THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Spring 2012

Candidacy Examination for
The Degree of Doctor of Philosophy

MACROECONOMIC THEORY

Please read the instructions carefully. You have 3-1/2 hours to complete the exam.
GOOD LUCK!
Macro Candidacy Exam: January, 2012

Directions. There are five 5) questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.

1. One-sector optimal growth with i.i.d. technology shocks

Time is discrete. The person maximizes expected discounted utility, with period utility \( u \) and discount factor \( \beta \in (0, 1) \). Here, \( u : \mathbb{R}_+ \to \mathbb{R} \), is twice differentiable, \( u'' < 0 < u' \), and \( u'(0) = \infty \). The timing at each date is as follows. The person enters a date with a capital stock, denoted \( x \), then realizes the technology shock \( z \in Z = \{ a_1, a_2, \ldots, a_n \} \), \( a_i \in \mathbb{R}_{++} \), \( \text{prob}(z = a_i) = \pi_i \in (0, 1) \), and \( 0 < a_i < a_{i+1} \). Then the person chooses consumption, denoted \( c \) and next period's capital stock, denoted \( y \), subject to \( c + y \leq zf(x) \). Here, \( f : \mathbb{R}_+ \to \mathbb{R}_+ \), is twice differentiable, \( f'' < 0 < f' \), \( f(0) = 0 \), and \( f(0) = \infty \), and is such that there exists \( \bar{x} > 0 \) and \( \bar{x} = a_n f(\bar{x}) \).

10) a) Describe a mapping with the property that the unique fixed point of the mapping is a continuous solution to the functional equation associated with the above problem.

10) b) Assume that the solution to the functional equation is attained by a function, denoted \( g(s) \), where \( s = (x, z) \). Assume that \( g \) is continuous and strictly increasing in both \( x \) and \( z \). Also, assume that \( zf(x) - g(z) \) consumption) is strictly increasing in both \( x \) and \( z \). Show that there exists \( x_{\min} > 0 \) such that \( g(x, z) > x \) for all \( x \in (0, x_{\min}) \) and all \( z \in Z \).

10) c) Let \( P(s, A) \) be the transition function for the above model, meaning that \( P(s, A) = \text{prob}(s_{t+1} \in A \mid s_t = s) \). Show that \( P \) is monotone.

10) d) The following mixing condition plays a role in showing that the above model has a unique stationary distribution: there exists a compact domain for \( s \) denoted \( [a, b] \ c \in (a, b) \ \epsilon > 0 \) and \( N \geq 1 \) such that \( P \left( \{ a, [c, b] \} \right) > \epsilon \) and \( P \left( \{ b, [a, c] \} \right) > \epsilon \). How would you choose \( a \) and \( b \) for the above model? Briefly defend your choice.

2. Money in a pure exchange model with idiosyncratic endowment shocks

There is a nonatomic unit measure of people and one good per discrete date. The per capita endowment of date-\( t \) good is constant. Each person maximizes expected discounted utility with discount factor \( \beta \in (0, 1) \) and period utility \( u \). Here, \( u : \mathbb{R}_+ \to \mathbb{R} \), is twice differentiable, \( u'' < 0 < u' \), and \( u'(0) = \infty \). A person's endowment at date \( t \) is \( z_t \in \{ a_1, a_2 \} \), \( 0 < a_1 < a_2 \), and \( \text{prob}(z_{t+1} = a_j \mid z_t = a_i) = \pi_{ij} > 0 \). Realizations of \( z_t \) are i.i.d. across people. There is a fixed stock of money.

At date \( t \) and after seeing the realization of \( z_t \), each person chooses \( (m_{t+1}, q_t) \in \mathbb{R}_+^2 \), subject to

\[ c_t + q_t m_{t+1} \leq u_t m_t + z_t, \]
while treating \( w_t \) the price of a unit of money in terms of the date-\( t \) good, as unaffected by the person's choice. Here \( c_t \), consumption, is the argument of \( u \) and \( n_t \) is a quantity of money.)

2) a) Define a recursive equilibrium and a steady state.

10) b) Are the assumptions sufficient for existence of a steady state in which \( w_t = v > 0 \) for all \( t \)? Defend your answer.

3. An OLG model with an asset

Consider the following pure-exchange model of two-date lived overlapping generations in which for \( t \geq 1 \) there is one consumption good per date, one person per generation, and one unit of a divisible asset which has a constant per date dividend, \( d > 0 \), an amount of the date \( t \) good.

Let us call generation \( t \) the person who is young at \( t \) and old at \( t + 1 \). Such a person has a utility function \( u(x, y) : R_+^2 \rightarrow R \), where the first argument is consumption when young and the second is consumption when old. The function \( u \) is strictly increasing, strictly concave, and continuously differentiable with derivatives denoted \( u_1(x, y) \) and \( u_2(x, y) \). Moreover, \( u(x, y) = u_1(x, y)/u_2(x, y) \) is strictly decreasing in \( x \), strictly increasing in \( y \), and \( \lim_{y \rightarrow 0} v(x, y) = \infty \). The old person at \( t = 1 \) maximizes current consumption.

The life-time endowment of a generation is \( (\omega_1, \omega_2) \in R_+^2 \), where the first component is endowment when young, the second is endowment when old. Notice that the social endowment of date-\( t \) good is \( \omega_1 + \omega_2 + d \), a constant.

At date-1, the asset is owned by the member of generation 0. Let \( p_t \) denote the price of the asset at date \( t \) in units of date-\( t \) good. In trading the asset, the agents treat the price as unaffected by their choices.

10) a) Show that there is a unique equilibrium in which \( p_t = p > 0 \). That is, show that such an equilibrium exists and that \( p \) is unique.

10) b) Is the part a) equilibrium Pareto efficient? Explain.

15) c) In order to express the dependence of the part a) equilibrium on \( d \), let \( p_t(d) \) denote that equilibrium. Assume that \( p(d) \) is continuous on \( R^+ \). Is \( \lim_{d \rightarrow 0} p(d) \) zero or positive? Defend your answer.

4. Pareto optima and competitive equilibria in infinite horizon economy.

Consider an infinite-horizon economy, \( t = 1, \ldots \). There is a single consumer and a single firm. There is one good per date. The commodity space \( S \) is \( l_\infty \). The production set is

\[
Y = \{y \in S \mid 0 \leq y_t \leq 1 + 1/t \text{ for all } t \geq 1 \},
\]

the consumption set is

\[
X = \{x \in S \mid x_t \geq 0 \text{ for all } t \geq 1 \},
\]

2
and the preference $u : X \rightarrow \mathbb{R}$ is

$$u(x) = \inf_{t \geq 1} x_t.$$

15) a) Consider the Pareto-optimal allocation, $x^* = \psi^* = 1 + 1/t$ for all $t \geq 1$. Show that the Second Welfare Theorem holds; that is, that there is a continuous linear functional $\phi : S \rightarrow \mathbb{R}$, not identically zero on $S$, such that i) for any $x \in X$ and $u(x) \geq u(x^*)$, $\phi(x) \geq \phi(x^*)$, and ii) for any $y \in Y$, $\phi(y) \leq \phi(y^*)$.

15) b) Show that the continuous linear functional $\phi$ defined in a) cannot be written as

$$\phi(x) = \sum_{t=1}^{\infty} p_t x_t$$

for some $p = (p_1, p_2, \ldots)$, and for any $x \in S$.

15) c) What additional assumption on preferences would rule out the situation in part b)? Explain.

5. Time-inconsistency.

There is a unit measure of people, each of whom maximizes discounted lifetime utility with discount factor $\beta \in [0, 1)$. Each person is both a potential inventor and a potential consumer of the inventions of others. At each date, each person must choose between developing his/her own invention (\( \xi = 1 \)) and not developing it (\( \xi = 0 \)). The cost of developing one's own invention is 1. There is a benevolent government that at each date chooses between granting patent protection to all inventions (\( y = 1 \)) or to none of them (\( y = 0 \)). If patent protection is granted, an invention yields net profit $a - 1 > 0$ to its owner and yields consumers' surplus $b > 0$ to all consumers. If patent protection is not granted, an invention yields net profit $-1$ to its owner and yields consumers' surplus $c > a + b$ to all consumers. If no invention is developed, then there are no costs or benefits to anyone. That is, if $x$ is the fraction of people who choose $\xi = 1$, then the period payoff to a person is

$$u(\xi, y, x) = \begin{cases} (a - 1)\xi + xb & \text{if } y = 1 \\ (-1)\xi + xc & \text{if } y = 0 \end{cases}.$$

15) a) Suppose the economy lasts one period (\( \beta = 0 \)), so households and the government make one-time choices. Solve for the following allocations: the first-best outcome; the Ramsey outcome; and the no-commitment outcome the government chooses $y$ after people choose $\xi$.

15) b) Suppose the economy is infinitely repeated (\( \beta \in (0, 1) \)). Assume that the government cannot commit to a patent policy i.e., at each date, the government chooses $y$ after people choose $\xi$. Derive a necessary and sufficient condition under which the infinite repetition of the one-period Ramsey outcome can be supported as a sustainable equilibrium.