THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Fall 2016

Candidacy Examination for The Degree of Doctor of Philosophy

MACROECONOMIC THEORY

Please read the instructions carefully. You have 3-1/2 hours to complete the exam.

GOOD LUCK!

Macro Candidacy Exam: August, 2016

Directions. There are four (4) equally weighted questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. Money-financed government expenditures

Consider an infinite horizon overlapping generations model. Agents live for two periods, producing in youth and consuming in old age. The only store of value is fiat money. The preferences of a representative generation t agent are given by: $u(c_{t+1}) - g(n_t)$ with $u(\cdot)$ strictly increasing and strictly concave and $g(\cdot)$ strictly increasing and strictly convex. Agents have a unit of time so $n_t \in [0, 1]$.

The non-standard part of the economy is the production function. The output of an agent supplying n units of time is $y = A(\bar{k})n$, where $\bar{k}$ is a fixed level of public capital and $A(\cdot)$ is an increasing function. Unless stated otherwise, $\bar{k}$ is given.

Public capital depreciates at a rate of $\delta \in (0, 1)$. To maintain a constant capital stock, the government prints money, buys goods, and converts these goods on a one-to-one basis into new capital. That is, this is a one sector economy.

(a) (10) Set out and explain the optimization problem of a representative generation t agent. Set out and explain the first order condition for this problem.

(b) (5) Set out and explain the government budget constraint.

(c) (5) Set out and explain the market clearing condition(s).

(d) (20) Define an equilibrium for this economy. Characterize the equilibrium through a difference equation in the labor input. Are there multiple steady states? What can you say about their stability properties? Which one, if any, coincides with the planner's optimal allocation given $\bar{k}$?

(e) (5) How would you study the choice of $\bar{k}$ by the government?

2. One-sector growth

Consider the following version of the one-sector growth model. A representative agent chooses a non-negative sequence of consumption and next period capital stock $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize lifetime utility,

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum \beta^t u(c_t)$$

subject to $c_t + k_{t+1} \leq f(k_t)$ with k_0 given. Here, $\beta \in (0,1)$ and both $u \colon \mathbb{R}_+ \to \mathbb{R}$ and $f \colon \mathbb{R}_+ \to \mathbb{R}_+$ are continuous, strictly increasing, strictly concave, and continuously differentiable, and satisfy $f(0) = 0$, $f'(0) = \infty$ and $\lim_{k \to \infty} f'(k) = 0$, and $u'(0) = \infty$.

(a) (7) Set out the functional equation associated with this sequential problem. Be explicit about the state space, the feasible correspondence, and the return function.

(b) (6) What properties does the value function satisfy? You do not need to prove your claim; just list all of them.

(c) (20) Let g be the policy function. Assume that g is a continuous and single-valued function. Show that g is increasing, and, furthermore, that consumption is increasing in the capital stock.

(d) (12) Use the assumptions and conclusions of part (c) to show that there exists a unique stationary point $k^* > 0$ such that $k^* = g(k^*)$.

3. Investment in education

Consider a two-period economy with a measure of two-period lived agents. An agent of type $(\alpha, a) \in [0,1] \times \mathbb{R}_+$ has endowment $\alpha\omega$ of the single good in youth and $(1-\alpha)\omega$ when old where $\omega > 0$, and gets $ah(e)$ units of additional output when old if the agent obtains e amount of education when young. Assume that $h(e)$ is strictly increasing and strictly concave. The cdf of (α, a) over the population is given by $G : [0,1] \times \mathbb{R}_+ \to [0,1]$. Lifetime utility is given by $u(c^y) + \beta u(c^o)$, where $u(\cdot)$ is strictly increasing and strictly concave and $\beta \in (0,1)$. Here c^y is consumption in youth and c^o is consumption when old.

Each agent can borrow and lend at a gross real rate of R, which they take as given. These are consumption loans as goods are not storable. Further, each agent can invest in education in youth. Let p denote the per unit cost of e, the input into h, in terms of the first-period endowment.

(a) (10) Set out and explain the optimization problem of an individual agent of type (α, a). Set out and explain the first-order conditions for the agent. Explain why the choice of e does not depend on α or on β. Explain how the choice of e depends on a.

(b) (15) Suppose that G is degenerate so that every agent has the same type. In a general equilibrium of this economy, what determines the equilibrium interest rate, R^*, and the equilibrium level of education, e^*? How do these equilibrium variables depend on β? Why does β influence e in general equilibrium but not in partial equilibrium?

(c) (10) How would you define an equilibrium, particularly the market clearing condition, when agents are heterogeneous?

(d) (10) Suppose that the individual faces a borrowing constraint: the most an agent can borrow is $\bar{b}$. Set out the optimization problem of an individual. Suppose the borrowing constraint binds. Discuss the first order condition(s) for this problem. How does the choice of e vary with a and the borrowing constraint?

4. A model of employment search

A discrete-time economy has a unit measure of workers. A worker who is unemployed has home production h. When employed, a worker produces y, which is the same for all workers. The values of y lie in the set $Y = \{y_1, y_2, ..., y_N\}$, where $y_N > y_{N-1} > ... > y_1 > h/\beta$ and $\beta \in (0, 1)$ is the discount factor. The Markov process for y is characterized by the transition probability $\phi(\hat{y}|y)$, where the caret indicates "next period" and where $\phi(\hat{y}|y)$ is monotone in the sense that $\int f(\hat{y})d\phi(\hat{y}|y)$ is increasing in y for all increasing functions $f(.)$. In a period, Nature breaks up a match with probability $\delta \in (0, 1)$. Vacancies are always available, but a match is not always found. A worker's matching probability is equal to the worker's search intensity, denoted x. The cost of search intensity is $k(x)$, where $k'(x) > 0$ and $k''(x) > 0$ for all $x > 0$; $k(0) = k'(0) = 0$, and $k'(1 - \delta) = \infty$.

The timing of events in a period is as follows. The state of the economy entering a period is (u, y), where u denotes the measure of workers who are unemployed at the start of the period. The measure of employed workers is denoted $e = 1 - u$. Then job destruction occurs. A worker whose job is destroyed cannot search until the next period. Then the planner chooses search intensity for workers who were unemployed at the beginning of the period. Each new match formed and each existing match produces y.

The planner's objective is to maximize the discounted sum of expected net output at each date, where net output is the sum of home production and output produced in matches minus the cost of search intensity.

(a) (6) Formulate the planner's problem recursively.

(b) (16) Show that a unique solution for the social welfare function exists and that the optimal choice x does not depend on u.

(c) (13) Is the optimal x increasing in y? Derive your claim and explain it.

(d) (10) Is the optimal x increasing in h? Derive your claim and explain it.