THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2015

Written Portion of the Candidacy Examination for the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

I.1 Define the function $c : \mathbb{R}_+ \times \mathbb{R}^2_{++} \to \mathbb{R}_+$ by

$$c(q; w_1, w_2) = q(w_1 + \sqrt{w_1 w_2} + w_2).$$

Can c be a cost function? If your answer is "No", prove that c cannot be a cost function for any technology. If your answer is "Yes", derive a production function $f : \mathbb{R}^2_+ \to \mathbb{R}_+$ for which c is the cost function.

I.2. Consider the following two-commodity, two-consumer exchange environment. Consumer preferences are represented by the utility functions

$$u_1(x_{11}, x_{21}) = x_{11} - (\frac{1}{4})(14 - x_{21})^2, \quad u_2(x_{12}, x_{22}) = x_{12} - (\frac{1}{2})(7 - x_{22})^2.$$

The endowments are

$$w_1 = (50, 0), \quad w_2 = (0, 12).$$

a) Find the set of Pareto efficient allocations.

b) Normalize prices by setting $p_1 = 1$, and find a competitive equilibrium.

c) Now suppose that consumer 2 acts as a monopolist in setting p_2, while consumer 1 continues to take prices as given. Find the monopoly equilibrium price and allocation.

I.3 Given an exchange economy $(\succeq_i, w_i)_i$ assume that each $\succeq_i$ is regular (complete and transitive), nondecreasing, locally nonsatiated, continuous and convex; and that $\bar{w} \in \mathbb{R}^L_{++}$. Let $(x_i^*)_i$ be a *weakly Pareto efficient* allocation, that is, $(x_i^*)_i$ is feasible and there does not exist a feasible allocation $(x_i)_i$ satisfying $x_i \succ_i x_i^*$ for all i. These assumptions apply to all three questions below.

a) Is $(x_i^*)_i$ necessarily Pareto efficient? If your answer is "Yes", give a proof. If your answer is "No", give a counterexample.

b) Does there necessarily exist a price $p^* \in \mathbb{R}^L_+ \backslash \{0\}$ such that $(p^*, (x_i^*)_i)$ is a competitive quasi-equilibrium? If your answer is "Yes," give a proof. If your answer is "No," give a counterexample.

c) Does there necessarily exist a price $p^* \in \mathbb{R}^L_+ \backslash \{0\}$ such that $(p^*, (x_i^*)_i, (m_i^*)_i)$ is a competitive equilibrium with transfers, where $m_i^* = p^* x_i^*$ for each i. If your answer is "Yes," give a proof. If your answer is "No," give a counterexample.

Section II

1. Consider three players, 1, 2 and 3, who are placed on a line with 1 connected to 2 but not to 3 but 2 connected to both 1 and 3. In each period, players are bilaterally matched if they are connected to play the Prisoners' Dilemma game below—that is, Player 2 plays two games, against 1 and 3 but 1 plays only against 2 and 3 also only against 2. In each game, each player chooses either C or D; Player 2 has to choose the same action for each of her games. The stage game is below.

	C	D
C	$3,3$	$0,4$
D	$4,0$	$1,1$

Future payoffs are discounted by the factor $\delta \in (0,1)$ Player 1's payoff is the discounted sum of his payoffs from the game with 2, as is Player 3's. Player 2's payoff is obtained by first adding her payoffs from the two games to get her stage game payoff and then taking the discounted sum of her stage game payoffs.

Consider the following strategy: Start by playing C. If D is observed in any play, play D thereafter.

(a) Using the one-deviation property, determine for what values of δ, if any, the strategy above, if followed by all three players, constitutes a subgame perfect equilibrium.

(b) Suppose now that Players 1 and 3 are also connected, so each player plays against both the others and gets a payoff in the stage game by adding the payoffs in the two games. Each player has to choose the same action in both games. Answer the same question as in (a) for this new set-up.

2. Consider the following strategic form games a and b :

a :

	A	B
A	$1,1$	$-1,0$
B	$0,-1$	$0,0$

b :

	A	B
A	$-1,-1$	$-2,0$
B	$0,-2$	$0,0$

The players, 1 and 2, know that a occurs with probability p and b with probability $1-p$ and $p < \frac{1}{2}$.

(a) Describe the equilibria of this game.

(b) Now suppose we modify the game so that the following timeline obtains:

i. Nature chooses a or b with the requisite probabilities.

ii. Player 1 learns Nature's choice with probability $q > 0$.

iii. Player 1 can choose to pay a cost $c > 0$ to communicate with Player 2. He can send a message from $\{a, b\}$. He can, of course, choose not to pay the cost and not send the message.

iv. Player 2 observes one of the following messages: $\{a, b, \text{ no message}\}$, depending on what Player 1 does.

v. Players 1 and 2 each choose one of the actions $\{A, B\}$ depending on whatever information they possess.

If a message in $\{a, b\}$ is sent, assume that it contains proof of the state. Construct an equilibrium for sufficiently small c. Are there any out-of-equilibriun probability zero outcomes? If so, be sure to specify what beliefs sustain the equilibrium. (Remember an equilibrium is a strategy profile, one for each player.)

3. Search engines like Google and Yahoo! sell links to advertisers who wish to advertise their product in response to a specific query. For instance, a search for "flowers" will produce a list of links to web sites of companies that offer flower delivery services. In what follows, suppose that there are only two (2) links for sale in response to a particular query: the link that is on top of the list and the one on the bottom, below the first. It has been estimated that the top link will produce c^T clicks per hour whereas the bottom link will produce $c^B < c^T$ clicks per hour. There are three companies who wish to buy links to their web sites. A single click on company i's link will produce an expected profit of x_i. Thus, if firm i's link is placed in position k (where $k = T, B$), then its expected profit is $c^k x_i$.

(a) First, suppose that the links are allocated using the Vickrey-Clarke-Groves (VCG) mechanism. Suppose also that $x_1 > x_2 > x_3$. What will be the allocation and what will each company pay to the search engine?

(b) Next, suppose that the links are allocated by means of an ascending auction. Specifically, the price per-click starts at zero and rises continuously. The current price is observed by all and companies indicate whether they are "in" or "out" by pressing a button. All three companies are "in" at a price of zero but can drop out whenever they wish. Once a company drops out, it cannot re-enter the auction. Suppose company 3 drops out at a price of p_3 and then company 2 drops out at a price of $p_2 > p_3$. Then company 1 is awarded the top link at a price of p_2 per-click and company 2 is awarded the bottom link at a price of p_3 per click.

i. Again assuming that $x_1 > x_2 > x_3$, show that the ascending auction has an equilibrium in which, when all three are "in", each company $i = 1, 2, 3$ decides to drop out when the price reaches x_i. When only two are "in", each remaining company $i = 1, 2$ decides to drop out at a price p_i such that

$$c^T (x_i - p_i) = c^B (x_i - p_3)$$

where p_3 is the price at which company 3 dropped out.

ii. Are the prices the companies pay in the equilibrium of the ascending auction the same as those in the VCG mechanism?