August 2015

Candidacy Examination: Macroeconomics

Directions. There are four questions worth a total of 180 possible points. The points assigned to each question, by part, are given in parentheses. Answer all questions. Be as specific as you can in your answers. Read the question: Credit will not be given for responses which are not directly related to the question. Write legibly: Credit will not be given for responses that cannot be read.

Question 1: (45 minutes)

Consider an infinitely-lived agent's consumption-saving problem. The agent receives e units of endowment every period, and can save via an asset with a constant return R. The agent is initially endowed with s_0 units of the asset. In period t, he chooses the amount to consume c_t and to save s_{t+1} among his available goods that period. This yields him period-t utility $u(c_t)$. In period $t+1$ his total resources are his endowment of e plus the total return on saving, Rs_{t+1}. The agent maximizes discounted lifetime utility $\sum_{t=0}^{\infty} \beta^t u(c_t)$ with discount factor β, subject to the constraints $c_t \geq 0$, $s_{t+1} \geq 0$ (cannot borrow) for all $t \geq 0$. The assumptions are $e > 0$, $\beta \in (0,1)$, $R \in [0, 1/\beta]$, and u strictly increasing, strictly concave, and differentiable.

(a) Write down the dynamic programming problem in both sequential and recursive formulation. State a set of sufficient conditions that a solution to the sequential problem should satisfy. [15]

(b) Suppose that $s_0 = 0$. Show that $s^*_{t+1} = 0$ for all $t \geq 0$ is the unique solution to the sequential problem. Is it a solution to the recursive problem? Justify your answer. [25]

(c) Can you use the set of sufficient conditions stated in (a) directly to show that $\{s^*_{t+1}\}_{t=0}^{\infty}$ is the solution to the sequential problem? If not, which condition fails? [5]

Question 2: (45 minutes)

Consider an infinite horizon overlapping generations model with two-period lived agents. Generation t agents work $n_t \in [0,1]$ units of time in period t and consume $c_{t+1} \geq 0$ in period $t+1$.

The preferences of the representative generation t agent are represented by $u(c_{t+1}) - \varepsilon_t g(n_t)$. Assume $u(\cdot)$ is strictly increasing and strictly concave and $g(\cdot)$ is strictly increasing and strictly convex. Here ε_t is the period t realization of an **aggregate** taste shock. Generation t young agents know the realization of ε_t prior to making their decision on work and output. Assume ε_t follows a stationary AR(1) process.

Individuals produce goods from their labor input, there are no firms. The production function, $f(\cdot)$, is strictly increasing and strictly concave. Agents sell the goods they produce for money in a competitive market. Fiat money is the only store of value.

The initial old are each endowed with $M_0 > 0$ units of fiat money. Population is constant. Normalize the size of each generation to 1. For now, the money supply is constant.

(a) Define a stationary rational expectations equilibrium (SREE) for **this** economy. [5]

(b) Assume that ε_t is iid. Prove that there exists a SREE with valued fiat money where output and employment are decreasing in ε_t. Does the result still hold when ε_t is positively correlated over time? [15]

(c) If young and old are equally valued in social welfare, does the SREE allocation maximize social welfare? [5]

(d) The central bank is concerned that output and employment are fluctuating in this economy. It decides to use monetary policy to stabilize the economy. The period t money supply is given by $M_t = M_{t-1}x_t$. Here $x_t = \xi(\varepsilon_{t-1})$ and is a **known deterministic** function of the period $t-1$ taste shock. Assume x_t only takes on positive values. Variations in the money supply take the form of proportional transfers to old agents. Can the monetary authority stabilize the economy? That is, does the stationary distribution of output and employment depend on the policy function, $\xi(\varepsilon_{t-1})$? Answer by characterizing the SREE for **this** economy. [20]

Question 3: (45 minutes)

Consider the one-sector stochastic growth model with a unit measure of identical agents. Each agent is endowed with one unit of labor per period and the same initial wealth in the form of capital. The period utility of an agent is $u(C) - g(N)$, where C is the agent's consumption and $N \in [0,1]$ is the time spent working. Assume $u(C)$ is strictly increasing, strictly concave, and differentiable. Assume $g(N)$ is strictly increasing, strictly convex, and differentiable. Each household maximizes discounted utility with discount factor $\beta \in (0,1)$. Household's save by holding capital which is rented to firms.

Output is produced by competitive firms who have access to a production function given by:

$$y = AF(\tilde{K}, \tilde{N}) \tag{1}$$

where $\tilde{K}$ is the capital rented from households and $\tilde{N}$ is the labor input supplied to an arbitrary firm. Assume $F(\tilde{K}, \tilde{N})$ exhibits constant returns to scale.

The productivity of a firm is given by $A = an_{-1}^{\gamma}$ and is composed of two terms. The first, denoted a, is a TFP shock to the production function. This is an aggregate shock and follows an AR(1) process. The second term, n_{-1}^{γ}, captures the effect of **average employment in the previous period** on productivity in the current period.[1] This dependence of current productivity on past employment is parameterized by γ. Each household is small and thus has no impact on average employment in any period.

(a) Suppose that each agent makes decisions taking n and its state dependent evolution as given. Define a recursive equilibrium. To do so, carefully and completely specify the individual household problem as a dynamic programming problem. Discuss state and control variables. Then describe the conditions of market clearing and other elements of a recursive competitive equilibrium. [20]

(b) Suppose the representative household believes that the aggregate capital stock, k, evolves according to $k' = h(a,k)$. Is this consistent with a recursive competitive equilibrium? [10]

(c) Assume there is a recursive equilibrium that satisfies your definition in part (a). Will it be coincide with the solution of the planner's problem? Defend your answer. [5]

(d) Assume a is iid and $\gamma > 0$. In this case, explain in words and through the model how variations in TFP in period t can impact the economy in future periods. In other words, explain how $\gamma > 0$ creates an endogenous propagation of TFP shocks. [10]

Question 4: (45 minutes)

Modify the model of Diamond and Dybvig (1983) as follows:

[1] Here capitalized words correspond to individual variables and lower case words are for aggregates and/or averages.

(i) An individual's utility function is $[\theta + (1-\theta)\phi]\, u(c_1) + (1-\theta)\beta u(c_2)$, where $\theta \in \{0,1\}$ is a taste shock and $\phi > 0$ is a known constant. Add a subscript H to the variables associated with individuals with $\theta = 1$ and a subscript L to the variables associated with individuals with $\theta = 0$.

(ii) Projects are divisible. If a fraction, say x, of a long-term project is liquidated in period 1, the liquidation yields x units of goods in period 1, while the remainder of the project will yield $R(1-x)$ units of goods in period 2.

As in the original paper, the gross rate of return to storing goods from one period to the next period is 1, and the bank cannot observe individuals' realizations of θ. Assume:

$$0 < \beta < 1 - \phi, \quad 0 < \phi < 1, \qquad R > \frac{1}{\phi + \beta}.$$

To simplify the analysis, assume that $u\,(c) = \ln c$. Please complete the following parts.

(a) Formulate and solve the problem of a social planner who can observe individuals' realizations of θ and who maximizes the expected utility of a representative individual in the economy. Denote the planner's choices of the consumption levels (i.e., the efficient allocation) as $(c_1^{*H}, c_1^{*L}, c_2^{*L})$, where the subscripts denote periods and the superscripts the realizations of θ. [10]

(b) In what sense does the efficient allocation feature risk sharing among the individuals? Support your answer with algebra. [5]

(c) Consider the following deposit contract offered by a bank. The individual deposits the entire unit of the endowment in the bank. (In the following description, the "fraction of the deposit" refers to the fraction of this unit, rather than the amount of goods that an individual obtains from a withdrawal.) The individual can withdraw any fraction $x \in [0,1]$ of his/her deposit in period 1, and keep the remaining fraction in the bank. The amount of goods obtained from withdrawing x fraction of the deposit is $x\delta$, where $\delta = c_1^{*H}$ and c_1^{*H} is characterized in part (2) above. In period 2, the individuals who still have deposits in the bank share the bank's assets in proportion to their deposits. That is, if $Y \in [0,1]$ is the fraction of total deposits withdrawn in period 1, then an individual who withdrew a fraction $x < 1$ of his/her deposit obtains the following amount of goods from the bank in period 2:

$$\max\left\{0, \frac{1-\delta Y}{1-Y} R\,(1-x)\right\}.$$

Let $W \in [0,1]$ denote the expectation in period 1 that every other individual with $\theta = 0$ will withdraw W fraction of the deposit in period 1. This expectation is formed after individuals have already deposited in the bank. Given this expectation, analyze the optimal withdrawal decisions of the two types of individuals. In particular, write x (the optimal fraction of withdrawal by an individual with $\theta = 0$) as a best response to W. [10]

(d) Define a symmetric equilibrium as one in which $x = W$, and define a bank-run equilibrium as one in which $x = W = 1$. Characterize all symmetric equilibria by solving W. Prove that the bank-run equilibrium is the only symmetric equilibrium if the following condition holds:

$$\frac{R\,(\phi+\beta) - 1}{R-1} < \frac{\phi}{\phi+\beta}. \tag{2}$$

[15]

(e) Explain the economic reason why there is no symmetric equilibrium with the above deposit contract that can implement the efficient allocation in part (a). [5]