THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

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Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

I.1 There are two individuals, A and B, with Bernoulli utility functions

$$u_A(w) = 1 - e^{-aw} \text{ and } u_B(w) = 1 - e^{-bw}$$

where $a > b > 0$. Suppose that their initial wealths have been normalized to zero.

(a) Show that there are gambles with positive expected value that would be accepted by one of the two individuals and not by the other. (You may indicate this in a picture, if needed.)

Now suppose that 1 and 2 form a partnership as a venture capital firm and agree to share all profits and losses according to a fixed sharing rule (s_1, s_2), where each $s_i : \mathbb{R} \to \mathbb{R}$ and for all x, $s_1(x) + s_2(x) = x$. This means that in the event the ex post profit (loss) from a gamble is x, then 1 will get (lose) $s_1(x)$ and 2 will get (lose) $s_2(x)$.

(b) Find a sharing rule that maximizes the sum of expected utilities of the two partners. Who has the bigger share of the profits and losses? Why is that?

(c) Show that in the sharing rule you found in part (b), either both partners will want to accept the gamble or both will want to reject it. In other words, the decision of whether or not to accept a particular gamble will be unanimous.

I.2 Consider a two-person, two-good pure exchange economy $\mathcal{E} = (u_i, \mathbf{w}_i)_{i=1}^2$ where both utility functions are "well-behaved" (continuous, strictly quasi-concave and strictly monotone) and the endowments of the two consumers are $\mathbf{w}_1 = (0, 5)$ and $\mathbf{w}_2 = (2, 1)$. Let $\overline{\mathcal{E}} = (u_i, \overline{\mathbf{w}}_i)_{i=1}^2$ be another exchange economy in which the utility functions are the *same* as in $\mathcal{E}$, but the endowments are now $\overline{\mathbf{w}}_1 = (5, 0)$ and $\overline{\mathbf{w}}_2 = (1, 2)$.

Show that if $\mathbf{p} = (5, 6)$ is a Walrasian equilibrium price vector for the economy $\mathcal{E}$, then $\overline{\mathbf{p}} = (6, 5)$ *cannot* be a Walrasian equilibrium price vector for the economy $\overline{\mathcal{E}}$.

(*Hint*: Draw Edgeworth boxes for the two economies with a common origin.)

I.3 Suppose there are two consumers, two states of nature and a single physical good. Consumers' utilities are common across states and given by

$$\begin{aligned} v_1(x_{1s}) &= \sqrt{x_{1s}} \\ v_2(x_{2s}) &= (x_{2s})^2 \end{aligned}$$

where x_{is} denotes i's consumption of the good in state s. Both consumers believe that the two states are equally likely and hence their expected utilities are

$$u_i(\mathbf{x}_i) = \frac{1}{2} \sum_s v_i(x_{is})$$

where $\mathbf{x}_i \in \mathbb{R}^2_+$ is i's consumption bundle. The endowment vectors of the two consumers are $\mathbf{w}_1 = (1, 1)$ and $\mathbf{w}_2 = (1, 1)$, respectively.

(a) First, suppose that there are no contingent claims markets and no asset markets. Does a Radner equilibrium exist?

(b) Next, suppose again that there are no claims markets but now there are two assets that can be traded. The matrix of returns of the two assets is

$$R = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Thus asset #1 pays 2 units of the good in state 1 and 1 unit in state 2. Asset #2 does the opposite. Does a Radner equilibrium exist with these assets?

SECTION II

II.1 Consider a two-player war of attrition between players 1 and 2. First, consider a simultaneous move game in which each player can choose whether to quit (Q) or to fight (F). If player i quits, then player j gets a prize for which her value is v_j. If both players quit, both players get 0. If both players fight, each player gets a payoff of $-c$. Suppose that $v_1 > v_2 > c > 0$.

(a) Write the normal-form of this simultaneous-move game.

(b) Describe the set of all pure and mixed Nash equilibria of the simultaneous-move game.

Now suppose that the war of attrition happens over an infinite horizon. At each $t = 0, 1, \ldots$, the two players simultaneously choose whether to quit or to fight, assuming no one has quit beforehand. If in period t, either player quits, then the other wins the prize and the game ends; if both quit, then the game ends with each obtaining 0. If both players fight in period t, then each player loses c in the current period and the game goes to the next period.

(c) Find a stationary, mixed strategy equilibrium of the game.

(d) Which player is more "aggressive" in pursuit of the prize?

(e) What are the expected payoffs in this mixed strategy equilibrium? Is the equilibrium selecting a Pareto efficient outcome?

(f) Does this game have other equilibria? If so, construct one, and if not, prove otherwise.

II.2 Recently, there has been concern about the lack of consumer privacy and its impact on firms' pricing decisions. This problem explores this issue in the context of a Bertrand duopoly with differentiated products. Bob needs to buy a telephone and is choosing between Apple and Google, each of whom makes him a non-negative price offer. Bob has to accept exactly one of these two offers. Bob's preferences between Apple and Google are represented by a location-parameter, θ, which is drawn uniformly from $[0, 1]$: when Bob is of type θ, his payoff from buying an Apple phone at price p_A is $-\theta - p_A$, and his payoff from buying a Google phone at price p_G is $-(1-\theta) - p_G$. Neither firm incurs any costs of production. In the problems below, restrict attention to pure strategy equilibria, and in these equilibria, you can break Bob's indifference as necessary.

(a) First, suppose that neither Apple nor Google learns θ.

i. Describe the strategy set for each firm and for Bob.

ii. Construct a pure strategy sub-game perfect equilibrium.

(b) Next, suppose that each of Apple and Google learn θ.

   i. What is the strategy set for each firm?

   ii. Describe the set of pure strategy sub-game perfect equilibria of this game.

   iii. For what types is Bob better off here relative to his payoffs in (a)?

(c) Finally, suppose that Google has been tracking Bob and so Google knows the value of θ. But Apple, in its respect for consumer privacy, has not been tracking Bob and so Apple does not know θ.

   i. Write down the strategy set for each firm.

   ii. Find a PBE of this game.

   iii. For what types is Bob better off here relative to his payoffs in (a)? For what types is he better off relative to his payoffs in (b)?

(d) Use the above to reflect on whether consumer privacy is valuable to Bob.

II.3 Two bidders participate in an auction. Their valuations are distributed over $[0, 1]$. The first bidder's value is distributed according to the CDF $F_1(\theta) = \theta^2$. The second bidder's value is distributed according to $F_2(\theta) = 2\theta - \theta^2$. Both players privately learn their own values before the auction. All this is common knowledge. Conditional on winning, their preferences are given by $\theta - p$, where θ is the privately learnt value and p is the payment. Assume throughout that any mechanism that the seller adopts must respect the (interim or Bayesian) individual rationality constraint of the buyer.

(a) Consider a second price auction.

   i. Briefly argue why bidding truthfully is a weakly dominant strategy.

   ii. An auction is efficient if the highest value bidder is always assigned the object. Would a second price auction allocate the object efficiently?

   iii. How much expected revenue would it raise for the seller?

(b) Characterize the Myersonian optimal auction, i.e., the direct mechanism that maximizes the expected revenue of the seller.

   i. What is the optimal allocation rule?

   ii. Graphically, exposit the optimal allocation rule with player 1's value on the x-axis and player 2's on the y-axis.

   iii. What is the expected revenue at the optimum?

   iv. Does this auction implement the efficient outcome? Provide brief intuition.

(c) Can the Myersonian optimal auction here be implemented as a second price auction with reserve prices? Explain.