \partial s_1}{\partial y^e_1} \right) \right),
\]
we can plug this into (A.29) and collect terms to give
\[
\tau^h_0 \Delta^{c_0}_{\theta_1} + \beta \tau^\theta_1 - \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \Delta^{AD}_{\theta_1} = 0,
\]
for \( \tau^\theta_1, \Delta^{AD}_{\theta_1}, \) and \( \Delta^{c_0}_{\theta_1} \) as defined in (34), (A.19), and (A.20), respectively. Re-arranging, we obtain
\[
\frac{\lambda_{RC2}}{m_1} \tau_{1,2} = \tau^h_0 \Delta^{c_0}_{\theta_1} + \beta \tau^\theta_1 \frac{1}{\Delta^{AD}_{\theta_1}} \Delta^{AD}_{\theta_1}, \tag{A.30}
\]
the stated result. \( \square \)

**B.5.3 Proposition B.1**

Proof. Plugging (A.30) into (A.27) for each \( i \in \{ e, u \} \) and collecting terms, we obtain
\[
\begin{align*}
\beta \frac{\partial v^i_1}{\partial y^e_1} + \tau^h_0 \left[ \frac{1}{p^1} \frac{\partial c_0}{\partial y^e_1} + \frac{\Delta^{c_0}_{\theta_1}}{\Delta^{AD}_{\theta_1}} \left( \frac{\partial c^e_1}{\partial y^e_1} - \frac{1}{p^1} \frac{\partial x_1 \partial s_1}{\partial y^e_1} \right) \right] & + \beta \tau^\theta_1 \left[ \frac{1}{\Delta^{AD}_{\theta_1}} \left( \frac{\partial c^e_1}{\partial y^e_1} - \frac{1}{p^1} \frac{\partial x_1 \partial s_1}{\partial y^e_1} \right) \right] = \frac{\lambda_{RC2}}{m_1},
\end{align*}
\]
the stated result. \hfill \Box

\section*{B.5.4 Proposition B.2}

\begin{proof}
The claim follows from the first order conditions of the generalized planning problem with respect to \( m_0 \) and \( m_1 \). First consider the first order condition with respect to \( m_0 \)

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial m_0} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial m_0} = \lambda_{ZLB_0}.
\]

Since

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial m_0} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial m_0} = \tau^h_0 \frac{\partial c_0}{\partial m_0},
\]

the above first order condition then implies

\[
\tau^h_0 \frac{\partial c_0}{\partial m_0} = \lambda_{ZLB_0},
\]

defining the first part of the claim given

\[
\zeta^0_0 \equiv \frac{\partial c_0}{\partial m_0}.
\]

A sufficient condition for \( \zeta^0_0 > 0 \) is \( \frac{d}{dc_0} u_0(c_0, f^{-1}_0(c_0)) < 0 \).

Next consider the first order condition with respect to \( m_1 \)

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial m_1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial m_1} + \beta \left[ p^e_1 \frac{\partial v^e_1}{\partial m_1} + (1 - p^e_1) \frac{\partial v^u_1}{\partial m_1} \right] + \lambda_{RC1} \left( \frac{\partial x_1}{\partial s_1} \frac{\partial s_1}{\partial m_1} - p^e_1 \frac{\partial c^e_1}{\partial m_1} - (1 - p^e_1) \frac{\partial c^u_1}{\partial m_1} \right) \\
+ \lambda_{RC2} \left( \frac{\partial x_2}{\partial s_1} \frac{\partial s_1}{\partial m_1} - p^e_1 \frac{\partial c^e_2}{\partial m_1} - (1 - p^e_1) \frac{\partial c^u_2}{\partial m_1} \right) = \lambda_{ZLB_1}. \quad (A.31)
\]

We can again use the definition of the labor wedge \( \tau^h_0 \) to conclude

\[
\frac{\partial u_0}{\partial c_0} \frac{\partial c_0}{\partial m_1} + \frac{\partial u_0}{\partial h_0} \frac{\partial h_0}{\partial m_1} = \tau^h_0 \frac{\partial c_0}{\partial m_1}.
\]
Now recall that by Roy’s identity
\[ \frac{\partial v^i_1}{\partial m_1} = -(c^i_2 - y^i_2) \frac{\partial v^i_1}{\partial y^i_1}, \]
\[ \frac{\partial s_1}{\partial m_1} = -(c^e_2 - y^e_2) \frac{\partial s_1}{\partial y^e_1} - (c^u_2 - y^u_2) \frac{\partial s_1}{\partial y^u_1}, \]

where the second follows from the first for \( i \in \{e,u\} \) as well as the comparative statics of workers’ optimal search effort \( s_1(m_1, y^e_1, y^u_1, \theta_1) \), and by Slutsky’s identity,
\[ \frac{\partial c^i_1}{\partial m_1} = \frac{\partial c^{i,h}_1}{\partial m_1} - (c^i_2 - y^i_2) \frac{\partial c^i_1}{\partial y^i_1} \]

for \( i \in \{e,u\} \) and \( t \in \{1,2\} \). Hence, multiplying (A.26) for \( i = e \) by \((c^e_2 - y^e_2)\), (A.26) for \( i = u \) by \((c^u_2 - y^u_2)\), and adding both to (A.31), these identities imply
\[ \tau^h_0 \left( \frac{\partial c_0}{\partial m_1} + (c^e_2 - y^e_2) \frac{\partial c_0}{\partial y^e_1} + (c^u_2 - y^u_2) \frac{\partial c_0}{\partial y^u_1} \right) \]
\[ - \lambda_{RC1} \left( p^e_1 \frac{\partial c_{e,h}^1}{\partial m_1} + (1 - p^e_1) \frac{\partial c_{e,h}^u}{\partial m_1} \right) \]
\[ - \lambda_{RC2} \left( p^u_1 \frac{\partial c_{u,h}^1}{\partial m_1} + (1 - p^u_1) \frac{\partial c_{u,h}^u}{\partial m_1} \right) = \lambda_{ZLB1}. \]

We can then use the definition of the relative price wedge \( \tau_{1,2} \) in (A.17) as well as the identity among compensated derivatives that
\[ \frac{\partial c^{i,h}_1}{\partial m_1} = -m_1 \frac{\partial c^{i,h}_1}{\partial m_1} \]

for \( i \in \{e,u\} \) to further simplify this as
\[ \tau^h_0 \left( \frac{\partial c_0}{\partial m_1} + (c^e_2 - y^e_2) \frac{\partial c_0}{\partial y^e_1} + (c^u_2 - y^u_2) \frac{\partial c_0}{\partial y^u_1} \right) \]
\[ + \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \left( p^e_1 \frac{\partial c_{e,h}^1}{\partial m_1} + (1 - p^e_1) \frac{\partial c_{e,h}^u}{\partial m_1} \right) \]
\[ = \lambda_{ZLB1}. \]

Finally, we can plug in the expression for \( \frac{\lambda_{RC2}}{m_1} \tau_{1,2} \) obtained in (A.18) to obtain the second part of the claim
\[ \tau^h_0 \left( \zeta^1_1 + \frac{\Delta c_0}{\Delta \theta_1} \zeta^1_1 \right) + \beta \tau^h_1 \frac{1}{\Delta \theta_1} \zeta^1_1 = \lambda_{ZLB1} \]
for
\[ \zeta_0^1 = \frac{\partial c_0}{\partial m_1} + (c_2^e - y_2^e) \frac{\partial c_0}{\partial y_1^e} + (c_2^u - y_2^u) \frac{\partial c_0}{\partial y_1^u}, \]
\[ \zeta_1^1 = p_e \frac{\partial c_1^{e,h}}{\partial m_1} + (1 - p_e) \frac{\partial c_1^{u,h}}{\partial m_1}. \]

Since compensated cross-price derivatives are non-negative, so must be \( \zeta_1^1 \). A sufficient set of conditions for \( \zeta_0^1 > 0 \) are \( \frac{\partial^2 u_i}{\partial (c_i^1)^2} < 0, \frac{\partial^2 u_i}{\partial (c_i^2)^2} < 0, \frac{\partial c_i^1}{\partial y_i^1} < 1, \) and \( \frac{d}{dc_0 u_0,c(c_0,f_0^{-1}(c_0))} < 0 \). □

**B.5.5 Lemma B.3**

*Proof.* The proof is very similar to that of Lemma 2. The only difference is that because retailer optimization now pins down the gross markup \( \frac{P_1}{P_1} \) (at 1, maintaining the assumption that \( \tau_{1R} = -\frac{1}{\varepsilon} \)), producers’ first-order condition with respect to \( \nu_1 \) pins down the real wage \( \frac{W}{P_1} \).\(^5\) It follows that the payroll tax \( \tau_1 \) is now determinate, ensuring that the real wage in (8) is consistent with the given allocation. Conversely, monetary policy is now indeterminate owing to a real-nominal dichotomy under flexible nominal prices and wages. □

**B.5.6 Proposition B.3**

*Proof.* Planning problem (A.23) is identical to the generalized planning problem of choosing monetary policy and UI without the constraints on monetary policy (A.22). The stated result then immediately follows from Proposition B.1 and Proposition B.2 with \( \lambda_{ZLB0} = \lambda_{ZLB1} = 0 \).

**B.5.7 Proposition B.4**

*Proof.* Ramsey optimal risk-sharing in Proposition B.3 implies (A.12) with \( \tau_h^0 = \tau_1^0 = 0 \). The remainder of the proof proceeds as in that of Proposition 4 with \( \mu = 1 \) under flexible prices and wages (given that the retailer subsidy eliminates the mark-up from monopolistic competition). □

**C Additional quantitative results**

In this section I describe supplementary results accompanying those in section 3 of the main text. I first provide a fuller discussion of the welfare effects of UI extensions across

\(^5\)In the same spirit as footnote 1, this argument does not apply in the limit of a fully rigid real wage in (8); but provided policymakers could levy the tax on producers rather than workers in that case, their first-order condition with respect to \( \nu_1 \) would pin it down, and the rest of the proof is unchanged.
the state space. I then describe the calibration of alternative steady-states and provide an alternative decomposition of the output effects of UI into redistribution and precautionary saving. Finally, I describe additional policy experiments comparing UI extensions in a slack labor market versus steady-state; assessing the effect of a positive macro wage elasticity to UI; and comparing replacement rate increases versus duration extensions.

C.1 Welfare effects of UI extensions

In my analysis of the welfare effects of UI extensions under sticky prices and fixed monetary policy (at the zero lower bound) in section 3.3.1, I focused for brevity on heterogeneity by employment status and the persistent component of income for employed agents. Here I present the full decomposition of welfare effects across the idiosyncratic state space.

For unemployed agents, the average welfare effects conditional on duration of unemployment, eligibility for UI, and discount factor are presented in Figure A.1. Duration has a
non-monotonic relationship with the change in welfare, as it is the initially medium-term unemployed who are likely to make use of the benefit extensions who gain the most. Recalling that eligibility and take-up of UI is modeled as an exogenous shock conditional on job loss, those who are able to receive UI obviously gain more than those who cannot. Finally, the most impatient unemployed agents gain the most.

For employed agents, the average welfare effects conditional on transitory income and discount factor are presented in Figure A.2. The transitory component of income naturally has a smaller effect on the change in welfare than the persistent component studied in the main text. As with unemployed agents, the most impatient employed agents gain the most.

The broad-based welfare gains from UI extensions, even among initially unemployed agents who do not receive UI or the initially employed, are further evidence of the positive demand externalities from transfers characterized in section 2.

C.2 Alternative calibrations of the steady-state

To further understand the roles of redistribution and precautionary saving in driving the macroeconomic effects of UI, and to obtain a broader range of estimates of these effects, in section 3.3.2 I described the effects of UI starting from three alternative calibrations. Here I describe each of these in further detail.

C.2.1 Dynamic selection

I first replace structural duration dependence with the literature’s other dominant explanation for negative duration dependence in observed job-finding rates: dynamic selection. As noted in the main text, I set $\lambda_0 = \lambda_1 = 0$ and instead assume a fraction $\mu$ have match
Table A.1: calibration results under dynamic selection

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Achieved</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wealth and income</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real interest rate (ann.)</td>
<td>2%</td>
<td>2.0%</td>
<td>$z^a/\bar{a}$</td>
<td>-1.548</td>
</tr>
<tr>
<td>Median wealth / monthly HH income</td>
<td>0.3</td>
<td>0.2</td>
<td>$\beta$</td>
<td>0.997</td>
</tr>
<tr>
<td>25pctile wealth / monthly HH income</td>
<td>0.0</td>
<td>-0.2</td>
<td>$z/\bar{a}$</td>
<td>-1.1</td>
</tr>
<tr>
<td>IQR wealth / monthly HH income</td>
<td>2.0</td>
<td>1.3</td>
<td>$\Delta^a$</td>
<td>0.0011</td>
</tr>
<tr>
<td>HH income with UI / pre job loss</td>
<td>0.75</td>
<td>0.75</td>
<td>$\omega/\bar{a}$</td>
<td>1.07</td>
</tr>
<tr>
<td>HH income without UI / pre job loss</td>
<td>0.60</td>
<td>0.61</td>
<td>$b^{SA}/\bar{a}$</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>Unemployment and job search</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>5%</td>
<td>5.0%</td>
<td>$\bar{w}/\bar{a}$</td>
<td>0.996</td>
</tr>
<tr>
<td>Fraction w/ duration 4-6 mos</td>
<td>0.14</td>
<td>0.18</td>
<td>$\mu$</td>
<td>0.8</td>
</tr>
<tr>
<td>Fraction w/ duration &gt; 6 mos</td>
<td>0.17</td>
<td>0.19</td>
<td>$\bar{m}_2/\bar{m}_1$</td>
<td>0.3</td>
</tr>
<tr>
<td>Share receiving UI</td>
<td>0.39</td>
<td>0.43</td>
<td>$\zeta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Duration elasticity to benefit duration</td>
<td>0.1</td>
<td>0.12</td>
<td>$\xi$</td>
<td>4</td>
</tr>
<tr>
<td>Conventional market tightness</td>
<td>0.634</td>
<td>0.639</td>
<td>$\bar{m}_1$</td>
<td>0.296</td>
</tr>
<tr>
<td>Fraction of monthly wage to hire worker</td>
<td>0.108</td>
<td>0.112</td>
<td>$k/\bar{a}$</td>
<td>0.069</td>
</tr>
</tbody>
</table>

efficiency $\bar{m}_1$ whenever they are unemployed, and a fraction $1 - \mu$ have match efficiency $\bar{m}_2$ whenever they are unemployed. Workers know their own match efficiencies. The parameters $\{\mu, \bar{m}_2\}$ replace $\{\lambda_1, \lambda_0\}$ in matching the share of medium-term and long-term unemployed, and the calibration targets remain fixed, yielding the parameter choices in Table A.1.

An important implication of this calibration is that the profile of MPCs by duration of unemployment flattens relative to the baseline model, as demonstrated in Table A.4 for a rebate of $500. Intuitively, a long-term unemployed pool consisting largely of workers with permanently low match efficiencies implies that their gap between temporary and permanent income is smaller than in the baseline model. This implies that such workers are less likely to spend the marginal dollar even when unemployed for several months.

### C.2.2 $\sigma = 4$

One of the assumed parameters in the calibration is the coefficient of relative risk aversion $\sigma$, set to 1 in the baseline case. Under CRRA utility this implies a coefficient of relative prudence $\sigma + 1$. The analytical results of section 2 confirm that prudence plays an important role in governing the macroeconomic effects of UI under nominal rigidities by controlling the strength of the precautionary savings response to changes in UI.

To understand how the quantitative results change under a higher coefficient of prudence and thus precautionary savings response, I set $\sigma = 4$ and re-calibrate the parameters of the model to match the same targets. This yields the parameter choices in Table A.2. Note
that a lower average discount factor is needed to match the same wealth distribution, as precautionary motives play a bigger role in driving wealth accumulation in this calibration.

While the primary purpose of this calibration is to understand the role of precautionary savings, the close connection between precautionary savings and the concavity of the consumption function (Carroll and Kimball [1996]) implies that the economy’s MPCs are affected as well. Table A.4 again provides the MPCs by duration of unemployment relative to the baseline model for a rebate of $500. As is evident, higher $\sigma$ tends to reduce the MPCs for the long-term unemployed, pushing the strength of the precautionary saving and redistribution channels in opposite directions.

C.2.3 Higher target for micro disincentive effect

Finally, another important behavioral elasticity in the model is the micro elasticity of unemployment duration with respect to potential duration of benefits. In the baseline calibration the elasticity of disutility from search $\xi = 9$ is used to target a micro elasticity of 0.1, within the range of estimates for the U.S. provided in the survey of Schmieder and von Wachter [2016]. However, as these authors note, a wide range of estimates for this elasticity have been obtained in the literature, reaching as high as roughly 0.4.

To understand how the quantitative results change under a higher disincentive effect of UI, I instead use $\xi$ to target an elasticity of unemployment duration with respect to benefit duration of 0.4, and re-calibrate the other parameters of the model to match the same
<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Achieved</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<td></td>
<td></td>
</tr>
<tr>
<td>Real interest rate (ann.)</td>
<td>2%</td>
<td>2.0%</td>
<td>$z^a/\bar{a}$</td>
<td>-1.5</td>
</tr>
<tr>
<td>Median wealth / monthly HH income</td>
<td>0.3</td>
<td>0.3</td>
<td>$\beta$</td>
<td>0.9969</td>
</tr>
<tr>
<td>25pctile wealth / monthly HH income</td>
<td>0.0</td>
<td>0.0</td>
<td>$\tilde{z}/\bar{a}$</td>
<td>-0.9</td>
</tr>
<tr>
<td>IQR wealth / monthly HH income</td>
<td>2.0</td>
<td>1.4</td>
<td>$\Delta^g$</td>
<td>0.0011</td>
</tr>
<tr>
<td>HH income with UI / pre job loss</td>
<td>0.75</td>
<td>0.75</td>
<td>$\omega/\bar{a}$</td>
<td>1.07</td>
</tr>
<tr>
<td>HH income without UI / pre job loss</td>
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<td>0.61</td>
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<td>0.21</td>
</tr>
<tr>
<td><strong>Unemployment and job search</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>5%</td>
<td>4.9%</td>
<td>$\tilde{w}/\bar{a}$</td>
<td>0.996</td>
</tr>
<tr>
<td>Fraction w/ duration 4-6 mos</td>
<td>0.14</td>
<td>0.17</td>
<td>$\lambda_0$</td>
<td>1.1</td>
</tr>
<tr>
<td>Fraction w/ duration &gt; 6 mos</td>
<td>0.17</td>
<td>0.22</td>
<td>$\lambda_1$</td>
<td>-0.14</td>
</tr>
<tr>
<td>Share receiving UI</td>
<td>0.39</td>
<td>0.43</td>
<td>$\zeta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Duration elasticity to benefit duration</td>
<td>0.4</td>
<td>0.41</td>
<td>$\xi$</td>
<td>2.01</td>
</tr>
<tr>
<td>Conventional market tightness</td>
<td>0.634</td>
<td>0.647</td>
<td>$\bar{m}(0)$</td>
<td>0.240</td>
</tr>
<tr>
<td>Fraction of monthly wage to hire worker</td>
<td>0.108</td>
<td>0.112</td>
<td>$k/\bar{a}$</td>
<td>0.069</td>
</tr>
</tbody>
</table>

Table A.3: calibration results targeting higher disincentive effect

<table>
<thead>
<tr>
<th>Group</th>
<th>Baseline</th>
<th>Dynamic selection</th>
<th>$\sigma = 4$</th>
<th>Higher disincentive effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>0.14</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>ST unemployed ($d \in {0, 1, 2}$)</td>
<td>0.36</td>
<td>0.33</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>MT unemployed ($d \in {3, 4, 5}$)</td>
<td>0.55</td>
<td>0.33</td>
<td>0.46</td>
<td>0.53</td>
</tr>
<tr>
<td>LT unemployed ($d \geq 6$)</td>
<td>0.83</td>
<td>0.52</td>
<td>0.72</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table A.4: model-generated quarterly MPC by duration of unemployment

targets. This yields the parameter choices in Table A.3. While the supply-side of the model has changed, Table A.4 implies that agents’ consumption behavior, such as their MPCs by duration of unemployment, are little changed relative to the baseline model.

### C.3 A decomposition into redistribution and precautionary saving

In the main text, the comparison of alternative calibrations in section 3.3.2 suggested the quantitative importance of redistribution and precautionary saving in driving the equilibrium effects of UI under sticky prices. Here I provide another window into the relative importance of these channels by decomposing the baseline dynamics into each.

First, I separately compare a 3-month extension of UI duration occurring in month $t = 0$ and in month $t = 9$ alone, though in both cases the increase in generosity is announced...
Figure A.3: decomposing the output response to UI shocks

in month $t = 0$. Second, for each of the dynamic output responses to these month-by-month shocks, I decompose the response into a partial equilibrium and (residual) general equilibrium component. The partial equilibrium component $c_t^{PE}$ is computed as described in section 3.3.2 of the main text.

The resulting decomposition in Figure A.3 implies that both the redistribution and precautionary saving channels are important, and that they interact and amplify each other in general equilibrium. The redistribution channel is isolated in the month 0 partial equilibrium response to extended UI in month 0, as this captures a pure unanticipated shift in resources from the employed to the long-term unemployed. The precautionary saving channel is (almost fully) isolated in the month 0 partial equilibrium response to extended UI in month 9, as employed agents’ permanent income is (almost completely) unchanged. As is evident, both channels stimulate aggregate demand in partial equilibrium. In general equilibrium, redistribution in month 9 then leads to higher tightness, further reducing incentives to precautionary save, and so on. The latter is particularly strong here because the initial negative shock to fundamentals means that long-term unemployment is particularly high several months into the simulation, as depicted in Figure 2 in the main text.

C.4 Additional policy experiments

Here I describe the effects of additional policy experiments comparing UI extensions in a slack labor market versus steady-state; assessing the effect of a positive macro wage elasticity to UI;

---

6The equilibrium response to the more realistic UI policy shock studied in the main text will be the sum of the equilibrium responses to month-by-month shocks such as these.

7I say “almost” because the probability of being unemployed in month 9 is not invariant across the population at month 0, so the policy change does lead to a small transfer in permanent income.
and comparing replacement rate increases versus duration extensions. Table A.5, building on Table 6 in the main text, summarizes these experiments.

The second column of Table A.5 summarizes the effects of UI extensions starting from the steady-state with no other fundamental shocks. Comparing these results to the baseline under a slack labor market, we see that the stimulus and welfare gains from UI are amplified when the labor market is slack. Since the output multipliers are little changed between environments, the amplification in the slack labor market appears to follow from the mechanical effect of more (long-term) unemployed agents being affected by the policy. The amplification of the welfare gains is further consistent with section 2’s result that macroeconomic slackness increases the wedge between the social and private value of UI.

The next column of Table A.5 demonstrates that if equilibrium wages increase in response to the UI extensions, the stimulus and welfare gains from UI only further increase. I use an elasticity of the average wage \( \bar{w} \) with respect to UI duration of 0.005, of the same order of magnitude recently estimated by Hagedorn et al. [2016].\(^8\) The temporary increase in wages implies a reallocation of income from dividends and towards wage payments. Because the asset-rich are disproportionate owners of firm equity, the induced reallocation to higher-MPC households further raises aggregate demand, analogous to the redistribution effect of UI.\(^9,10\)

The final column of Table A.5 compares UI extensions to an alternative policy of raising the replacement rate, reinforcing the point that the long-term unemployed are an especially good “tag” for transfers in macroeconomic stabilization. Rather than increasing the duration of benefits, I simulate a 10pp increase in the replacement rate. While the replacement rate increase offers larger overall stimulus, UI extensions offer greater bang-for-buck as evidenced by their higher multipliers. This is consistent with the long-term unemployed being a group

---

\(^8\)These authors’ exact estimate is 0.0099 (Table 4). I do not use this exact value in my setting because this would raise the level of wages above productivity in the flexible price case.

\(^9\)I treat the change in wages as a general equilibrium response to the change in UI, so that it is not incorporated in the PE output multiplier. Hence, the latter is identical to the baseline case.

\(^10\)To appreciate how different this is from the standard effect on vacancy creation under flexible prices, I simulate the same UI extensions and increase in wages in the latter setting. Because of the small profit share in this economy, the unemployment rate rises by a dramatic 3.5pp during the year of extended benefits.
with especially high MPCs and long-term unemployment being an important risk against which agents save. It suggests that policymakers should continue to focus on duration rather than level as the primary lever to vary within the UI system for stabilization purposes.

D Computational algorithm

In this section I describe the computational algorithm used to numerically study the infinite horizon environment in section 3 of the main text. I first characterize the equilibrium conditions in this environment as well as the conditions for bilateral efficiency of wages. I then outline the algorithms used to solve for the stationary RCE and transitional dynamics in response to unanticipated macroeconomic shocks. The latter algorithm builds most directly on that in Guerrieri and Lorenzoni [2017], extending it to a frictional labor market with endogenous unemployment.

D.1 Equilibrium conditions in the infinite horizon

I begin with worker-consumers’ optimality conditions. The optimal search effort of unemployed workers at the beginning of period \( t \) facing \((3')\) solves

\[
p(\theta_t; \zeta^u_t) s_t(z_t; \zeta^u_t) \left( \int \psi(z_t, \zeta^e_t) \Gamma_t(\zeta^e_t | \zeta^u_t) d\zeta^e_t - v^u_t(z_t; \zeta^u_t) \right) = \psi'(s_t(z_t; \zeta^u_t)). \tag{A.32}
\]

The optimal consumption and savings decisions of agents in the middle of period \( t \) facing \((4e')\) and \((4u')\) solve the standard Euler equations

\[
\begin{align*}
&u'(c^e_t(z_t; \zeta^e_t)) \geq \beta_t \frac{1}{m_t} v^e_t(z_{t+1}^e(z_t; \zeta^e_t); \zeta^e_t), \tag{A.33} \\
&u'(c^u_t(z_t; \zeta^u_t)) \geq \beta_t \frac{1}{m_t} v^u_t(z_{t+1}^u(z_t; \zeta^u_t); \zeta^e_t), \tag{A.34}
\end{align*}
\]

which hold with equality if \( z_{t+1}^e(z_t; \zeta^e_t) > z_t \) or \( z_{t+1}^u(z_t; \zeta^u_t) > z_t \), respectively, and where the inverse real interest rate is defined to be

\[
m_t \equiv M_t \frac{P_{t+1}}{P_t}
\]
given standard CES price indices \(P_t\) and \(P_{t+1}\). On the right-hand side, \(\psi_t(z^e_{t+1}; \zeta^e_t)\) and \(\psi_t(z^u_{t+1}; \zeta^u_t)\) denote the continuation values

\[
\psi_t(z^e_{t+1}; \zeta^e_t) = (1 - \delta_t) \int_{\zeta^e_{t+1}} \tilde{v}^e_{t+1}(z^e_{t+1}; \zeta^e_t) \Gamma_t(z^e_{t+1}|\zeta^e_t) d\zeta^e_{t+1}
+ \delta_t \int_{\zeta^e_{t+1}} \tilde{v}^u_{t+1}(z^e_{t+1}; \zeta^u_t) \Gamma_t(z^u_{t+1}|\zeta^u_t) d\zeta^u_{t+1};
\]

(A.35)

\[
\psi_t(z^u_{t+1}; \zeta^u_t) = \int_{\zeta^u_{t+1}} \tilde{v}^u_{t+1}(z^u_{t+1}; \zeta^u_t) \Gamma_t(z^u_{t+1}|\zeta^u_t) d\zeta^u_{t+1};
\]

(A.36)

with \(\psi_t(z^e_{t+1}; \zeta^e_t)\) and \(\psi_t(z^u_{t+1}; \zeta^u_t)\) denoting the marginal value of higher wealth. If the value functions and policy functions are locally differentiable, the Envelope Theorem further implies

\[
\psi_t(z^e_{t+1}; \zeta^e_t) = (1 - \delta_t) \int_{\zeta^e_{t+1}} u'(e^e_{t+1}(z^e_{t+1}; \zeta^e_t)) \Gamma_t(z^e_{t+1}|\zeta^e_t) d\zeta^e_{t+1}
+ \delta_t \int_{\zeta^e_{t+1}} \tilde{v}^u_{t+1}(z^e_{t+1}; \zeta^u_t) \Gamma_t(z^u_{t+1}|\zeta^u_t) d\zeta^u_{t+1},
\]

(A.37)

\[
\psi_t(z^u_{t+1}; \zeta^u_t) = \int_{\zeta^u_{t+1}} \tilde{v}^u_{t+1}(z^u_{t+1}; \zeta^u_t) \Gamma_t(z^u_{t+1}|\zeta^u_t) d\zeta^u_{t+1};
\]

(A.38)

where

\[
\tilde{v}^u_{t+1,z}(z_{t+1}; \zeta^u_t) = p(\theta_{t+1}; \zeta^u_t) s_t(z_{t+1}; \zeta^u_t) \int_{\zeta^u_{t+1}} \tilde{v}^u_{t+1}(z^e_{t+1}; \zeta^e_t) \Gamma_t(z^e_{t+1}|\zeta^e_t) d\zeta^e_{t+1}
+ (1 - p(\theta_{t+1}; \zeta^u_t) s_t(z_{t+1}; \zeta^u_t)) u'(e^u_{t+1}(z_{t+1}; \zeta^u_t)).
\]

(A.39)

Turning to firms, retailer \(j\) facing (7) optimally sets

\[
P_{tj} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) (1 + \tau^R) P_t^I = P_t^I
\]

when it can flexibly set prices each period (recalling that \(\tau^R\) is passively set at \(-\frac{1}{\varepsilon}\)), and meets desired demand at each \(t\) provided \(P_{tj} \geq (1 + \tau^R) P_t^I\) when it cannot.

Finally, producers facing (6′) face a rich problem in view of the heterogeneity in worker productivity. It is helpful to rewrite (6′) so that the firm only has one state variable, the composite \(\phi^u_t(\zeta^e_t) = n_t \phi^u_t(\zeta^u_t)\) giving the measure of workers of type \(\zeta^e_t\) employed by the firm.
Then the constraint summarizing the evolution of $\phi^n_t$ is

$$
\phi^n_{t+1}(\zeta^e_{t+1}) = (1 - \delta_t) \int_{\zeta^e_t} \Gamma_t(\zeta^e_t | \zeta^e_t) \left( \phi^n_t(\zeta^e_t) + q(\theta_t) \nu_t \int_{\zeta^e_t} \Gamma_t(\zeta^e_t | \zeta^e_t) \tilde{\varphi}_t(\zeta^e_t) d\zeta^e_t \right) d\zeta^e_t,
$$

with associated Lagrange multiplier $\lambda^\phi_t(\zeta^e_{t+1})$. Employing the calculus of variations, producer optimality is characterized by

$$
\int_{\zeta^e_t} s^f_t(\zeta^e_t) \left( \int_{\zeta^e_t} \Gamma_t(\zeta^e_t | \zeta^e_t) \tilde{\varphi}_t(\zeta^e_t) d\zeta^e_t \right) d\zeta^e_t - \mu_t - 1\bar{a}_t q(\theta_t) = 0, \tag{A.40}
$$

$$
\lambda^\phi_t(\zeta^e_{t+1}) = m_t s^f_{t+1}(\zeta^e_{t+1}), \tag{A.41}
$$

for gross mark-up $\mu_t \equiv \frac{P_t}{P_t^I}$ and real firm surplus from employing a marginal worker of type $\zeta^e_t$ in period $t$

$$
s^f_t(\zeta^e_t) \equiv \mu_t^{-1} a_t(\zeta^e_t) - w_t(\zeta^e_t) + (1 - \delta_t) \int_{\zeta^e_{t+1}} \lambda^\phi_{t+1}(\zeta^e_{t+1}) \Gamma_t(\zeta^e_{t+1} | \zeta^e_t) d\zeta^e_{t+1}. \tag{A.42}
$$

In the fully sticky price equilibrium, (A.40)-(A.42) will pin down the path of the gross mark-up $\mu_t$ required to implement the rest of the equilibrium allocation, as is described further in the algorithm for the sticky price dynamics later in this appendix. But when prices are flexible and the gross mark-up is one, (A.40)-(A.42) collapse to

$$
\bar{a}_t \left( 1 - \frac{k}{q(\theta_t)} \right) - \bar{w}_t + m_t(1 - \delta_t) \bar{a}_{t+1} \frac{k}{q(\theta_{t+1})} = 0 \tag{A.43}
$$

owing to the assumption on wages in (39) ensuring that all workers generate the same marginal surplus for the firm.

The preceding optimality conditions characterize equilibrium along with agents’ resource constraints and the market clearing conditions (9′)-(14′). To fully characterize the real allocation in both the flexible and fully sticky price equilibria of interest to this paper, it only remains to scale these constraints and market clearing conditions by the price level.

In particular, worker-consumers’ resource constraints imply

$$
c^i_t + m_t z^i_{t+1} = y^i_t(\zeta^i_t) + z^i_t \tag{A.44}
$$
at each $t$, where

$$y^e_t(\zeta^e_t) \equiv w_t(\zeta^e_t) + \omega_t - t_t, \quad (A.45)$$

$$y^u_t(\zeta^u_t) \equiv b_t(\zeta^u_t) + \omega_t. \quad (A.46)$$

The real value of a claim to economy-wide profits at the end of period $t$ is given by

$$q_t \equiv \frac{1}{P_t} Q_t = m_t [\pi_{t+1} + q_{t+1}] \quad (A.47)$$

where real dividends net of $\frac{R_t}{P_{t+1}}$ financing the retailer subsidy are

$$\pi_{t+1} = y_{t+1} - p^e_{t+1} \int_{\zeta^e_{t+1}} w_{t+1}(\zeta^e_{t+1}) \phi^e_{t+1}(\zeta^e_{t+1}) d\zeta^e_{t+1}, \quad (A.48)$$

using the fact that the output of each variety will be the same in any equilibrium without price dispersion. The same observation implies that combining retailers’ pure pass through technology with intermediate goods market clearing and equilibrium in the labor market yields

$$y_t = p^e_t \int_{\zeta^e_t} a_t(\zeta^e_t) \phi^e_t(\zeta^e_t) d\zeta^e_t - \bar{a}_t k \theta_t \bar{s}_t, \quad (A.49)$$

while final goods market clearing implies

$$p^e_t \int_{\zeta^e_t} \int_{z_t} c^e_t(z_t; \zeta^e_t) \phi^e_t(z_t; \zeta^e_t) dz_t d\zeta^e_t + (1 - p^e_t) \int_{\zeta^e_t} \int_{z_t} c^u_t(z_t; \zeta^u_t) \phi^u_t(z_t; \zeta^u_t) dz_t d\zeta^u_t = y_t + \omega_t. \quad (A.50)$$

Asset market clearing implies

$$p^e_t \int_{\zeta^e_t} \int_{z_t} z^e_t(z_t; \zeta^e_t) \phi^e_t(z_t; \zeta^e_t) dz_t d\zeta^e_t + (1 - p^e_t) \int_{\zeta^u_t} \int_{z_t} z^u_t(z_t; \zeta^u_t) \phi^u_t(z_t; \zeta^u_t) dz_t d\zeta^u_t = -z^g + \frac{1}{m_t} q_t. \quad (A.51)$$

Finally, budget balance for the government implies

$$p^e_t t_t + (1 - m_t) z^g = (1 - p^e_t) \int_{\zeta^u_t} \int_{z_t} b_t(\zeta^u_t) \phi_t(z_t; \zeta^u_t) dz_t d\zeta^u_t. \quad (A.52)$$

### D.2 Bilateral efficiency of wages

We can use the equilibrium conditions of the prior subsection to describe when the assumed wage process in (39) is consistent with bilateral efficiency absent commitment to long-term
contracts. The firm’s real surplus from employing a marginal worker of type \( \zeta^e_t \) at the
arbitrary wage \( \tilde{W}_t \) in period \( t \) and equilibrium wage \( P_t w_t(\cdot) \) thereafter is

\[
s^f_t (\zeta^e_t; \tilde{W}_t) = \mu_t^{-1} a_t(\zeta^e_t) - \frac{\tilde{W}_t}{P_t} + (1 - \delta_t) \int_{\zeta^e_{t+1}} \lambda_{t+1}^e(\zeta^e_{t+1}) \Gamma_t(\zeta^e_{t+1}|\zeta^e_t) d\zeta^e_{t+1},
\]

where the real shadow value of an incumbent \( \lambda_{t+1}^e(\zeta^e_{t+1}) \) is characterized in (A.40)-(A.42).\(^{11}\)

The surplus for an unemployed worker with wealth \( z_t \) and of type \( \zeta^u_t \) who matches with a
firm, becomes type \( \zeta^e_t \), and receives wage \( \tilde{W}_t \) in period \( t \) and the equilibrium wage \( P_t w_t(\cdot) \)
thereafter is

\[
s^w_t (z_t; \zeta^u_t; \zeta^e_t; \tilde{W}_t) = \hat{v}_t^e(z_t; \zeta^e_t; \tilde{W}_t) - v_t^u(z_t; \zeta^u_t),
\]

where \( \hat{v}_t^e(z_t; \zeta^e_t; \tilde{W}_t) \) is identical to (4e') except with period \( t \) wage \( \tilde{W}_t \) rather than \( P_t w_t(\zeta^e_t) \).

It follows that the real wage process \( w_t(\cdot) \) in (39) is bilaterally efficient for all agents in the
economy (absent commitment to long-term contracts) if and only if

\[
s^f_t (\zeta^e_t; P_t w_t(\zeta^e_t)) \geq 0 \Rightarrow \mu_t^{-1} a_t(\zeta^e_t) - w_t(\zeta^e_t)
\]

\[
+ (1 - \delta_t) \int_{\zeta^e_{t+1}} \lambda_{t+1}^e(\zeta^e_{t+1}) \Gamma_t(\zeta^e_{t+1}|\zeta^e_t) d\zeta^e_{t+1} \geq 0, \tag{A.53}
\]

\[
s^w_t (z_t; \zeta^u_t; \zeta^e_t; P_t w_t(\zeta^e_t)) \geq 0 \Rightarrow v_t^e(z_t; \zeta^e_t) - v_t^u(z_t; \zeta^u_t) \geq 0, \tag{A.54}
\]

for all \( \zeta^e_t \) employed by the firm in (A.53), and all \( \zeta^e_t \) such that \( \Gamma_t(\zeta^e_t|\zeta^u_t) > 0 \) in (A.54).

Wages less than labor productivity make it easy to satisfy (A.53), while the assumed
absence of disutility from labor and replacement rates less than 100% of an agent’s persistent
component of income make it easy to satisfy (A.54). I verify that these conditions are satisfied
for all workers in the stationary RCE and all periods of the flexible price transitional dynamics
described in the main text.\(^{12}\) They are further satisfied for all workers in all periods in the
sticky price transitional dynamics, except for the initial period of the shock \( t = 0 \) when the
binding zero lower bound leads to a collapse in aggregate demand and firms lay off workers.
As described in footnote 44 in the main text, an interior equilibrium in that case can be
obtained by letting firms lay off workers. For simplicity, I assume firms randomly lay off
workers; since this requires that the average surplus of the marginal incumbent (A.42) is
zero, condition (A.53) is necessarily violated for some worker types. If instead we assume
that firms target their layoffs on employees from which they earn the lowest surplus, we can
ensure (A.53) is satisfied in this period also.

\(^{11}\)Note that when evaluated at the equilibrium wage, this is indeed the firm’s surplus defined in (A.42).

\(^{12}\)Indeed, (A.53) is in fact implied by interiority in firms’ vacancy posting (A.43) in this case.
D.3 Algorithm to solve for the stationary RCE

The goal is to find a fixed point in the tuple

\[ \{m, t, p^e\}. \]

This generalizes the algorithm of simpler heterogeneous agent models where only a fixed point in \( m \) (or its inverse, the real interest rate) needs to be obtained. Here, a conjecture of \( t \) is needed to calculate agents’ real income when employed, owing to the frictions in labor markets and government intervention via UI. A conjecture of \( p^e \) is further needed to compute \( \omega_t \) according to (41), capturing spousal and other sources of household income.

The idiosyncratic state space is simplified and approximated as follows. The functional forms of UI (42) and duration-dependence in matching (44) together define a duration \( \bar{d} \equiv \max\{\bar{d}, 8\} \) after which unemployed agents face identical problems. It follows that the state space along the duration margin can be limited to \( \{0, 1, \ldots, \bar{d} - 1, \geq \bar{d}\} \). I use the Rouwenhorst procedure as described in Kopecky and Suen [2010] to discretize the persistent component of worker productivity into 3 values, and I use the Gauss-Hermite procedure to discretize the transitory component of worker productivity into 3 values. Following (40), the discount factors take on 3 values \( \{\bar{\beta} - \Delta \beta, \bar{\beta}, \bar{\beta} + \Delta \beta\} \). Finally, I discretize assets using a grid of 201 points, denser near the lower bound \( z \).

I then solve for the stationary RCE as follows:

1. Initialize small, positive tolerance levels \( \{\epsilon_{z+1}, \epsilon_t, \epsilon_{p^e}\} \) and step lengths \( \{\Delta m, \Delta t, \Delta p^e\} \).
2. Conjecture \( \{m, t, p^e\} \).
3. Use (41) to compute the steady-state endowment \( \omega \), and then (A.45)-(A.46) to compute worker real incomes \( \{y^e(\zeta^e), y^u(\zeta^u)\} \).
4. Use (A.43) along with stationarity to compute \( \theta \).
5. Iterate worker-consumers’ value functions backward using optimality conditions (A.32)-(A.39) and resource constraints (A.44), obtaining approximations of the value functions \( \{\hat{v}^e, \hat{v}^u\} \) and policy functions \( \{\hat{s}, \hat{c}^e, \hat{c}^u\} \). Here Carroll [2006]’s endogenous gridpoint method substantially speeds up convergence.
6. Iterate the resulting policy functions forward, obtaining approximations of the beginning-of-period distribution \( \{\hat{p}^e, \hat{\phi}^e, \hat{\phi}^u\} \) and middle-of-period distribution \( \{\hat{p}^e, \hat{\phi}^e, \hat{\phi}^u\} \).
7. Using the approximated policy functions and ergodic distribution, assess market clearing and consistency conditions and update \( \{m', t', p'^e\} \) accordingly:
(a) Compute the end-of-period market value of firm equity $q$ using (A.47)-(A.49) and stationarity.

(b) Approximate steady-state net asset demand $\hat{z}_{t+1}$, given by the left-hand side less the right-hand side of (A.51).

(c) Approximate steady-state taxes solving (A.52).

(d) Set $\{m', t', p^e\}$ based on the deviations in $\{\hat{z}_{t+1}, \hat{t}, \hat{p}^e\}$ from $\{0, t, p^e\}$:

\[
m' = \begin{cases} 
  m + \Delta_m \hat{z}_{t+1} & \text{if } |\hat{z}_{t+1}| > \epsilon_{z_{t+1}}, \\
  m & \text{otherwise}
\end{cases}, \\
\]
\[
t' = \begin{cases} 
  t + \Delta_t (\hat{t} - t) & \text{if } |\hat{t} - t| > \epsilon_t, \\
  t & \text{otherwise}
\end{cases}, \\
\]
\[
p^e' = \begin{cases} 
  p^e + \Delta_{p^e} (\hat{p}^e - p^e) & \text{if } |\hat{p}^e - p^e| > \epsilon_{p^e}, \\
  p^e & \text{otherwise}
\end{cases}.
\]

8. If $\{m, t, p^e\} = \{m', t', p^e'\}$, stop. Else, return to step 2 with $\{m', t', p^e'\}$.

D.4 Algorithm to solve for transitional dynamics

When prices are flexible, the goal is to find a fixed point in the sequence

\[
\{q_0, \{m_0, t_0, p^e_0\}, \ldots, \{m_T, t_T, p^e_T\}\}
\]

for $T$ very large, at which point it is assumed that the initial stationary RCE is again reached. The rationale for iterating over $\{m_t, t_t, p^e_t\}$ was explained in the prior subsection. We also need to iterate over the beginning-of-period-0 market value of equity $q_0$, given by

\[
q_0 = \pi_0 + q_0; \quad (A.55)
\]

which is needed to compute agents’ initial capital gain/loss on equity claims given the unanticipated macroeconomic shock.

When prices are fully sticky, the path of real interest rates is determined by monetary policy, so the goal is instead to find a fixed point in the sequence

\[
\{q_0, \{\theta_0, t_0, p^e_0\}, \ldots, \{\theta_T, t_T, p^e_T\}\},
\]

where labor market tightness replaces the (inverse) real interest rate in equilibrating markets.

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The idiosyncratic state space remains characterized as in the prior subsection, except for the fact that \( \bar{d} \) needs to be as large as the maximal duration of UI throughout the simulation.

I then solve for the equilibrium as follows:

1. Initialize small, positive tolerance levels \( \left\{ \epsilon_q, \epsilon_t, \epsilon_p \right\} \) and step lengths \( \left\{ \Delta_q, \Delta_t, \Delta_p \right\} \).

Under flexible prices, further initialize \( \left\{ \epsilon_{z+1}, \Delta_m \right\} \), while under sticky prices, further initialize \( \left\{ \epsilon_c, \Delta_\theta \right\} \).

2. Conjecture \( \left\{ \tilde{q}_0, \left\{ m_t, t_t, p_t^e \right\}_{t=0}^T \right\} \) in the flexible price case or \( \left\{ \tilde{q}_0, \left\{ \theta_t, t_t, p_t^e \right\}_{t=0}^T \right\} \) in the sticky price case.

3. Use (41) to compute the endowments \( \left\{ \omega_t \right\}_{t=0}^T \), and then (A.45)-(A.46) to compute worker real incomes \( \left\{ y_t^e(\zeta_t^e), y_t^u(\zeta_t^u) \right\} \).

4. Under flexible prices, iterate backward on (A.43) to compute \( \left\{ \theta_t \right\}_{t=0}^T \) given \( \theta_{T+1} = \theta^{ss} \).

5. Iterate worker-consumers’ value functions backward using optimality conditions (A.32)-(A.39) and resource constraints (A.44), obtaining approximations of the value functions \( \left\{ \hat{v}_t^e, \hat{v}_t^u \right\}_{t=0}^T \) and policy functions \( \left\{ \hat{s}_t, \hat{c}_t^e, \hat{c}_t^u \right\}_{t=0}^T \). Carroll [2006]’s endogenous gridpoint method again speeds up convergence.

6. Re-value agents’ initial wealth given the conjectured \( \tilde{q}_0 \) and assumed equity shares in asset portfolios across the idiosyncratic state space.

7. Using the policy functions from step 5 with the re-valued wealth distribution from step 6, iterate forward to obtain approximations of the beginning-of-period distributions \( \left\{ \hat{p}_t^e, \hat{\varphi}_t^e, \hat{\varphi}_t^u \right\}_{t=0}^T \) and middle-of-period distributions \( \left\{ \tilde{p}_t^e, \tilde{\varphi}_t^e, \tilde{\varphi}_t^u \right\}_{t=0}^T \).

8. Using the approximated policy functions and distributions, assess market clearing and consistency conditions and update \( \left\{ \tilde{q}_0, \left\{ m_t, t_t, p_t^e \right\}_{t=0}^T \right\} \) (under flexible prices) or \( \left\{ \tilde{q}_0, \left\{ \theta_t, t_t, p_t^e \right\}_{t=0}^T \right\} \) (under sticky prices) accordingly:

   (a) Compute the end-of-period market value of firm equity by iterating backward on (A.47)-(A.49) given \( q_{T+1} = q^{ss} \).

   (b) Compute the beginning-of-period-0 market value of firm equity \( \hat{q}_0 \) using (A.55) and \( \tilde{q}_0 \) from the previous step.

   (c) Under flexible prices, approximate net asset demand \( \hat{z}_{t+1} \), given by the left-hand side less the right-hand side of (A.51).

Under sticky prices, approximate net excess goods demand \( \hat{c}_t \), given by the left-hand side less the right-hand side of (A.50).
I find that focusing on the goods market facilitates convergence under sticky prices, while focusing on the asset market facilitates convergence under flexible prices, though of course by Walras’ Law equilibrium in one market implies the other (conditional on all other resource constraints being satisfied exactly).

(d) Approximate taxes \( \hat{t}_t \) solving (A.52).

(e) Under flexible prices, set \( \{ \hat{q}_0', m_t', t_t', p_t'' \}_{t=0}^T \):

\[
\begin{align*}
\hat{q}_0' &= \begin{cases} 
\hat{q}_0 + \Delta_q(\hat{q}_0 - \hat{q}_0) & \text{if } |\hat{q}_0 - \hat{q}_0| > \epsilon_q, \\
\hat{q}_0 & \text{otherwise}
\end{cases}, \\
m_t' &= \begin{cases} 
m_t + \phi_t \Delta_m \hat{z}_{t+1} & \text{if } \max_{t \in \{0, \ldots, T\}} |\hat{z}_{t+1}| > \epsilon_z, \\
m_t & \text{otherwise}
\end{cases}, \\
t_t' &= \begin{cases} 
t_t + \phi_t \Delta_t (\hat{t}_t - t_t) & \text{if } \max_{t \in \{0, \ldots, T\}} |\hat{t}_t - t_t| > \epsilon_t, \\
t_t & \text{otherwise}
\end{cases}, \\
p_t'' &= \begin{cases} 
p_t + \phi_t \Delta_p (\hat{p}_t - p_t) & \text{if } \max_{t \in \{0, \ldots, T\}} |\hat{p}_t - p_t| > \epsilon_p, \\
p_t & \text{otherwise}
\end{cases}.
\end{align*}
\]

As in Guerrieri and Lorenzoni [2017], I find that a weighting function \( \phi_t = \exp(-\gamma t) \) with \( \gamma > 0 \) aids in convergence.

Analogously, under sticky prices, set \( \{ \hat{q}_0', \theta_t', t_t', p_t'' \}_{t=0}^T \). In this case the second condition above is replaced by

\[
\theta_t' = \begin{cases} 
\theta_t + \phi_t \Delta \hat{c}_t & \text{if } \max_{t \in \{0, \ldots, T\}} |\hat{c}_t| > \epsilon_c, \\
\theta_t & \text{otherwise}
\end{cases},
\]

and the others remain unchanged.

9. Under flexible prices, if \( \{ \hat{q}_0, \{ m_t, t_t, p_t' \}_{t=0}^T \} = \{ \hat{q}_0', m_t', t_t', p_t'' \}_{t=0}^T \} \), stop. Else, return to step 2 with \( \{ \hat{q}_0', m_t', t_t', p_t'' \}_{t=0}^T \} \).

Under sticky prices, if \( \{ \hat{q}_0, \{ \theta_t, t_t, p_t'' \}_{t=0}^T \} = \{ \theta_t', t_t', p_t'' \}_{t=0}^T \} \), stop. Else, return to step 2 with \( \{ \hat{q}_0', \theta_t', t_t', p_t'' \}_{t=0}^T \} \).

Under sticky prices, it remains to solve for the mark-ups \( \{ \mu_0, \ldots, \mu_T \} \) consistent with producer optimality. Since the stationary RCE is reached by period \( T + 1 \), these can be calculated by iterating backwards on (A.40)-(A.42) starting from \( \lambda_{T+1}^\phi (\zeta_{T+2}^p) = m_{ss} a_{ss} k z_{ss}^{\theta_0} \) and \( \mu_{T+1} = 1 \), given the sequence \( \{ \theta_t, \phi_t^\Lambda (\zeta_t^u) \}_{t=0}^T \) found in the above algorithm. Given these
mark-ups, we must ensure that retailers indeed earn non-negative profits

\[
\mu_t \frac{1}{1 + \tau_t R_t} \geq 1 \Rightarrow \mu_t \frac{\varepsilon}{\varepsilon - 1} \geq 1,
\]

where the second expression uses the retailer tax at rate \(-\frac{1}{\varepsilon}\). The lowest mark-up I find in any month across all policy experiments studied in the paper is just above 0.9979 (when I consider the UI extension under sticky prices and a positive wage elasticity, described further in appendix C). Hence, provided \(\varepsilon \leq \frac{1}{0.9979 - 1} \approx 476\), the above condition is satisfied.

References


