Macro Candidacy Exam: August, 2018

Directions. There are four (4) questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. An OLG model with money

Consider an infinite horizon overlapping generations model. Agents live for two periods, producing in youth and consuming in old age. The only store of value is fiat money. The preferences of a representative generation t agent are given by: $u(c_{t+1}) - g(n_t)$ with $u(\cdot)$ strictly increasing and strictly concave and $g(\cdot)$ strictly increasing and strictly convex. Agents have a unit of time so $n_t \in [0, 1]$.

Each agent produces the single consumption good from a technology where output is equal to labor input. There are no firms in this economy.

In each generation, two types of agents are born. A fraction λ of agents are forward looking and have rational expectations of future variables. A fraction $1 - \lambda$ of agents hold static beliefs: these agents believe that the future money price of goods will equal the money price of goods in the current period. Throughout, agents make decisions taking prices as given.

Assume, until part (d), that there is a fixed stock of fiat money endowed to the initial old. There is no population growth.

(15) (a) Describe the optimization problem of a representative forward looking generation t agent. Set out and interpret the first-order condition for this problem. Do the same for the agents who hold static expectations.

(5) (b) What are the market clearing condition(s)?

(15) (c) Define an equilibrium for this economy. Characterize the equilibrium through a difference equation in the labor input. Are there multiple steady states? What can you say about their stability properties?

(15) (d) Suppose the money supply is stochastic, with transfers of new money proportional to existing holdings. Is money neutral?

2. A consumption-saving problem

Consider an economy populated by a continuum of infinitely-lived agents with measure 1. There is one perishable endowment/consumption good per period. Each agent's preference over random sequences of consumption is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0,1)$, $u\colon \mathbb{R}_+ \to [0, \bar{u}]$ is bounded, continuous, strictly increasing, strictly concave, and continuously differentiable, and satisfies $u(0) = 0$, $u'(0) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. At the beginning of each period, each agent receives an idiosyncratic random endowment $y \in Y = \{y_l, y_h\}$ with $0 < y_l < y_h$. The endowment y is i.i.d. across agents and across periods: the probability of $y = y_j$ is $\pi_j \in (0,1)$, $j = l, h$, and $\pi_l + \pi_h = 1$.

There is an intrinsically useless durable asset called money. The initial distribution of money among agents is $\mu_0 \in \Delta(\mathbb{R}_+)$. In each period after the endowment shock is realized, agents may trade their money holdings against the consumption good in a competitive market. There is no borrowing and lending. Agents can save only in the form of money.

At the end of each period after trading, the government levies a proportional tax on each individual's money balance at the constant rate $\tau \in [0,1]$. It redistributes the tax proceeds evenly among all agents (i.e., via lump-sum transfers). This implies that the aggregate stock of money is constant over time. Assume, as a normalization, that the aggregate stock of money is unity per capita. It follows that each agent's lump-sum transfer is τ.

(18) (a) Given a government policy variable τ, define a stationary recursive competitive equilibrium. What does it mean that such an equilibrium is monetary?

(12) (b) Consider an individual agent's recursive problem in a stationary monetary equilibrium. State a set of sufficient conditions that guarantee the existence and uniqueness of the value function, and the additional conditions that are sufficient for the existence and uniqueness of the optimal policy function (i.e., solution to the value function). Verify whether or not those conditions are satisfied.

(5) (c) Consider the individual agent's problem in (b). Rewrite the Bellman equation so that the individual state is represented by a single variable.

(10) (d) Set out the Euler equation for the reformulated Bellman equation in part (c). Use it to show that a stationary monetary equilibrium does not exist if τ is too high.

3. An oil producer

Consider an oil producer with a given amount of reserves in the ground. The producer can extract the oil at a cost which depends on a random variable, z. Suppose that the extraction cost is increasing and convex in the amount extracted and the marginal extraction cost is increasing in z. The producer faces a stochastic downward sloping demand curve. The objective of the producer is to maximize discounted expected profits. The shocks to costs and to demand are serially correlated over time but not correlated with each other.

(10) (a) Describe the optimization problem of the oil producer as a dynamic programming problem.

(10) (b) Set out and interpret the Euler equation of the producer.

(10) (c) How would you amend the dynamic programming problem if the extraction cost was increasing the amount of oil left in the ground?

(10) (d) If you were going to program this dynamic optimization problem, how would you do so? Would you be able to find a solution through value-function iteration? What is meant by "a solution"?

4. A search model

A discrete-time economy has a unit measure of workers. A worker who is unemployed has home production h. When employed, a worker produces y in the market, which is the same for all workers. The values of y lie in the set $Y = \{y_1, y_2, ..., y_N\}$. The Markov process for y is characterized by the transition probability $\phi(\hat{y}|y)$, where the caret indicates "next period". In a period, Nature breaks up a match with probability $\delta \in (0,1)$. Vacancies can be created <u>with no cost</u> but a match is not always found. A worker's matching probability is equal to the worker's search intensity denoted x. The cost of search intensity is $k(x)$. Search intensity is publicly observable.

The timing of events in a period is as follows. The state of the economy entering a period is (u, y), where u denotes the measure of workers who are unemployed at the start of the period. The measure of employed workers is denoted $e = 1 - u$. Then job destruction occurs. A worker whose job is destroyed cannot search until the next period. Then the planner chooses search intensity for workers who were unemployed at the beginning of the period. Each new match formed and each existing match produces y. Assume:

$$k'(x) > 0 \text{ and } k''(x) > 0 \text{ for all } x > 0; \; k(0) = k'(0) = 0,$$
$$k'(1-\delta) = \infty; \quad y_N > y_{N-1} > ... > y_1 > h/\beta;$$

and that the transition function $\phi(\hat{y}|y)$ is monotone in the sense that $\int f(\hat{y}) d\phi(\hat{y}|y)$ is increasing in y for all increasing functions $f(.)$.

The planner's objective is to maximize the discounted sum of expected net output at each date, where net output is the sum of home production and output produced in matches minus the cost of search intensity.

(6) (a) Formulate the planner's problem recursively.

(16) (b) Show that a unique solution for the social welfare function exists and that the optimal choice x does not depend on u.

(13) (c) Characterize the optimal choice x. Does this choice help reduce the fluctuations in aggregate output caused by the fluctuations in y relative to an economy where x is constant at the mean of the optimal x? Prove and explain.

(10) (d) Is the optimal x increasing in h? Prove and explain.