Macro Candidacy Exam: August, 2019

Directions. There are four (4) questions worth a total of 180 possible points. You should answer all the questions. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. One-sector growth in an OLG model

In discrete time, there is one good per date. The production function is constant-returns-to scale in two inputs: capital and labor. Expressed in terms of the capital-labor ratio, denoted k, output per unit of labor is $f(k)$, where $f : R_+ \to R_+$, is twice differentiable and satisfies $f''(k) < 0 < f'(k)$ and $f'(0) = \infty$ and $f'(\infty) = 0$. Let c_t denote consumption per unit of labor and let the first date be $t = 1$, with $k_1 > 0$ given. We say that a sequence $\{c_t\}_{t=1}^{\infty}$ is *feasible* if there exists a non-negative sequence $\{k_t\}_{t=2}^{\infty}$ that satisfies

$$c_t + k_{t+1} \leq f(k_t).$$

(We are assuming that capital depreciates fully in a period.) Given k_1, we say that a feasible sequence $\{c_t\}_{t=1}^{\infty}$ is *inefficient* if there exists another feasible sequence $\{c'_t\}_{t=1}^{\infty}$ satisfying $c_t \leq c'_t$ and strictly at some date.

(15) (a) Let k^* satisfy $f'(k^*) = 1$. Prove the following. If $\{\hat{k}_t\}_{t=2}^{\infty}$ converges to $\hat{k} > k^*$, then any $\{c_t\}_{t=1}^{\infty}$ that is feasible for $\{k_t\}_{t=2}^{\infty} = \{\hat{k}_t\}_{t=2}^{\infty}$ is inefficient.

For the remainder of the question, assume the following OLG setting. There are two-period lived people with a large and constant number of people per generation. Each person when young supplies one unit of labor, earns their marginal product, and saves it in a form that becomes capital when old. (They consume only when old and their consumption when old is the product of the marginal product of capital times the amount they save.) Assume here that $f(k) = k^{\alpha}$, where $\alpha \in (0, 1)$.

(5) (b) Express the implied law of motion for k_t as a first-order difference equation in k_t.

(10) (c) Show that the sequence for k_t implied by your to answer to (b) converges to a steady state and give an explicit formula for the steady state. (Hint: You may find it useful to express the difference equation in terms of $\ln k_t$.)

(5) (d) For what values of α is the consumption path implied by part (c) inefficient?

(10) (e) Assume that the part (c) consumption path is inefficient. Suggest a government policy that would eliminate the inefficiency.

2. Borrowing and lending in general equilibrium

Consider the following infinite-horizon exchange economy with a single non-storable good in each period. There is a measure-1 of agents, divided into two types based on their discount factor. Half of the agents are type P, and the other half are type I. Type P are more patient than type I. The preference of a type-s agent, $s \in S \equiv \{P, I\}$, is given by $\sum_{t=0}^{\infty} (\beta_s)^t \ln(c_t^s)$, where c_t^s is consumption of a type-s agent at date-t, $\beta_s \in (0, 1)$, and $\beta_P > \beta_I$. Both types of consumer have the same constant endowment y in each period. At each period, they can borrow and lend in the form of risk-free, one-period loans on a competitive loan market. Let a_t^s be the net loan position of a type s agent at the start of date-t. The initial net loan position of both types is $a_0^s = 0$. In addition, all agents face the same borrowing constraint: at any t, $a_{t+1}^s \geq \underline{a}$.

(6) (a) Display the Bellman equation for an arbitrary type-s agent, $s \in S$. Be explicit about the state variables and the feasible set.

(8) (b) Define a symmetric recursive competitive equilibrium (where symmetry means that all agents of the same type take the same actions).

(12) (c) Is there a steady-state equilibrium where both types of agents' borrowing constraint are not binding? If so, characterize (i.e. solve) it explicitly. If not, explain why, and characterize the dynamic path of consumption of each type of agent.

Now, instead of permanent types, assume that each agent's discount factor follows a Markov process, with transition probability

$$\text{Prob}(\beta_{t+1} = \beta_l | \beta_t = \beta_s) = \pi_{sl} \in (0, 1), \quad \forall\, s, l \in S = \{P, I\}$$

and $\pi_{sP} + \pi_{sI} = 1$, for any $s \in S$. The discount-factor process is independent across agents. Each agent maximizes expected discounted lifetime utility given initial discount factor β_0, $E_0[\sum_{t=0}^{\infty} (\beta_t)^t \ln(c_t) \,|\, \beta_0]$. The rest of the model is as above.

(7) (d) Assume the economy is stationary. Formulate an agent's problem in recursive form, be explicit about the state variables and feasible set.

(12) (e) Define a stationary recursive competitive equilibrium.

3. Job search in continuous time

Time is continuous. Workers are infinitely lived, risk neutral, and have discount rate ρ. There is no borrowing or lending. Workers can be either unemployed or employed. During unemployment, a worker gets flow utility b; while employed, a worker gets flow utility equal to the wage. Job offers arrive at Poisson rate λ. A job offer consists of a wage w, which is an i.i.d. draw from a distribution with differentiable CDF $F(\cdot)$. The support of $F(\cdot)$ is $[0, \bar{w}]$. Assume that $b < \bar{w}$.

There is no on-the-job search, but when employed, a worker can experience two types of shocks. At Poisson rate δ, the worker exogenously becomes unemployed. At Poisson rate γ, the job switches from paying w to paying w', where w' is an i.i.d. draw from a distribution with differentiable CDF $G(\cdot)$. The support of $G(\cdot)$ is also $[0, \bar{w}]$. When a wage shock arrives, the worker does not have the option to stay employed at the old wage w; instead, the worker must choose whether to keep working for the new wage w' or to quit into unemployment.

(6) (a) Let v_u denote the value associated with being unemployed, and let $v_e(w)$ denote the value associated with being employed at wage w. Display the continuous-time Bellman equations for v_u and for $v_e(w)$. (You don't have to derive them.)

(6) (b) Show that $v_e(\cdot)$ is strictly increasing, and provide an expression for $v_e'(\cdot)$. (It will be clear that $v_e(\cdot)$ is continuous and differentiable.)

(6) (c) Argue that a worker's optimal behavior takes the following form: (I) An unemployed worker follows a reservation-wage strategy when deciding whether to accept a new job; (II) An employed worker who experiences a wage shock also uses a reservation-wage strategy when deciding whether to quit or to keep working; (III) The reservation wage is the same for employed and unemployed workers. For the remainder of the problem, denote this reservation wage by w^*.

(10) (d) Derive a single equation that implicitly characterizes w^* in terms of model primitives (namely, b, ρ, δ, γ, λ, $G(\cdot)$, and $F(\cdot)$). Your expression should not contain v_u, $v_e(\cdot)$, or $v_e'(\cdot)$.

(5) (e) Let $\phi(w^*)$ denote the rate at which unemployed workers become employed as a function of the reservation wage; let $\sigma(w^*)$ denote the rate at which employed workers become unemployed as a function of the reservation wage. Provide expressions for $\phi(w^*)$ and $\sigma(w^*)$.

(5) (f) Suppose that the economy is inhabited by a continuum of workers solving the job-search problem described above. Let u_t be the fraction of workers who are unemployed at date t, so $1 - u_t$ is the fraction of workers who are employed at date t. Provide an expression for $\dot{u}_t$, the time derivative of the unemployment rate, in terms of $\phi(w^*)$ and $\sigma(w^*)$.

(7) (g) Now, assume that $\lambda > \gamma$ and that $G(\cdot) = F(\cdot)$. Let $\bar{u}$ denote the steady-state unemployment rate; i.e., $\dot{u}_t = 0$ when $u_t = \bar{u}$. Show what happens to $\bar{u}$ when b goes up. (Hint: What happens to w^*? How does $\bar{u}$ depend on $\phi(w^*)$ and $\sigma(w^*)$?)

4. A variant of Lucas and Moll (2014, JPE)

Consider the following variant of the continuous-time model of knowledge diffusion in Lucas and Moll (2014, JPE). Individuals in the economy at time t are distributed over the knowledge level $z \in [0, \infty)$ according to the density $f(z,t)$. Before meeting someone, an individual z chooses the time in teaching, h, and the time in learning, ℓ. The time in production is $1 - h - \ell - \frac{1}{2}\left(h^2 + \ell^2\right)$, where the last term is the additional cost of teaching and learning. At the aggregate level, total teaching time is $H = \int_0^\infty h(z) f(z,t)\, dz$, and total learning time is $L = \int_0^\infty \ell(z) f(z,t)\, dz$. Knowledge diffusion occurs only when a teacher meets a student and only when the teacher's knowledge is higher than the student's. A teacher with h randomly meets a student at the Poisson rate $\frac{\alpha h L}{H+L}$, and a student with ℓ randomly meets a teacher at the rate $\frac{\alpha \ell H}{H+L}$, where $\alpha \in (0, \infty)$ is a constant. In a meeting between a teacher with knowledge y and a student with knowledge $z < y$, the student pays the teacher an amount $p(z,y,t) = \frac{1}{2}\left[V(y,t) - V(z,t)\right]$, where $V(y,t)$ and $V(z,t)$ are the two individuals' value functions at t.

(11) (a) For any arbitrary z, derive the Bellman equation for $V(z,t)$ and explain the terms in the equation.

For the remaining parts, assume that $V(y,t)$ is differentiable and strictly increasing in y and that $h + \ell + \frac{h^2+\ell^2}{2} < 1$. Denote the optimal choices of individual z as $h(z)$ and $\ell(z)$.

(18) (b) Derive the conditions for the optimal choices $h(z)$ and $\ell(z)$. Prove that there exists a unique $z_1 \in (0, \infty)$ such that $\ell(z) > 0$ if and only if $z < z_1$. Moreover, show that $\ell(z)$ is strictly decreasing for $z < z_1$. Explain these results intuitively.

(7) (c) Conjecture that there exists $z_2 \in (0, \infty)$ such that $h(z) > 0$ if and only if $z > z_2$. Explain when this conjecture is true and relate the condition to the features of $V(z,t)$. (No proof is needed.)

(9) (d) Discuss social (in)efficiency of the equilibrium relative to that in Lucas and Moll (2014).