THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2013

Candidacy Examination for

The Degree of Doctor of Philosophy

MACROECONOMIC THEORY

Please read the instructions carefully. You have 3-1/2 hours to complete the exam.

GOOD LUCK!

Macro Candidacy Exam: January, 2013

Directions. There are five 5) questions worth a total of 180 possible points. The points assigned to each question and each part of each question are given in parentheses. If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. One-sector optimal growth

Time is discrete. A person maximizes expected discounted utility, with period utility u and discount factor $\beta \in (0, 1)$. Here, $u : \mathbb{R}_+ \to \mathbb{R}$, is twice differentiable, $u'' < 0 < u'$, and $u'(0) = \infty$. The person enters a date with capital stock x and chooses consumption, denoted c, and next period's capital stock, denoted y, subject to $c + y \leq f(x)$. Here, $f : \mathbb{R}_+ \to \mathbb{R}_+$, is twice differentiable, $f'' < 0 < f'$, $f(0) = 0$, and $f'(0) = \infty$, and is such that there exists $\bar{x} > 0$ and $\bar{x} = f(\bar{x})$.

15) a) Let $S = [0, \bar{x}]$ and let $C(S)$ be the set of continuous functions on S with the sup norm. Also, let the mapping T be defined as follows: for $h \in C(S)$,

$$Th(x) = \sup_{y \in [0, f(x)]} [u(f(x) - y) + \beta h(y)]. \qquad 1)$$

Show that if h is concave, then Th is strictly concave.

For the remaining two parts, make the following two assumptions. i) The mapping T defined by 1), has a unique fixed point, denoted v, which is increasing and strictly concave. ii) v is attained by a policy function $g(x)$, which is continuous.

15) b) Show that $g(x) \in (0, f(x))$ for all $x \in (0, \bar{x})$.

10) c) Sketch the so-called *tr pping* argument used to show that v is differentiable on $(0, \bar{x})$.

2. Recursive equilibrium in an incomplete-markets model with heterogeneous endowment shocks

There is a nonatomic unit measure of people and one good per discrete date. The per capita endowment of date-t good is constant no aggregate risk). Each person maximizes expected discounted utility with discount factor $\beta \in (0,1)$ and period utility $u(c)$, where $u : \mathbb{R}_+ \to \mathbb{R}$, is twice differentiable, bounded, and satisfies $u'' < 0 < u'$.

Each person's endowment of the date-t good is a drawing z_t from the set $Z = \{a_1, a_2, ..., a_n\}$, where $a_i \in \mathbb{R}_{++}$ and $prob(z = a_i) = \pi_i \in (0,1)$. That is, realizations of z_t are i.i.d. across people and over time. There is a fixed stock of a divisible asset, whose quantity is normalized to be unity per capita. For any $q \geq 0$, q amount of the asset throws off a dividend at each date equal to qd in the form of the date-t good, where $d > 0$.

At date t and after seeing the date$-t$ realization of the endowment z_t, each person chooses $(q_{t+1}, c_t) \in \mathbb{R}^2_+$ subject to

$$c_t + p_t q_{t+1} \leq p_t q_t + q_t d + z_t,$$

while treating p_t, the ex-dividend price of the asset in terms of the date-t good, as unaffected by the person's choice. Here c_t, date-t consumption, is the argument of u, q_t is the quantity of the asset brought into date t, and z_t is the endowment realization.

15) a) Define a recursive equilibrium.

5) b) Define a steady state.

15) c) Consider the special case with $a_i = a$ for all i no shocks). That is, $z_t = a$, a constant, for all dates and all people.) Does a steady state exist? Is there one steady state or are there many? Defend your answer.

25) 3. Money in an OLG model with a random end-of-the-world

Consider the following pure-exchange model of two-date lived overlapping generations in which for $t \geq 1$ there is one consumption good per date and at most one person per generation. Let us call generation t the person who is young at t and old at $t+1$. At each date, there is a realization from the set $\{g, b\}$ g for good, b for bad). Let h^t be the history of realizations from date-1 up through date-t. If $h^t = (g, g, ..., g)$, then generation t, a person who is young at t and old at $t+1$ appears; in any other history there is no generation t the world ends or has already ended). At each date, g occurs with probability $\pi \in (0, 1)$.

For $t \geq 1$, a member of t maximizes expected utility, where the utility function is $u(x, y) : \mathbb{R}^2_+ \to \mathbb{R}$. Here, the first argument is consumption when young and the second is consumption when old. The function u is strictly increasing, strictly quasi-concave, continuously differentiable, and satisfies $\lim_{x \to 0} u_1(x, y) = \infty$ for any $y > 0$. The old person at $t = 1$ maximizes current consumption.

The life-time endowment of a member of generation t for $t \geq 1$ is $(\omega_1, \omega_2) \in \mathbb{R}^2_{++}$, where the first component is endowment when young endowment in the form of date-t good), the second is endowment when old in the form of the date-$t+1$ good. The endowment of the old person at $t = 1$ is $\omega_2 > 0$ in units of date-1 good and 1 unit of money. Notice that the social endowment of the date-t good, denoted ω_t satisfies

$$\omega_t = \begin{cases} \omega_1 + \omega_2 \text{ if } h^t = (g, g, ..., g, g) \\ \omega_2 \text{ if } h^t = (g, g, ..., g, b) \\ 0 \text{ otherwise} \end{cases}.$$

The timing is as follows. Consider date t in history $h^{t-1} = (g, g, ..., g)$. The old person enters date-t. Then there is a realization from the set $\{g, b\}$. Finally, there is price-taking spot trade of the date-t good for money if the realization is g).

Let p_t be the price of money at date t in units of the date$-t$ good in history $h^t = (g, g, ..., g, g)$. Is there a price-taking equilibrium in which $p_t = p > 0$? Defend your answer.

4. Risk-sharing in an endowment economy

Consider a one-good per date, infinite-horizon, endowment economy with I agents. In each period $t \geq 0$, there is a realization of a stochastic event $s_t \in S$, a finite set. The history of events up to time t is denoted by $s^t = (s_0, s_1, \ldots, s_t)$. The aggregate endowment of the date-t good is $y_t(s^t)$.

Agent i orders a stochastic process for consumption according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\left[c_t^i(s^t)\right] \pi_t(s^t),$$

where $\beta \in (0,1)$ is the discount factor, $\pi_t(s^t)$ is the probability of history s^t, and

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma > 0.$$

30) a) Characterize Pareto optimal allocations.

10) b) Would your answer to a) change if there were diverse subjective probability distributions over the histories? In other words, what if $\pi_t(s^t)$ is replaced by a person-specific probability, $\pi_t^i(s^t)$, and if $\pi_t^i(s^t)$, person i's subjective probability of history s^t, could differ from $\pi_t^j(s^t)$, person j's subjective probability of history s^t?

40) 5. Optimal monetary policy with and without commitment

The central bank's objective is to minimize the loss function

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \lambda x_{t+j}^2 + \gamma i_{t+j}^2\right],$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \epsilon_t,$$

and

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{t+1}) + \eta_t,$$

where π_t is inflation, x_t is the output gap, ϵ_t and η_t are *iid* shocks, and $\lambda > 0,\ \gamma \geq 0,\ 0 < \beta < 1,$ and $\kappa > 0$ are model parameters. Finally, i_t is the nominal interest rate, which is the central bank's instrument that is chosen after the shocks ϵ_t and η_t are realized and observed. All variables are expressed in logarithms so that there are no sign restrictions on them.)

Characterize and compare the solutions to two versions of the above problem: in one, the central bank can commit at date 0 to a contingent path for i_t; in the other, it cannot commit and, instead, chooses i_t at each date.