THE PENNSYLVANIA STATE UNIVERSITY

Department of Economics

Spring 2013

Written Portion of the Candidacy Examination for

the Degree of Doctor of Philosophy

MICROECONOMIC THEORY

Instructions: This examination contains two sections, each of which contains three questions. You must answer two questions from each section. You will not receive additional credit, and may receive **less credit**, if you answer more than four questions. You have $3\frac{1}{2}$ hours to complete this exam.

Section I

1. Let Z_+ denote the nonnegative integers, and let X denote the n-fold cartesian product of Z_+. Suppose that a consumer has a preference relation $\succeq$ on X that is regular (total, transitive and reflexive) and strictly increasing ($x \geq x'$ and $x \neq x' \Rightarrow x \succ x'$). For each $(p, m) \in I\!R_+^{n+1}$, let $D(p, m)$ denote the consumer's demand set, defined in the usual way except that the consumption set is X, *not* $I\!R_+^n$. Let $(p^a, m) \in I\!R_+^{n+1}$ and $x^a \in D(p^a, m)$. Suppose that the price of commodity j changes by the amount $\Delta \neq 0$. That is,

$$p^b = (p_1^a, \ldots, p_{j-1}^a, p_j^a + \Delta, p_{j+1}^a, \ldots, p_n^a).$$

a) Define the *Slutsky substitution effect* of the change in p_j on the demand for commodity j.

b) Must the Slutsky substitution effect be either zero or opposite in sign to Δ? If your answer is "yes", give a proof. If your answer is "no", give a counter-example.

2. Consider a pure exchange economy with two traders and two commodities. The traders have the endowments

$$w_1 = (4, 4), \quad w_2 = (4, 5).$$

The traders' preferences are represented by the utility function,

$$u_i(x_{1i}, x_{2i}) = \begin{cases} \min\{x_{1i}, x_{2i}\} & \text{if } \min\{x_{1i}, x_{2i}\} \leq 3; \\ 3 \quad \text{if } 3 \leq \min\{x_{1i}, x_{2i}\} \leq 5; \\ \min\{x_{1i}, x_{2i}\} - 2 & \text{if } \min\{x_{1i}, x_{2i}\} \geq 5. \end{cases}$$

a) Find the set of Pareto efficient allocations.

b) Find the set of competitive equilibrium allocations and prices.

c) Find the set of core allocations.

d) Find the set of competitive quasiequilibrium prices and allocations..

Be sure to illustrate your answers by drawing clearly labeled Edgeworth Box diagrams on the graph paper provided (additional graph paper is available if needed). Your diagrams must be **clearly readable**.

Question 3 is on the next page.

3. Consider an exchange economy $(u_i, w_i)_{i=1}^I$ in which there are S states of nature at date 1 and only one physical good. The consumers' utility functions are of the expected utility form; that is,

$$u_i(x_i) = \sum_{s=1}^{S} \pi_s v_i(x_{is}),$$

where x_{is} denotes $i's$ consumption of the good in state s and π_s is the probability of state s (for all s, $\pi_s > 0$ and $\Sigma_s \pi_s = 1$). Notice that in this specification all consumers agree on the probabilities of the different states. Further, assume that for each i, v_i is increasing, strictly concave, differentiable, and satisfies $v_i(0) = 0$. Suppose that the total endowment of the economy is the same in all states, that is, there is a constant $w > 0$ such that for all s, $\Sigma_{i=1}^I w_{is} = w$.

Let (p^*, x^*) be a Walrasian equilibrium (with complete markets at date 0) such that the price of the good in each state $p_s^* > 0$ and $\Sigma_s p_s^* = 1$. Moreover, each $x_{is}^* > 0$.

a) Show that the equilibrium consumption of each consumer is independent of the state, that is, for any two states s, s', $x_{is}^* = x_{is'}^*$.

b) Show that the price of the good in each state s, $p_s^* = \pi_s$, the probability of state s.

Section II

1. A start-up company A has uncertain profits $\pi = \pi_H$ or π_L, depending on the quality of its product. With probability p, its product is of high quality (H) and with probabilty $1-p$, it is of low quality (L). The quality, H or L, is private information known only to the company. It is considering buying another firm B whose technology will improve the quality its product and increase A's profits by a constant amount R. Since A has no cash-flow at present, however, it will try to buy firm B by offering it an equity share $s \in [0,1]$. If the offer is accepted, the total profits will be $\pi + R$ and B will get a share s of the total profits, whereas company A will get a share $1-s$. If the offer is rejected, the total profits will be π, all of which will be kept by company A and firm B will get profits of $r < R$ from its current business.

   (a) Under which conditions on the parameter values does a pooling perfect Bayesian equilibrium (PBE) exist? Carefully specify the strategies and beliefs.

   (b) Characterize all separating PBEs.

2. There are three (3) sellers (farmers, say) who own one unit of land each. The value V_i that seller i currently derives from his land is private information that is drawn independently from the same distribution F over $[0,1]$, which has a density $f = F'$ that is positive on $(0,1)$. A buyer (a real estate developer, say) wishes to acquire two (2) units of land for a project (to build shopping mall, say). From the buyer's perspective all units are identical—she does not care which two units are acquired—but needs a minimum of two units of land and no more. Thus, one unit of land is of no value to the buyer and the third unit is of no additional value over two units.

   (a) Suppose that the buyer has a commonly known value $v_0 = 2$ for two units of land. Consider the following mechanism. Each seller i submits a "bid" b_i. The two lowest bids are accepted and these two sellers are each paid an amount equal to the *highest* bid (ties are resolved by randomizing equally). Thus, for instance, if the three bids are $(b_1, b_2, b_3) = (\$0.4, \$0.7, \$0.6)$, then the buyer acquires land from sellers 1 and 3 at a price of \$0.7 each. What

are equilibrium bidding strategies in the resulting game? Is this mechanism efficient?

(b) Now suppose that the value V_0 derived by the seller from ownership of two units of land is also private information. Specifically, it is drawn from a distribution G over $[0, 2]$ with density $g = G'$ that is positive over $(0, 2)$. Exhibit a mechanism such that is efficient, incentive compatible and individually rational. Is the mechanism budget-balanced—that is, is the amount paid by the buyer equal to the total amount received by the sellers?

3. Per-period demand for a homogeneous product is given by the function $q = \max(a - p, 0)$, where $a > 0$. There are N identical firms with a constant per-unit cost of c and for simplicity, suppose that $c = 0$. The firms engage in repeated (Bertrand) price competition over an infinite horizon $t = 1, 2, ...$ and use a common discount factor of $\delta = \frac{2}{3}$ to evaluate future profits. In any given period, firms set prices simultaneously and customers buy from the firm with the lowest price. If two or more firms charge the same lowest price, customers are indifferent and the market is shared, and how this is done is a decision for the firms. All past prices are perfectly observable by all firms.

(a) Suppose that there are two completely separate markets A and B, each with the same demand and cost functions as specified above. The markets differ, however, in the number of operating firms. There are only $N_A = 2$ firms in market A but $N_B = 4$ firms in market B. Show that in market A there is a subgame perfect equilibrium of the resulting infinitely repeated game in which the two firms "collude" and set the monopoly price in every period. Specify the strategies carefully. Argue that such an equilibrium does not exist in the game in market B.

(b) Next, suppose that firm 1 in market A merges with firm 1 in market B and firm 2 in market A merges with firm 2 in market B. As a result, there are a total of $N = 4$ firms. The merged firms 1 and 2 operate in both markets A and B, whereas firms 3 and 4 operate only in market B. Show that with this structure there is a subgame perfect equilibrium of the infinitely repeated game in which all firms set the monopoly price in both markets in every

period. (Note that in specifying the strategies you should also specify how the market is to be shared. Recall that market shares of firms setting the same lowest price do not have to be equal.)