THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

August 2019

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section (precise instructions are below)—100 points in each section—for a total of 200 points. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are six (6) pages, including this one.

SECTION I

From this section, please answer any **two** of the three questions.

I.1 (50 points) Of the two goods available to a consumer, one is indivisible and can only be consumed in integer amounts (e.g., movies) so that $x_1 \in \mathbb{Z}_+ = \{0, 1, 2, 3, ...\}$. The other good (e.g., food), can, as usual, be consumed in any non-negative real amount so that $x_2 \in \mathbb{R}_+$. The consumer faces prices p_1 and p_2 for the two goods and has income w. A typical budget set is depicted below. Notice that because the first good is available only in integer amounts, the consumer must choose a consumption bundle on one of the vertical lines that are depicted.

(a) Suppose that the consumer has a demand function $\mathbf{x}(\mathbf{p}, w)$ and that this satisfies the budget equality and is homogeneous of degree 0. Furthermore, the demand function satisfies the weak axiom of revealed preference (WARP). Will the consumer satisfy the "law of compensated demand"?

(b) Now suppose that the consumer is known to have a utility function $u(x_1, x_2)$. In the presence of such indivisibilities, is the resulting expenditure function $e(\mathbf{p}, u)$ a concave function of prices?
[*Note:* Be as precise as possible, carefully stating any assumptions about u that you are making.]

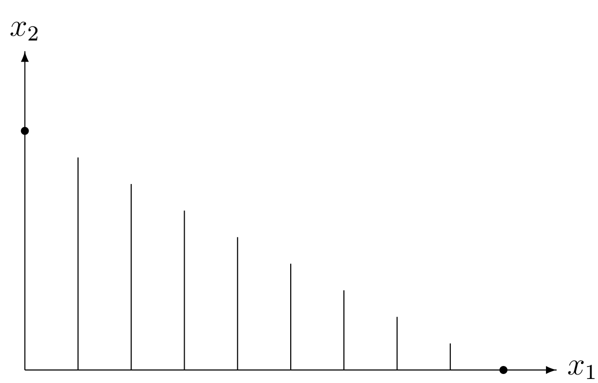

Figure 1: A budget set with indivisibilities

I.2 (50 points) There are two separate parts to this question.

(a) Consider a two-person exchange situation with two goods, denoted by x and y. Suppose that the utility functions of the two consumers are

$$u_1(x, y) = x^\alpha y^{1-\alpha} \text{ and } u_2(x, y) = \min(x, y)$$

where $0 < \alpha < 1$. There is a total of 2 units of good x and a total of 2 units of good y in the economy. Thus, the *total* endowment vector is $(2, 2)$. Consider the allocation

$$\mathbf{x}_1^* = (1, 1) \text{ and } \mathbf{x}_2^* = (1, 1)$$

in which each consumer gets 1 unit of each good.

i. Is this allocation Pareto efficient?

ii. Are there initial endowments $\mathbf{w}_1$ and $\mathbf{w}_2$ for the two consumers satisfying $\mathbf{w}_1 + \mathbf{w}_2 = (2, 2)$ such that the allocation $(\mathbf{x}_1, \mathbf{x}_2)$ is a Walrasian equilibrium allocation for the resulting exchange economy?

iii. Find all such endowments.

(b) In general, suppose that there is a two-person, two-good exchange situation with a total endowment vector $\mathbf{w}$ and both consumers have continuous and convex preferences. Suppose that $\mathbf{x}^*$ is a Walrasian equilibrium allocation for two initial endowments $(\mathbf{w}_1, \mathbf{w}_2)$ and $(\overline{\mathbf{w}}_1, \overline{\mathbf{w}}_2)$. (In both cases, the total endowment is $\mathbf{w}$.)

i. Does this imply that $\mathbf{x}^*$ is also a Walrasian allocation when the endowments are $\frac{1}{2}(\mathbf{w}_1, \mathbf{w}_2) + \frac{1}{2}(\overline{\mathbf{w}}_1, \overline{\mathbf{w}}_2)$? In other words, is the set of endowments that result in a particular allocation $\mathbf{x}^*$ being Walrasian a convex set? Prove this or show a counterexample.

I.3 (50 points) Consider a two-person exchange economy $\mathcal{E}$ with uncertainty: there are two states of nature S, "rain" (r) and "shine" (s). There is only one physical good, denoted by x. The agents have identical utility functions

$$u_i(x_{ir}, x_{is}) = \ln(1 + x_{ir}) + \ln(1 + x_{is})$$

where x_{iS} denotes i's consumption of the good in state S. The initial endowments are

$$(w_{1r}, w_{1s}) = (2, 0) \text{ and } (w_{2r}, w_{2s}) = (0, 2)$$

where w_{iS} denotes i's endowment of the good in state S.

In addition, there are two assets that the agents can buy and sell before the state of nature is known. The structure of asset returns is

$$R = \begin{bmatrix} 1+\varepsilon & 1 \\ 1 & 1+\varepsilon \end{bmatrix}$$

where r_{Sk} denotes the return of asset k in state S.

(a) First, suppose $\varepsilon > 0$. Derive a Radner equilibrium, explicitly stating all equilibrium objects as a function of ε.

(b) Now, suppose $\varepsilon = 0$. Derive a Radner equilibrium.

(c) How do your findings in part (a) compare to those in part (b)?

SECTION II

In this section please answer **either** II.1 or II.2 (not both) and then answer II.3.

II.1 (40 points) There are two players and one object. Player 1's privately-known valuation (willingness to pay) for the object is either 0 or 1/2, with equal probability. Player 2's valuation for the object is known to be 1. After observing his own valuation, player 1 posts a non-negative price for the object. Player 2 observes the price and decides whether or not to buy the object at that price, and then the game ends.

(a) Draw the game in extensive form. Label all parts clearly. What are players' strategies?

(b) What are all the Nash equilibrium outcomes?

(c) What are all the subgame perfect equilibrium outcomes?

(d) What are all the perfect Bayesian equilibrium outcomes?

II.2 (40 points) Two players are about to compete in an all-pay auction. Each player i's valuation v_i for the object is drawn independently from the uniform distribution on $[0, 1]$. Each player knows his own valuation but not that of the other player. If player i bids $m \geq 0$ and gets the object his payoff is $v_i - m$; if he does not get the object his payoff is $-m$.

(a) Solve for the symmetric Bayesian Nash equilibrium (using first-order conditions). What is each player's expected payoff as a function of his type? What is the expected revenue?

Now suppose that the auction designer learns both players' valuations and makes them public before the players bid in the all-pay auction.

(b) Solve for the Nash equilibrium in the subgame that follows the designer's revelation (when each player i's valuation is some $v_i > 0$). (Prove that the pair of strategies you specified form a Nash equilibrium - you do not need to prove uniqueness.)

(c) What is each player's expected payoff in the subgame? What is each player's expected payoff as a function of his type in any subgame-perfect equilibrium?

(d) If the designer wants to maximize the expected revenue, should he reveal players' valuations or not? (*Hint*: no calculations are needed at this point.)

II.3 (60 points) This question studies a labor setting with signaling. There is a worker whose privately-known productivity is either low (0) or high (1), each with probability 1/2. Two risk-neutral firms compete in wages for the worker and hold the same beliefs (which are determined in equilibrium), and therefore the worker's wage is equal to his expected productivity (given all the available information). Instead of acquiring education, the worker chooses whether to take a costly test or not. The fee for taking the test is c. The test is perfectly accurate: if the worker takes the test the firms learn his type. If he does not take the test, the firms do not learn the worker's type and offer wages based on the equilibrium inferences they make.

(a) Argue that in any Bayesian Nash equilibrium, the revenue from the test is at most 1/2 (a precise informal explanation suffices). Show that for $c = 1$ there is a perfect Bayesian equilibrium in which the revenue from the test is 1/2.

(b) Show that for any $c \geq 1/2$, there is a perfect Bayesian equilibrium in which the revenue from the test is 0.

(c) For any $c < 1/2$ solve for the unique perfect Bayesian equilibrium. What is the revenue from the test in this equilibrium?

Now suppose that the test is noisy: if a high productivity worker takes the test, then with a small probability ε the test indicates he is a low productivity worker (and with the remaining probability $1 - \varepsilon$ the test indicates that he is a high productivity worker). If a low productivity worker takes the test, the test always indicates that he is a low productivity worker. Suppose the test fee c is slightly less than $1/2 - \varepsilon$.

(d) Prove that in any perfect Bayesian equilibrium the high productivity worker takes the test with probability 1.

(e) Prove that in any perfect Bayesian equilibrium it cannot be the case that the low productivity worker takes the test with probability 0 or with probability 1.

(f) Give an expression for the probability with which the low type takes the test. Use this expression to derive the revenue from the test as ε approaches 0 and c approaches $1/2 - \varepsilon$.