THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Fall 2013

Written Portion of the Comprehensive Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer two questions from each section. You will not receive additional credit, and may receive less credit, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

1. Given a continuous and monotonic utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$, the associated *indirect utility function* $v : \mathbb{R}^L_{++} \times \mathbb{R}_+ \to \mathbb{R}$ indicates the maximum utility that can be obtained at prices $\mathbf{p} \in \mathbb{R}^L_{++}$ and income $y \in \mathbb{R}_+$. Formally,
$$v(\mathbf{p}, y) = \max\left\{u(\mathbf{x}) : \mathbf{x} \in \mathbb{R}^L_+,\ \mathbf{p}\cdot\mathbf{x} \le y\right\}$$

   (a) Show that v is a quasi-convex function.

   (b) Find the indirect utility function v associated with the utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ defined by $u(\mathbf{x}) = (x_1)^\alpha (x_2)^\beta$, where $\alpha, \beta > 0$. When is the function $v(\mathbf{p}, \cdot)$ a concave function of income?

   (c) Now consider a consumer with a general utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ that is *concave.* Show that for any fixed $\mathbf{p} \in \mathbb{R}^L_{++}$, the function $v(\mathbf{p}, \cdot)$ is a *concave* function of income. What does this tell you about the consumer's attitude to variabiity in income?

   (d) What can you say in general about a consumer's attitude to variability in prices?

2. Consider an economy $\mathcal{E} = (u_i, \mathbf{w}_i)_{i=1}^I$ in which there are two (2) states of nature at date 1 and only one physical good. The consumers' utility functions are of the expected utility form. Specifically, consumer i's utility is
$$u_i(\mathbf{x}_i) = \pi_i \ln x_{i1} + (1 - \pi_i)\ln x_{i2}$$
where x_{is} denotes i's consumption of the good in state s and π_i is consumer i's subjective probability of state 1 (for all i, $0 < \pi_i < 1$). Notice that in this specification all consumers have the same state contingent utility function in every state but may disagree on the probabilities of the different states (it is possible that $\pi_i \neq \pi_j$). Suppose also that each consumer is endowed with one unit of the good in each state, that is, for all i, $w_{is} = 1$.

   (a) Find a Walrasian equilibrium (with complete contingent markets at date 0) for $\mathcal{E}$. What role do contingent markets play in this economy?

   (b) Now suppose that there are no contingent markets available at date 0. There are, however, two assets that can be bought and sold. One asset pays one unit of the good in state 1 and none in state 0. The other asset pays one unit of the good in both states 1 and 2. Find a Radner equilibrium (with incomplete markets) for $\mathcal{E}$.

3. Consider an economy $\mathcal{E} = (\succeq_i, \mathbf{w}_i)_{i=1}^I$ in which all consumers' preferences are regular (complete and transitive) and strictly increasing (if $\mathbf{x}'_i \ge \mathbf{x}_i$ and $\mathbf{x}'_i \neq \mathbf{x}_i$ then $\mathbf{x}'_i \succ_i \mathbf{x}_i$). Let $(\mathbf{p}^*, (\mathbf{x}^*_i)_i)$ be a Walrasian equilibrium.

   (a) Assume that commodities $1, \ldots, L-1$ are divisible, as usual, but commodity L is discrete (exists only in integer amounts). Is $(\mathbf{x}^*_i)_i$ necessarily Pareto efficient? If your answer is "No," give a counterexample. If your answer is "Yes," give a proof.

   (b) Same as (a), except that now, *all* commodities are discrete.

Section II

1. The following is a game between three players, 1,2 and 3. In stage 1, Players 1 and 2 decide whether to establish a *link.* This happens by a randomly chosen player, either 1 or 2, proposing to form a link or passing; if she proposes, the other player can accept or reject. If accept, the link forms and each player pays a small cost $\epsilon > 0$. In either case, the game moves to stage 2. In stage 2 , the process is repeated with Players 2 and 3. In stage 3, the process is repeated with Players 3 and 1. At each stage, the formation or non-formation of a link is known to all players. In stage 4, every pair that has a link connecting them play a Prisoners' Dilemma, that is, the following game with the *direct* payoffs for a player being shown below as a function of the action choices of the pair:

$$\begin{array}{ccc} & C & D \\ C & 3 & -1 \\ D & 4 & 0 \end{array} .$$

   The other player's payoff is symmetric.

   There is also an indirect payoff for Player i from her link to Player j. This is denoted by x_{ij} and is equal to $j's$ payoff from the game with k,if the link jk exists *and* if i and j both play C. (Otherwise it is 0.) Note players move simultaneously in stage 4 and a player can choose to play separate actions against her two opponents, if she has two.

   The game ends after stage 4. If Player i is not linked to either of the two other players, his payoff from the game is 0. A player's payoff is the sum of her payoffs from all her links minus the costs of the links she has formed in the first three stages of the game.

   Suppose Player 3 is a "crazy" type, who is addicted to playing C, whenever he finds himself in a game where the direct payoffs correspond to a Prisoners' Dilemma. However, Player 3 is rational in his acceptance/rejection decision for links. Describe a subgame perfect equilibrium in this modified game. Is it unique?

2. (a) **Sure-thing principle for lotteries.** In the von Neumann-Morgenstern expected utility framework, consider lotteries x, y, z, w on a finite set of consequences Z. The sure-thing principle for lotteries is the following axiom:

   For any $\alpha \in (0, 1]$ and for any lotteries x, y, z, w,

   $\alpha x + (1-\alpha)z \succ \alpha y + (1-\alpha)z \iff \alpha x + (1-\alpha)w \succ \alpha y + (1-\alpha)w.$

   Show that the vNM axioms imply the sure-thing principle. (You can use any version, but use the axioms themselves, not the expected utility characterisation. State the axioms you are using. For example, you can use the ones we used in class, defined in terms of strict preference, where Independence is stated as follows, for $\alpha \in (0, 1]$ and for any lotteries x, y, z; if $x \succ y$, then $\alpha x + (1-\alpha)z \succ \alpha y + (1-\alpha)z$.)

   (b) Assume that a risk averse consumer initially has wealth $w > 0$. There is some probability $p \in (0, 1)$ that he will lose an amount L, where $0 < L < w$. (For example, his house may burn down with probability p) The consumer has the chance to purchase an insurance. He can choose any coverage level $q \in [0, L]$: If he purchases coverage level q, the insurance company will pay q if the consumer faces a loss but consumer has to pay πq to purchase a coverage level q, where $\pi \in (0, 1]$ and is determined by the insurance company.

   Assume that the insurance industry is perfectly competitive so that the insurance company is making zero expected profit for any coverage level. In this case how much coverage will the consumer purchase?

3. Consider a sealed bid auction with n bidders, with bidder i having a private value $v_{i,}$which is her private information. The v_i are i.i.d draws with a cdf $F(.)$ on [0,1]. All players are risk neutral and the seller value is 0. Suppose the seller sets a reserve price $r \geq 0$. This means no bid below r will be accepted and, in a second-price auction, if the highest bidder wins by bidding above r and the second-highest bid is below r, she pays r. Find the symmetric increasing strategy equilibrium for the first-price auction. (You may use revenue equivalence with the second-price auction if you want.)