Patrik Guggenberger, Department of Economics, Penn State, Econ 501, 8/18/2017

Comprehensive Exam

This is a closed-book exam. The total number of points is 100. The number of points per question is indicated in round brackets. Please provide complete explanations for your answers. Write clearly.

1. **(15)** You are in Las Vegas with \$2 and you need \$4 to pay off a debt. You consider two strategies: (a) bet all you have on Black; (b) bet \$1 at a time on Black until you either go bankrupt or end up with \$4. You can assume that the casino pays out twice the wager for a winning roll. Assume the probability of Black is p, where $0 < p < 1$. Compare the two strategies and determine which has the larger probability of succeding (as a function of p).

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2. **(25)** Suppose $X_1, ..., X_n$ are i.i.d. $Bern(p)$ random variables.

(a) State the lemma by Neyman and Pearson.

(b) What is the likelihood function in this case?

(c) Find a UMP test for $H_0 : p = p_0$ versus $H_1 : p = p_1$, where $p_0 \neq p_1$.

(d) Show that the test from part (c) is UMP for $H_0 : p = p_0$ versus $H_1 : p > p_0$.

(e) Is the test from part (c) UMP for $H_0 : p \leq p_0$ versus $H_1 : p > p_0$. Why, why not?

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3. **(20)** Suppose (Y_i, X_i) are i.i.d. with $Y_i = X_i'\beta + U_i$ for $X_i, \beta \in R^k$, $EX_iU_i = 0$, EX_iX_i' is positive definite, and $E||U_iX_i||^2 < \infty$. Show that the least squares estimator $\widehat{\beta}_n$ of β satisfies $n^{1/2}(\widehat{\beta}_n - \beta) \to_d Z \sim N(0, V)$, where

$$V = (EX_iX_i')^{-1}EU_i^2X_iX_i'(EX_iX_i')^{-1}.$$

Find a consistent estimator $\widehat{V}_n$ of V and show that it is consistent. State any additional assumptions needed to prove asymptotic normality of $\widehat{\beta}_n$ and consistency of $\widehat{V}_n$.

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4. **(15)** Let X be a random variable on the probability space $(\Omega, \mathcal{F}, P)$ and $g(X)$ a scalar function of X.

(a) State the definition of $Eg(X)$. [We assume $E|g(X)| < \infty$.] (b) Show that $Eg(X) = \int g dP_X$.

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5. **(15)** Assume $X = (X_1', X_2')' \sim N(\mu, \Sigma)$, where $\mu = (\mu_1', \mu_2')'$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$, and $X_i \in R^{p_i}$ for $i = 1, 2$ for some integers $p_i > 0$. Derive the conditional distribution of X_1 given $X_2 = x_2$. (In your derivation, you can use the facts that linear transformations of X are again normally distributed and that two jointly normal random vectors are independent iff they have zero covariance.)

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6. (10) State and prove the invariance property of the maximum likelihood estimator.

Andres Aradillas-Lopez, Department of Economics, Penn State, Econ 510, 8/18/2017

Comprehensive Exam

This is a closed-book exam. The total number of points is 100. The number of points per question is indicated in round brackets. Please provide complete explanations for your answers. Write clearly.

Let Y be a real-valued random variable and let X be a vector of variables. Let $Z \equiv (Y, X)$ and define

$$\rho(Z, \theta) = Y - f(X, \theta). \tag{1}$$

θ is a finite-dimensional parameter vector, with k elements. Suppose that f has known functional form, and assume throughout that f is as smooth (i.e, differentiable) in θ as you need it to be. The parameter space for θ is denoted by Θ, a compact subset of $\mathbb{R}^k$. Assume throughout that we observe a random sample $(Z_i)_{i=1}^n$. Suppose that the following conditional moment restriction is satisfied at the true parameter value $\theta_0 \in \Theta$:

$$E\left[\rho(Z, \theta_0)|X\right] = 0 \quad \text{w.p.1.} \tag{2}$$

Suppose we are interested in the class of GMM estimators based on the following unconditional moment restriction implied by (2):

$$E\left[A(X)\rho(Z, \theta_0)\right] = 0, \tag{3}$$

where $A(X)$ is a $k \times 1$ vector of functions of X (recall that $k = dim(\theta)$).

1. (20) Describe the generic form for the asymptotic variance of the class of GMM estimators described by Equation (3).

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2. (40) Using your answer to part (1), characterize the efficient (optimal) choice of instruments $\overline{A}(X)$. That is, the instruments $\overline{A}(X)$ that achieve the smallest possible variance within the class of GMM estimators described by Equation (3).

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3. (20) Consider a special case of the model described in Equations (1)-(3), where $f(X,\theta) = X'\theta$ and therefore $\rho(Z,\theta) = Y - X'\theta$. Suppose we assume that $E\left[\rho(Z,\theta_0)^2|X\right] = W(X)'\alpha_0$ for some $W(X)$ that is a (known) function of X, and α_0 is a vector of parameters. How do the optimal instruments $\overline{A}(X)$ look like in this case?

4. (10) For the model described in part (3), describe a *feasible* efficient GMM estimator (i.e, a GMM estimator that uses a first-step estimator for the optimal instruments $\overline{A}(X)$).

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5. (10) For the feasible efficient GMM estimator you proposed in part (4), suppose the assumption that $E\left[\rho(Z,\theta_0)^2|X\right] = W(X)'\alpha_0$ is incorrect, but the assumption $E\left[\rho(Z,\theta_0)|X\right] = 0$ is still correct. Will your feasible estimator still be consistent? Prove your claim.