THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

August 2016

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

SECTION I

I.1 Given a continuous and increasing utility function $U : \mathbb{R}^L_+ \to \mathbb{R}_+$ which is onto, define the corresponding expenditure function $e : \mathbb{R}^L_{++} \times \mathbb{R}_+ \to \mathbb{R}_+$ by

$$e(\mathrm{p}, u) = \min_{\mathrm{x}} \mathrm{p} \cdot \mathrm{x} \text{ subject to } U(\mathrm{x}) \geq u$$

and the corresponding indirect utility function $v : \mathbb{R}^L_{++} \times \mathbb{R}_+ \to \mathbb{R}_+$ by

$$v(\mathrm{p}, w) = \max_{\mathrm{x}} U(\mathrm{x}) \text{ subject to } \mathrm{p} \cdot \mathrm{x} \leq w$$

Show rigorously that for any $\mathrm{p} \in \mathbb{R}^L_{++}$, the functions $e(\mathrm{p}, \cdot)$ and $v(\mathrm{p}, \cdot)$ are inverses of each other.

I.2 Consider the following two-consumer, two-good exchange economy $\mathcal{E}$. The consumers have utility functions

$$u_1(x, y) = xy \qquad \text{and} \qquad u_2(x, y) = \max\{x, y\}$$

respectively, and their endowment vectors are: $\mathrm{w}_1 = (3, 2)$ and $\mathrm{w}_2 = (2, 3)$.

(a) Show that this economy does not have a Walrasian equilibrium. Does it have a core allocation?

(b) Now consider a replica economy $\mathcal{E}(2)$ in which there are two clones of each individual—that is, there are two consumers with utility functions and endowments (u_1, w_1) and two with (u_2, w_2). Show that $\mathcal{E}(2)$ does have a Walrasian equilibrium. Is the Walrasian equilibrium allocation also a core allocation? Why?

(c) Finally, consider a replica economy $\mathcal{E}(n)$ in which there are n clones of each individual. Show that if n is odd, then $\mathcal{E}(n)$ does not have a Walrasian equilibrium but if n is even, then it does.

I.3 Suppose there are two consumers, two states of nature—"sun" and "rain"—and a single physical good. Consumers' utilities are common across states and given by

$$v_i(x_{is}) = \sqrt{x_{is}}$$

where x_{is} denotes i's consumption of the good in state s. Both consumers believe that the two states are equally likely and hence the expected utility is

$$u_i(\mathrm{x}_i) = \frac{1}{2} \sum_s \sqrt{x_{is}}$$

where $\mathrm{x}_i \in \mathbb{R}^2_+$ is i's consumption bundle. Suppose that the endowment vectors of the two consumers are $\mathrm{w}_1 = (2, 0)$ and $\mathrm{w}_2 = (0, 2)$, respectively. Thus, for instance, consumer 1 has an endowment of 2 units if the state is "sun" and 0 if the state is "rain."

(a) Find a Walrasian equilibrium with complete markets.

(b) Now suppose that both consumers have access to a weather forecast prior to any trading. The forecast is accurate with probability $p \geq \frac{1}{2}$; that is, if the true state is "sun", it predicts "sun" with probability p and "rain" with probability $1 - p$. Similarly, if the true state is "rain", it predicts "rain" with probability p and "sun" with probability $1 - p$. How does the accuracy of the weather forecast affect the welfare of the consumers?

SECTION II

II.1 Given a Bernoulli utility function u, its local *risk aversion function* is defined by

$$r(x) = -\frac{u''(x)}{u'(x)}$$

(a) Show that any two utility functions with the same risk aversion function must have the same ranking over lotteries. In other words, the two utility functions must be positive affine transformations of one another.

(b) There are two assets available to investors: a safe asset with no return (say cash) and a risky asset with a random return (say a stock). Suppose that the rate of return from the risky asset is $\rho_1 > 0$ with probability π and $\rho_2 < 0$ with probability $(1 - \pi)$. The risky asset has positive expected return, that is, $\pi\rho_1 + (1 - \pi)\rho_2 > 0$. Suppose that there are two investors, 1 and 2, with the same wealth w, with Bernoulli utility functions u_1 and u_2, respectively. Suppose that the corresponding risk aversion functions r_1 and r_2 are such that for all x, $r_1(x) < r_2(x)$. Show that investor 1 will invest a greater amount in the risky asset than will investor 2.

II.2 Suppose Players A and B have to agree on dividing a pie of size 1. They both discount the future using discount factor δ and player i's utility from obtaining x_i at time t is $\delta^{t-1} x_i$. If the players cannot agree, each gets 0. The bargaining proceeds as follows: Player 1 makes an offer in periods $1, 3, 5, 7...$; Player 2 either accepts or rejects the offer. Player 2 makes offers (if no earlier offer has been accepted, of course) in periods $2, 4, 6, 8...$ and Player 1 responds by accepting or rejecting. The one difference from the usual Rubinstein bargaining model is that the amount offered to the other player must be drawn from $\{0, .01, .02, ...0.99, 1\}$ with the proposer keeping 1 minus the amount offered to the other player (if the offer is accepted). Suppose $\delta = .995$. Show that the allocations $(1, 0)$ and $(0, 1)$ can both be sustained as subgame perfect equilibria.

II.3 Suppose there are four players, A, B, C, D playing an infinitely repeated Prisoners' Dilemma in the following way: In each period, $t = 1, 2, 3...$ one of the following match-ups is randomly chosen with probability 0.5 each: $\{(A, C), (B, D)\}$ or $\{(A, D), (B, C)\}$. That is, A has a 0.5 chance of meeting C and a 0.5 chance of meeting D in any given period. However, players are anonymously matched (suppose they are playing online) so that no one recognizes his or her opponent. The game each pair plays in each period is the following:

	c	d
c	$3, 3$	$-1, 4$
d	$4, -1$	$0, 0$

Each player chooses c or d and observes her payoff. Then she is rematched the following period with an equal chance of meeting either player on the other side. All players discount the future with the discount factor δ. Check and show whether the following strategy is a subgame perfect equilibrium for sufficiently high δ : Each player begins by playing c. If, in any period, he observes a payoff other than 3, he plays d in every period thereafter. Would things change if a person can recognize the person he is playing with in a given period?