THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2011

Candidacy Examination for

The Degree of Doctor of Philosophy

MACROECONOMIC THEORY

Please read the instructions carefully. You have 3-1/2 hours to complete the exam.

GOOD LUCK!

Macro Candidacy Exam: January, 2011[1]

Directions. There are four (4) questions, each of which is worth 50 points. The points assigned to each part of each question are given in parentheses.

1. One-sector optimal growth with i.i.d. preference shocks

Time is discrete. The person maximizes expected discounted utility, where utility in a period is $u(\cdot)/z_t$. Here, $u : \mathbb{R}_+ \to \mathbb{R}$, is twice differentiable, $u'' < 0 < u'$, and $u'(0) = \infty$; while $z_t \in Z = \{a_1, a_2, ..., a_n\}$, $prob(z_t = a_i) = \pi_i \in (0,1)$, and $0 < a_i < a_{i+1}$. The discount factor is $\beta \in (0,1)$. The timing at each date is as follows. The person enters a date with a capital stock, denoted x, then realizes the preference shock, and then chooses consumption, denoted c and next period's capital stock, denoted y, subject to $c + y \leq f(x)$. Here, $f : \mathbb{R}_+ \to \mathbb{R}_+$, is twice differentiable, $f'' < 0 < f'$, $f(0) = 0$, and $f'(0) = \infty$, and is such that there exists $\bar{x} > 0$ and $\bar{x} = f(\bar{x})$.

Assume that the functional equation associated with the above problem has a unique solution, denoted $v : S \to \mathbb{R}$, where $S = [0, \bar{x}] \times Z$ and that v is strictly concave and differentiable in its first argument. Assume also that the associated optimal policy giving next period's capital stock is a function, denoted $g(x, a_i)$, that is continuous and strictly increasing (in both its arguments), and that $x > 0$ implies $g(x, a_i) < f(x)$.

(25) (a) show that if $a_j > a_i$, then

$$u'[f(x) - g(x, a_j)]/a_j < u'[f(x) - g(x, a_i)]/a_i.$$

(25) (b) Let $K_i = \{x \in (0, \bar{x}) \mid g(x, a_i) = x\}$. Show that if $x_1^* \in K_1$ and $x_n^* \in K_n$, then $x_1^* < x_n^*$.

[1] A typo in question 1a of the original version has been corrected.

2. An OLG model with money

Consider the following pure-exchange model of two-date lived overlapping generations in which for $t \geq 1$ there is one consumption good per date and one person per generation. There is a constant social endowment of the good at each date and a constant endowment of one unit of a divisible asset, money.

The utility function of a two-dated lived person is $u(x, y) : \mathbf{R}^2_{++} \to \mathbf{R}$, where the first argument is consumption when young and the second is consumption when old. The function u is strictly increasing, strictly concave, and continuously differentiable with derivatives denoted $u_1(x, y)$ and $u_2(x, y)$. Moreover, $v(x, y) = u_1(x, y)/u_2(x, y)$ is strictly decreasing in x, strictly increasing in y, and $\lim_{x \to 0} v(x, y) = \infty$. (The old person at $t = 1$ maximizes current consumption.)

The life-time endowment of a two-dated lived person is $(\omega_1, \omega_2) \in \mathbf{R}^2_{++}$, where the first component is endowment when young and the second is endowment when old. Assume that $v(\omega_1, \omega_2) < 1$.

The equilibrium concept is competitive equilibrium (CE). At date-1, the unit of money is owned by the member of generation 0. Let p_t denote the price of the asset at date t in units of date-t good.

(15) (a) A nonnegative sequence $\{p_t\}_{t=1}^{\infty}$ is a CE price sequence if it satisfies an implicit (and autonomous) first-order difference equation in p_t. Display that implicit difference equation.

(15) (b) Show that there exists a CE with $p_t = p > 0$ for all t and that p is unique.

(20) (c) Now let $u(x, y) = \ln x + \ln y$ and let $H(p_t, p_{t+1}) = 0$ be the special form that your answer to part (a) takes for this specification. Moreover, let p^* denote the unique p from part (b). Show that if $p_t \in (0, p^*)$, then there exists a unique $p_{t+1} \in (0, p_t)$ such that $H(p_t, p_{t+1}) = 0$.

3. Risk sharing without commitment

Consider a firm and a worker. Jointly, the firm and the worker can produce 1 unit of a perishable and divisible consumption good at each discrete date. Each maximizes expected discounted utility with the common discount factor $\beta \in (0,1)$. The period utility function of the firm is the identity function c, while that of the worker is $u(c)$, where $u'' < 0 < u'$, for consuming c units of good. (That is, the firm is risk neutral and the worker is risk averse.)

At each date, each has permanent autarky as an outside option. The worker's outside option at date t is a sequence of consumption $\{s_\tau\}_{\tau=t}^{\infty}$, where $s_\tau \in S = \{s^1, \ldots s^n\}$, $0 < s^1 < \ldots < s^n < 1$, and $prob\{s_t = s^i\} = \pi_i$, for $i = 1, \ldots n$, where $\sum_{i=1}^{n} \pi_i = 1$. The firm's outside option at date t is the sequence of consumption $\{1 - s_\tau\}_{\tau=t}^{\infty}$. (That is, the sum of the firm's and the worker's outside options is a sequence all of whose terms are unity.)

At date 0 and given s_0, a firm and a worker can agree to a long-term contract. At any date t and given s_t, either can defect to permanent autarky.

(25) (a) Formulate recursively the dynamic programming problem for the efficient self-enforcing contract (taking into account the fact that each agent can defect permanently to autarky).

(25) (b) Assume that the value function formulated in (a) exists and is monotone, strictly concave, and differentiable. Assume also that the solution is interior in the sense that it gives both agents positive consumption at each date in each state. Show that an efficient contract has the following property. Let c_t denote the worker's consumption at t. If the worker's defection constraint is not binding at $t+1$, then $c_{t+1} \leq c_t$; if it is binding at $t+1$, then $c_{t+1} > c_t$.

4. Credible government policy

Consider an economy with a nonatomic unit measure of agents and a government. There are two kinds of investments: the output of one can be taxed, while the output of the other cannot be taxed. Each agent has an endowment of 1 unit of good that can be invested in any proportion between the two kinds of investment. The net (before tax) rate of return on the taxable investment is $r > 0$, while that on the nontaxable investment is 0. The government uses the tax proceeds to produce a public good.

Each agent maximizes $u(c) + v(g)$, where c is consumption of the private good and g is consumption of the public good. If the tax rate on the output of taxable investment is τ, if *aggregate* investment in the taxable good is k, and if an agent invests ξ in the taxable investment, then the agent's consumption of the private good is $c = (1 - \xi) + \xi(1 + r)(1 - \tau)$ and that of the public good is $g = \tau(1+r)k$. The functions u and v are strictly increasing, strictly concave, and differentiable with $u'(1) < v'(r)$. Assume that if an agent is indifferent between several different choices for ξ, then the agent chooses the largest ξ among those. The government chooses τ to maximize $u(c) + v(g)$.

(25) (a) Suppose that the government chooses a tax rate τ first and can commit to it, and then simultaneously each agent chooses ξ. Find τ and k. (Be explicit about the notion of equilibrium that you use.)

(25) (b) Suppose that first and simultaneously each agent chooses ξ, and then the government chooses the tax rate τ. Find τ and k. (Be explicit about the notion of equilibrium that you use.)