Macro Candidacy Exam: August, 2017

Directions. There are four (4) equally weighted questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. Monetary and fiscal policy in an OLG flexible-price economy.

There are overlapping-generations of two-period lived people and there is one good per discrete date $t \geq 1$. Each two-period lived person has an endowment of the good when young, denoted w, and nothing when old. There are N identical people in each generation. Each two-date lived person maximizes the expected value of $u(x, y)$, where x is consumption when young, y is consumption when old, and u is strictly increasing, strictly concave, and satisfies the usual Inada conditions.

There are two assets available. One of them is a real asset with an exogenous real payoff distribution which is risky in the aggregate: per unit invested in the real asset at t, the payoff is r units of date $t+1$ good, where $r \in \{r_1, r_2, ..., r_K\}$ and $r = r_k$ with probability π_k, with realizations that are *i.i.d.* over time. The other is a nominal asset which at date t bears a nominal interest rate i_t and which has a price, p_t, in units of the date-t good. (The nominal asset, interpreted as deposits held at the central bank, is a safe asset in the sense that both i_t and its price at $t+1$ are known when the young person at t makes decisions.) The young person at date t takes prices as given and maximizes $\sum_{k=1}^{K} \pi_k u(x_t, y_{k,t})$ by choice of $s_{r,t}$ and $s_{n,t}$, where

$$x_t = w - s_{r,t} - s_{n,t} \tag{1}$$

and

$$y_{k,t+1} = s_{r,t} r_k + s_{n,t}(1 + i_t)(p_{t+1}/p_t) - p_{t+1}\tau_{t+1}. \tag{2}$$

Here, $s_{r,t}$ is real saving at t in the form of the real asset, $s_{n,t}$ is real saving at t in the form of the nominal asset, and τ_{t+1} is the nominal lump-sum tax levied at date $t+1$ which is known at t. The government budget constraint is $B_{t+1} = (1 + i_t)B_t - N\tau_{t+1}$, where $B_1 > 0$ is a given initial condition. Given policy in the form of sequences for B_{t+1}, i_t, and τ_{t+1} that satisfy the government budget constraint, the equilibrium condition is $Ns_{n,t} = p_t B_t$ for all $t \geq 1$.

Given monetary policy of the form $i_t = i$ for all t, there are two alternative accompanying fiscal policies: (i) $\tau_{t+1} = 0$ for all t; (ii) $B_{t+1} = B_1$ for all t.

(30) (a) Let i be in a small interval around zero. Are there specifications of the model consistent with equilibria in which $s_{r,t} = s_r > 0$ and $s_{n,t} = s_n > 0$ for any such i and under either fiscal policy? Explain.

(15) (b) Assume that i and the model are consistent with both assets being held under either fiscal policy. What are the consequences of different settings for i under the two alternative accompanying fiscal policies?

2. A consumption-saving problem.

Consider the following finite-horizon consumption-saving problem with uncertain labor income. A consumer is born at date 0 with some initial asset $a_0 \geq 0$, and dies at the end of period T. At an arbitrary period t, he receives uncertain labor income y_t, then decides how much to consume c_t and how much to save a_{t+1} in order to maximize his expected discounted utility over his lifetime. So the agent problem is

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{T}} E \sum_{t=0}^{T} \beta^t u(c_t)$$

subject to

$$c_t + a_{t+1} = Ra_t + y_t, \quad t = 0, \ldots, T,$$
$$a_0 \geq 0 \text{ given}$$

where $R > 0$ is the gross interest rate, $\beta \in (0,1)$ is the discount factor, y_t is an i.i.d random variable with a distribution over $[y_{min}, y_{max}]$, and the expectation is taken with respect to $\{y_t\}_{t=0}^{T}$. The agent's period utility function $u(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable, and satisfies $\lim_{c \to 0} u'(c) = \infty$. In addition, the agent faces an exogenous borrowing constraint each period: he can borrow up to $b \geq 0$, i.e. $a_{t+1} \geq -b$ for $t = 0, \ldots T-1$, but cannot be in debt when he dies, $a_{T+1} \geq 0$.

(7) (a) Formulate the dynamic programming problem recursively (the Bellman equation). Be explicit about the state variables, the state space, and the feasibility correspondence.

(18) (b) Argue that the value function in (a) is strictly increasing, strictly concave, and continuously differentiable in asset holding. (You can either prove directly that these properties are satisfied or you can apply existing theorems by verifying that the conditions in those theorems are satisfied.)

(20) (c) Show that the optimal consumption function and optimal saving function are continuous and increasing in asset holding.

3. The behavior of a monopolist who faces a cost of changing its price

Consider the dynamic optimization problem of a monopolist. Each period the monopolist earns a profit, in terms of the numeraire good, that is the difference between its revenue and production costs.

The monopolist faces a constant downward sloping demand curve and satisfies the quantity demanded at its price. Each period, the monopolist chooses to either charge the price from last period or to set a different price. The monopolist faces a stochastic, non-negative, cost, denoted F, of changing its price, where F follows a first-order Markov process.

Labor is the only input into production and the cost per worker, denoted ω, is constant and taken as given. There is a production technology, given by $y = zf(n)$, specifying that output, y, is produced stochastically from labor, n, where the function $f(\cdot)$ is strictly increasing and strictly concave. Productivity, denoted z, is random and follows a first-order Markov process.

At each date, after the realizations of F and z are observed, the monopolist chooses the current price and employs enough labor to satisfy demand at that price.

The monopolist is risk neutral and discounts future expected profits at a rate of $\beta \in (0,1)$. There is no exit in the model.

(10) (a) Express the optimization problem of the monopolist as a dynamic programming problem. What are the state variables? What are the control variables?

(10) (b) What do the policy functions depend on? Explain in words the monopolist's optimal pricing policy, being careful to distinguish between its choice about whether to change the price and its choice of a price conditional on changing the price.

(5) (c) How does the demand for labor depend on z? Is it possible that an increase in productivity leads to a reduction in employment?

(20) (d) Suppose that instead of there being a monopolist, this is a monopolistically competitive economy, with multiple sellers of differentiated products. Let P denote the aggregate price index and suppose that demand for an individual product depends both on the price of the seller, p, as well as on P. Define a recursive equilibrium for this economy. To do so, be sure to describe household and firm behavior.

4. On-the-job training in a search model.

Consider the following continuous-time economy. Workers and firms are risk neutral and have the time discount rate $r > 0$. The measure of workers is fixed at one. When a worker is unemployed, human capital remains at $h_0 = 0$ and home production is $b > 0$. When a worker is employed, the worker supplies one unit of time inelastically, human capital h accumulates through training, and output is equal to h. Let $x(t)$ be the input (of goods) into training for a worker with tenure t. Human capital evolves according to

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = [x(t)]^{\frac{1}{2}} - \rho\, h(t), \tag{3}$$

where $\rho \in (0, \infty)$ is a constant. The firm contributes a fraction ϕ of the training input and the worker the fraction $1 - \phi$. An employed worker does not separate into unemployment. Instead, a worker (employed or unemployed) exits the economy exogenously at a rate $\delta > 0$ and is replaced by a new worker who enters the economy unemployed with h_0.

Firms enter the market competitively to create vacancies. The flow cost of a vacancy is $k > 0$. A recruiting firm offers two numbers, (w, ϕ), where w is the ratio of the wage to output. These are constant during a worker's employment in a firm. Only unemployed workers can search for jobs, and search is directed. In submarket (w, ϕ), the matching rate is $q(w, \phi)$ for a vacancy and $p(w, \phi) = A - q(w, \phi)$ for a worker, where $A \in (0, \infty)$ is a constant. The function $q(.,.)$ is determined by competitive entry of vacancies, but firms and workers take the function as given.

(21) (a) After a match is formed, the firm chooses the training schedule $\{x(t)\}_{t\geq 0}$ to maximize the firm value denoted as $J(h_0, w, \phi)$, where w and ϕ are taken as given. Solve for the firm's optimal choice of $\{x(t)\}_{t\geq 0}$.

(17) (b) Solve for the equilibrium function $q(w, \phi)$. Given this function, formulate a recruiting firm's optimal choice of (w, ϕ).

(7) (c) Consider an alternative environment in which $\{x(t)\}_{t\geq 0}$ is chosen to maximize the sum $J(h_0, w, \phi) + V(h_0, w, \phi)$, where V is the present value of the worker. Prove that the optimal training schedule is the same as that in the equilibrium of the original environment.