THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

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Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions.

There are five (5) pages, including this one.

SECTION I

I.1 This question concerns price fluctuations.

(a) A consumer with a utility function $u(x_1, x_2) = \sqrt{x_1 + x_2}$ faces uncertain prices for two goods. The consumer has a fixed income $w > 0$ and makes her consumption choice *after* prices are realized (and so knows the prices at the time she makes her choices). Which of the following situations does the consumer prefer: (i) the prices are uncertain and equal $\mathbf{p} = (3, 2)$ and $\mathbf{p}' = (1, 4)$ each with probability $\frac{1}{2}$; (ii) the prices are $\overline{\mathbf{p}} = (2, 3)$ for sure? Assume that the consumer evaluates each situation according to the expected utility she derives.

(b) Next consider a competitive firm with a well-behaved production set Y. Prices fluctuate but all production decisions are made after prices are realized. Would the firm prefer price fluctuations or a certain price (equal to the expected price with fluctuations)?

(c) Provide explanations for your findings in part (a) and (b).

I.2 Consider a two-good pure exchange economy $\mathcal{E}$ with two consumers A and B. Consumer A's utility function is $u^A(x_1^A, x_2^A) = \ln(x_1^A + 1) + \ln(x_2^A + 1)$. Consumer B's utility function is $u^B(x_1^B, x_2^B) = \ln(x_1^B + 1) + \ln(x_2^B + 1) + \frac{1}{2}\ln(x_1^A + 1)$ and so B cares about A's consumption of good 1. The endowments of the two consumers are $\mathbf{w}^A = (4, 0)$ and $\mathbf{w}^B = (0, 4)$.

a) Find all Pareto efficient allocations for this economy.

b) Find a "Walrasian" equilibrium for this economy—that is, find $(\overline{\mathbf{p}}, \overline{\mathbf{x}}^A, \overline{\mathbf{x}}^B)$ such that each consumer maximizes utility given prices $\overline{\mathbf{p}}$ and the choices of the other consumer and all markets clear.

(c) Is the "Walrasian" equilibrium Pareto efficient? If not, find some transfers and taxes that would result in efficiency.

I.3 Consider an exchange economy with uncertainty: there are two states of the world, $S \in \{1,2\}$, two goods, $l \in \{1,2\}$, and three agents, Alice (A), Bob (B), and Carol (C). As usual, x_{ils} denotes the quantity of good l that agent i consumes in state s. The utility functions of the three agents are

$$\begin{aligned} u_A(\mathbf{x}_A) &= x_{A21} \\ u_B(\mathbf{x}_B) &= \sqrt{x_{B11}} + \sqrt{x_{B12}} \\ u_C(\mathbf{x}_C) &= \sqrt{x_{C21}} + \sqrt{x_{C12}} \end{aligned}$$

and their endowments are

	$S=1$	$S=2$
$\mathbf{w}_A$	$(1,0)$	$(0,0)$
$\mathbf{w}_B$	$(0,2)$	$(0,0)$
$\mathbf{w}_C$	$(0,0)$	$(2,0)$

Thus, Alice has an endowment of $(1,0)$ if $S=1$ and $(0,0)$ if $S=2$, etc.

(a) First, suppose that there are no assets whatsoever. So all that Alice, Bob, and Carol face are spot markets after the state is realized. Derive the equilibrium allocations in each state of the world, prices that support those allocations, and each agent's utility.

(b) Now consider an Arrow-Debreu economy in which players can buy and sell contingent contracts. Find an equilibrium in which Alice is worse off relative to when there are no assets whatsoever.

SECTION II

II.1 Suppose there is a monopolistic seller of a good and a continuum of possible buyers labelled $a \in [0,1]$. Buyer a is willing to pay up to \$$a$ for the good. The monopolist, whose cost per unit is 0, offers the good in each of two periods $t = 0$ and 1 at prices p_0 and p_1, respectively. Each buyer must decide whether to buy or not at $t = 0$. Those who buy at $t = 0$ exit the market immediately. Those who choose not to buy at $t = 0$ can then decide if they want to buy at $t = 1$. All share a common discount factor $\delta < 1$. If buyer a buys at time t for a price p_t, her payoff is $\delta^t(a - p_t)$. The monopolist's payoff if his prices are p_0 and p_1 and a fraction f_0 of buyers buy in the first period and a fraction f_1 in the second is $f_0 p_0 + \delta f_1 p_1$.

(a) Consider the following strategies. **Seller**: charge $p_0 = \frac{2}{4-\delta}$. If all buyers with $v \geq p_0$ buy at $t = 0$, then $p_1 = \frac{p_0}{2} = \frac{1}{4-\delta}$. If some buyer with $v \geq p_0$ fails to buy in period 0, then set $p_1 = 1$. **Buyer** a: buy in period 0 if $p_0 \leq a$. If did not buy in period 0, buy in period 1 if $p_1 \leq a$. Show that these strategies constitute a Nash equilibrium of the game. Why is this Nash equilibrium not subgame perfect?

(b) Construct a subgame perfect equilibrium in this game, limiting yourself to equilibria that satisfy the following assumption: if two buyers a and a' buy in period 0, so must every buyer in between.

I2 Player 1 and Player 2 have agreed to bargain using the following (strange) procedure: Player 1 makes an offer to split the pie of size one—let this offer be denoted by $(x, 1-x)$, where x is Player 1's share. Player 2 accepts or rejects. If Player 2 accepts the offer, the payoffs are x and $1-x$, respectively. If Player 2 rejects the offer, Player 1 makes a *new* offer $(y, 1-y)$. If Player 2 accepts the new offer, the payoffs are y and $1-y$. If Player 2 rejects the new offer, then Player 2 makes an offer $(z, 1-z)$. The game proceeds in this way. Thus, Player 1 makes two offers in a row followed by one offer from Player 2, followed by two offers from Player 1 and so on. The game is infinite horizon—there is no fixed end date. Players use a common discount factor $\delta < 1$.

(a) Show that there is a subgame perfect equilibrium in which Player 2's payoff is $\frac{\delta^2}{1+\delta+\delta^2}$.

(b) Starting from a period in which Player 2 is the proposer, show that the maximum subgame perfect equilibrium payoff for Player 2 is $\frac{1}{1+\delta+\delta^2}$.

II.3 Consider the overlapping generations repeated Prisoners' Dilemma-type game below. Each player t lives for two periods, t and $t+1$: In each period t, there are two living agents, person $t-1$ (who is **old** in period t) and person t (who is **young** in period t): Each player's payoff is the sum of her (two-period) lifetime payoffs. Only the young player in a period takes an action, choosing to co-operate (C) or defect (D). The payoffs to the players in that period are as follows:

	Payoff to **young**	Payoff to **old**
Young chooses C	2	2
Young chooses D	3	0

Consider pure strategies conditioning only on the choices of the last period. That is, the young player in each period t chooses an action based only on the choices made by the young in the preceding period $t-1$:(Each player conditions her choice only on the choices of her parent.) For example: Play D, if $t-1$ played D; play C otherwise.

(a) How many such pure strategies are there?

(b) Suppose every player t plays the same pure strategy of this kind. Is there a subgame perfect equilibrium (in such strategies) in which C is always played on the equilibrium path? Explain why or why not.