THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2012

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer **two** questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

1. Let $d : I\!R^L_{++} \times I\!R_+ \to I\!R^L_+$ satisfy $pd(p, m) \leq m$ for all $(p, m) \in I\!R^L_{++} \times I\!R_+$.

   a) State the *strong axiom of revealed preference* (SARP).

   b) Prove that SARP is necessary and sufficient for d to be the demand function for a regular (total, reflexive and transitive) preference relation on $I\!R^L_+$.

2.

   a) Define a *competitive quasi-equilibrium* for an economy with production.

   b) State assumptions sufficient to ensure that a competitive quasi-equilibrium allocation is Pareto efficient. Be as general as you can.

   c) Prove that your assumptions are sufficient. Your proof should be self-contained. For example, if you believe that a duality result is relevant, instead of invoking the duality result your proof should contain the relevant elements of the proof of the duality result.

3. Let $((\succeq_i, \omega_i)_i, R)$ be an asset exchange economy with two traders and three states. The traders have identical preferences on $I\!R^3_+$, which are represented by the utility function

$$u_i(x_i) = \frac{1}{2}\sqrt{x_{1i}} + \frac{1}{3}\sqrt{x_{2i}} + \frac{1}{6}\sqrt{x_{3i}}.$$

Their endowments are

$$w_1 = (2, 0, 0), \qquad w_2 = (0, 2, 2).$$

   a) Suppose there are three assets, with the return vectors (columns of R),

$$r_1 = (1, 1, 0), \qquad r_2 = (1, 0, 1), \qquad r_3 = (0, 1, 1).$$

   Find a competitive equilibrium $(q; z_1, z_2)$, where $q \in I\!R^3_+$ is the vector of asset prices and $z_i \in I\!R^3$ is trader i's asset portfolio (short sales are permitted).

   b) Suppose there are only two assets, with the return vectors

$$r_1 = (1, 0, 0), \qquad r_2 = (0, 1, 1).$$

   Find a competitive equilibrium.

   c) Suppose there are only two assets, with the return vectors

$$r_1 = (1, 1, 0), \qquad r_2 = (0, 0, 1).$$

   Does a competitive equilibrium exist? Justify your answer.

Section II

1. A single object is for sale to *two* bidders. Each bidder's value X_i is a random variable that is independently distributed on $[0, 1]$ according to the distribution function F with density f. Bidder i knows the realization x_i of X_i and only that other bidders' values are independently and uniformly distributed. The object is sold via an *all-pay* auction—each bidder i submits a sealed-bid of b_i, the object is awarded to the highest bidder and all bidders pay what they bid.

   (a) Find a symmetric equilibrium of the all-pay auction. What is the expected revenue of the seller?

   (b) (Due to E. Akyol). Let H denote the bid distribution of a player in the symmetric equilibrium derived in part (a). Thus H $b)$ is the probability that a bidder submits a bid not exceeding b. Show that H is a *concave* function.

   (*Note*: You may assume that F is twice-continuously differentiable).

2. Consider a dynamic pricing problem facing a monopoly seller of goods that last at most two periods. There are four potential customers, H_1, H_2 and L_1 and L_2, each of whom wants at most one good. The respective (*per period* utilities (in money units) they get are in the table below:

	Period 1	Period 2
H_1, H_2	1200	500
L_1, L_2	500	200

   So if H_1, say, buys in the first period at a price of p^1, she gets a total utility of $1200 + 500)$ $- p^1$. If she waits until the second period and buys at a price of p^2, she gets $500 - p^2$. There is no discounting by any of the parties and the monopolist's costs are zero.

   (a) First, suppose that the monopolist can irrevocably commit to a pricing policy; that is, in period 1 the monopolist announces both prices p^1 and p^2 which are then fixed. What is the optimal pricing policy under commitment? What are the monopolist's profits?

   (b) Now suppose that the monopolist cannot commit to a pricing policy. The monopolist announces a first-period price p^1; all buyers H_i, L_i simultaneously and independently, decide whether to buy or not. The monopolist observes who has bought and who is left to buy and then chooses a second-period price p^2.What is the subgame perfect Nash equilibrium pricing policy for the monopolist? What are his profits?

(c) What would happen if the monopolist could lease (rent) the product in each period instead of making a once-and-for-all sale? What are his profits from an optimal leasing policy?

3. (Due to Chae and Yang) Let $G_t\ \pi; P_i, P_j)$ denote the two-player alternating offer Rubinstein bargaining game in which players P_i and P_j bargain over a pie of size π with P_i making the first offer in period t. Now let $G_t\ \pi; P_i, P_j, P_k)$ denote a bargaining game in which *three* players P_i, P_j, P_k bargain over a pie of size π beginning in period t as follows: player P_i makes an offer $x_j \in [0, \pi]$ to P_j (the *next* player in the sequence). If P_j says yes , then his share is fixed at x_j and then the game $G_t\ \pi - x_j; P_i, P_k)$ is played in which P_j is no longer active and the two remaining players bargain over the remaining pie of size $1 - x_j$. If P_j says no , then the game $G_{t+1}\ \pi; P_j, P_k, P_i)$ is played, beginning in period $t + 1$. All players discount future payoffs using a discount factor δ and payoffs are obtained only when *all* players have agreed to a division of the pie. So, for instance, if player 2 agrees to x_2 in period 1 but players 1 and 3 agree to a division $x_1, x_3)$ of the remaining pie $1 - x_2$ only in period $t > 1$, then the payoffs of the three players, discounted back to the beginning of period 1, are $\left(\delta^{t-1}x_1, \delta^{t-1}x_2, \delta^{t-1}x_3\right)$.

(a) Draw a schematic tree to depict the game $G_1\ 1;\ P_1, P_2, P_3)$. Find a *stationary* subgame perfect equilibrium of the game. What is the equilibrium division of the pie?

(b) Now consider $G_t^*\ \pi; P_i, P_j, P_k)$, a variant of the game above, defined as follows: in period t, player P_i makes an offer $x_k \in [0, \pi]$ to P_k (the *last* player in the sequence). Again, if P_k says yes , then his share is fixed at x_k and then the game $G_t\ \pi - x_k; P_i, P_j)$ is played in which P_k is no longer active and the two remaining players bargain over the remaining pie of size $1 - x_k$. If P_k says no , then the game $G_{t+1}^*\ \pi; P_j, P_k, P_i)$ is played beginning in period $t + 1$. As before, players discount future payoffs using a discount factor δ and payoffs are obtained only when *all* players have agreed to a division of the pie.

Draw a schematic tree to depict the game $G_1^*\ 1;\ P_1, P_2, P_3)$. Find a *stationary* subgame perfect equilibrium of the game. What is the equilibrium division of the pie?