THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2011

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer two questions from each section. You will not receive additional credit, and may receive less credit, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

1. Let $\succeq$ be a regular preference relation on $I\!R^L_+$, let $p \in I\!R^L_{++}$, and define the expenditure function $e(p, \cdot) : I\!R^L_+ \to I\!R_+$ by

$$e(p, x) = \inf\{px' : x' \succeq x\}.$$

a) State assumptions on $\succeq$ sufficient to imply that $e(p, \cdot)$ is a utility function for $\succeq$. Be as general as you can

b) Prove the sufficiency of your assumptions.

2. Consider a pure exchange economy with two traders and two commodities. Commodity 1 is available *only in integer amounts* while commodity 2 is divisible as usual. The traders have the endowments

$$w_1 = (4, 0), \quad w_2 = (0, 4).$$

The traders' preferences are represented by the utility function,

$$u_i(x_{i1}, x_{i2}) = 2\sqrt{x}_{i1} + x_{i2}, \quad i = 1, 2.$$

a) Find the set of Pareto efficient allocations.

b) Find the set of competitive equilibrium allocations and prices.

Be sure to illustrate your answers by drawing a clearly labeled Edgeworth Box diagram.

c) In this setting, with one indivisible commodity and one divisible commodity, what general assumptions on the preferences $\succeq_1$ and $\succeq_2$ ensure that competitive equilibrium allocations are Pareto efficient? Without giving a complete proof, explain why your assumptions are sufficient.

3. Let $((\succeq_i, \omega_i)_i, R)$ be an asset exchange environment with three states ($S = 3$) and two assets ($K = 2$). The asset return vectors are $r_1 = (1, 0, 0)$ and $r_2 = (1, 1, 0)$. All consumers' preferences on R^S_+ are regular and strictly increasing. Let $(q^*, (z_i^*)_i)$ be a competitive equilibrium.

a) Give an example in which the allocation $(w_i + z_i^*)_i$ is (unconstrained) Pareto efficient.

b) Give an example in which the allocation $(w_i + z_i^*)_i$ is *not* (unconstrained) Pareto efficient.

Section II

1. Two candidates, A and B, are running in a majority-rule election (with ties to be decided by the toss of a fair coin). There are two states of nature, α and β. All voters agree that candidate A is the best choice in state α and candidate B is the best choice in state β. Specifically, in state α, all voters get a payoff of 1 if A is elected and 0 if B is elected. In state β, the situation is reversed.

   (a) First, suppose that there are only *two (2)* voters who are asymmetrically informed. Voter 1 is informed—he knows whether the state is α or β. Voter 2 is uninformed—he assigns a probability $\pi = 0.9$ that the state is α. Each voter has three choices: vote for A, vote for B or abstain (not vote at all). Voters' choices are made simultaneously.

      i. Write the matrix of payoffs in each of the two states.

      ii. Show that the game has two pure strategy Nash equilibria.

      iii. Show that one of the two equilibria from (i) does not involve the use of a *weakly dominated* strategy.

   (b) Now suppose that there are *three (3)* voters who are symmetrically but imperfectly informed. Each voter assigns a prior probability $\pi = \frac{1}{2}$ that the state is α but receives a private signal correlated with the state. In state α, each voter receives an a signal with probability 1. In state β, each voter receives a b signal with probability s strictly between $\frac{1}{2}$ and 1 and an a signal with probability $1 - s$. Conditional on the state, voters' signals are independent. Each voter has only two choices: vote for A or vote for B—it is not possible to abstain.

      i. Argue that the following "sincere voting" strategies do *not* constitute an equilibrium: each voter with a signal of a votes for A and each voter with a signal of b votes for B.

2. Two (2) bidders are competing in an open ascending (English) auction for a single object—the auctioneer continuously raises the price from zero until one of the bidders decides to drop out at a price p, say. The other bidder then wins the object at the price p. The object is of uncertain value but each bidder i receives a private signal $X_i \in [0, 1]$ and the value of the object to bidder i is:

$$v_i(x_i, x_j) = \gamma x_i + (1 - \gamma) x_j$$

   where x_i is the signal received by bidder i and x_j is the signal received by bidder $j \neq i$ and $\frac{1}{2} < \gamma < 1$. Notice that the value to bidder i depends on both signals. At the time of the auction, bidder i knows only x_i and not the precise value of the object.

(a) Show that the following strategies constitute an equilibrium: bidder i with signal x_i drops out when the price reaches $\beta(x_i) = x_i$.

(b) Argue that the equilibrium of part (a) can be achieved by iterated elimination of *weakly dominated* strategies.

3. A real-estate developer, labelled 0, wants to build a shopping mall on land which consists of two lots. The smaller of the two lots, lot 1, is owned by farmer 1 and the larger lot 2 by by farmer 2. The mall can be built only if both lots are bought. At present, lot 1 results in a flow payoff of v_1 per period to farmer 1 and lot 2 results in a payoff of v_2 per period to farmer 2. Suppose that the developer first bargains with farmer 1 using a standard alternating offer protocol (with 0 moving first) until an agreement is reached. If an agreement is reached with farmer 1 in period t, say, then the negotiated price is paid immediately, and then in period $t+1$, the developer 0 and farmer 2 bargain, again using a standard alternating offer protocol (with 0 moving first). All parties use a common discount factor of δ per period. The present discounted value of the mall to the developer is \$1 and suppose that $v_1 < v_2$.

   (a) Suppose that farmer 1 has agreed to sell his land at some price in period t. Show that in the subgame beginning in period $t+1$ in which 0 and 2 bargain over the price of lot 2, an equilibrium is characterized by the following:

$$\begin{aligned} p_2 &= v_2 + \delta q_2 \\ 1 - q_2 &= \delta(1 - p_2) \end{aligned}$$

   where p_2 is the price offered by the developer to farmer 2 and q_2 is a price demanded by farmer 2.

   (b) Suppose that $(1+\delta)v_1 + \delta v_2 \leq \delta$. Show that in the game beginning in period 1, an equilibrium of the bargaining game is characterized by:

$$\begin{aligned} p_1 &= v_1 + \delta q_1 \\ \delta(1 - p_2) - q_1 &= \delta(\delta(1 - p_2) - p_1). \end{aligned}$$

   where p_1 is the price offered by the developer to farmer 1 and q_1 is the price demanded by farmer 1 (p_2 is as determined in part (a)).

   (c) What are the equilibrium payoffs of the different parties?

   (d) Would the developer be better off bargaining with farmer 2 first and then with farmer 1?