Macro Candidacy Exam: January, 2015

Directions. There are four (4) questions worth a total of 180 possible points. The points assigned to each part of each question are given in parentheses. (If you can proceed at the rate of a point per minute, then you can complete the entire exam with time to spare.)

1. The Huggett (1993) model with trade in a Lucas tree and no borrowing

There is a nonatomic unit measure of people and one good per discrete date. Each person maximizes expected discounted utility with discount factor $\beta \in (0,1)$ and date-t period utility $u(c_t)$. Here, $u : \mathbb{R}_+ \to \mathbb{R}$ and is bounded above, strictly increasing, strictly concave, and satisfies $u'(0) = \infty$. Let $Y = \{y_1, y_2\}$ where $0 < y_1 < y_2$. Each person's endowment of the good follows a Markov process over the set Y, with realizations that are independent across people.

There exist uniform and divisible lucas trees, the per capita amount of which is normalized to be unity. A unit of the tree throws off a dividend d at each date in units of the consumption good. The consumption good is perishable. The only trade is spot trade of holdings of the tree and there is no borrowing.

(5) (a) At the start of a date, whether in steady state or not, specify the *state* of this economy.

(20) (b) Define a recursive equilibrium for the above economy. (If we knew such an equilibrium and if it were unique, then we would know how the economy evolves from an arbitrary state as specified in part (a).)

(5) (c) Define a steady state (sometimes called a steady-state equilibrium) for this model.

(15) (d) Let p denote the steady state price of the tree. Assume that a unique steady state exists for each $d > 0$ and that p is a continuous function of d. Do you think $\lim_{d \to 0} p = 0$? Explain.

2. An OLG model with shocks to productivity and the money supply

Consider an infinite horizon overlapping generations model with two-period lived agents. Each member of generation t is endowed with a unit of time in period t, works $n_t \in [0,1]$ units of time in period t and consumes $c_{t+1} \geq 0$ in period $t+1$. The initial old are each endowed with $M_0 > 0$ units of fiat money. Population is constant. Normalize the size of each generation to 1.

When young, agents have access to a technology to convert labor time into the single good: $y_t = A_t f(n_t)$. There are no firms and production is undertaken by each agent individually. A_t is the period t realization of an aggregate technology shock. Assume that A_t follows a finite-state Markov process with a positive support. The production function, $f(\cdot)$, is strictly increasing and strictly concave.

The representative generation-t agent maximizes the expected value of $u(c_{t+1}) - g(n_t)$, where $u(\cdot)$ is strictly increasing and strictly concave and $g(\cdot)$ is strictly increasing and strictly convex.

The aggregate money supply is stochastic. The period t money supply is given by $M_t = M_{t-1} x_t$, where x_t is *i.i.d.* with a finite and positive support. Changes in the money supply occur by way of proportional transfers. Generation-t agents take actions knowing the current money supply, M_t and the current productivity, but not future realizations of the shocks.

(15) (a) Assume that goods prices are predetermined one period in advance, so that the money price of goods in period t does not depend on the realizations of (A_t, x_t). In particular, assume that $p_t = Z M_{t-1}$ where Z is some positive constant. Also, assume that young agents produce and meet all demand at the predetermined price. Show that economic activity is increasing in x_t. Also, show that employment is decreasing in A_t.

(10) (b) Evidence suggests that monetary shocks in period t can influence real economic activity in future periods. Is that a feature of the part (a) model? How do models with staggered price setting create dynamic effects of monetary shocks?

(20) Assume here that prices are flexible. Define a stationary rational expectations equilibrium (SREE). Is there a SREE in which the price in period t does not depend on (A_t, x_t)?

3. Investment in training.

Consider the choice problem of a household that lives for T periods. For periods $(1, 2...T^r)$, the household is endowed with a unit of time that can be split between work and training. Here $T^r < T$ is an exogenous retirement age. The amount of output produced by the worker is given by $\omega h n$ where ω is the agent's productivity, h is the stock of human capital and n is the time spent working. Assume that ω is an iid random variable.

The time spent training, denoted e, leads to human capital accumulation according to the transition equation: $h' = \phi(h, e)$. Assume that $\phi(\cdot)$ is increasing and strictly concave in each of its arguments.

For periods $(T^r + 1, T^r + 2, ...T)$, the household is retired and, thus, does not work and does not train. The household consumes the single good each period. The household can lend at a real gross interest rate of R, but cannot borrow.

(20) (a) Based upon this verbal description, formulate the household's problem as a dynamic programming problem. To do so, you may find it necessary to put additional structure on the problem. Be specific and make your assumptions clear. What are the state variables? What are the controls?

(15) (b) Find the Euler equations characterizing the choices of the household. Explain them in words.

(10) (c) Evaluate the following conjecture: all else the same, younger workers train more than older workers.

4. Risk-sharing without commitment

Consider an infinite-horizon endowment economy with two agents. Both agents have preference $E \sum_{t=0}^{\infty} \beta^t u(c_t)$, where E is the expectation operator, $\beta \in (0, 1)$ is the discount factor, and $u(\cdot)$ is strictly increasing, strictly concave, and satisfies the usual Inada conditions. The endowments of the two agents are random and perfectly negatively correlated. At date t, if agent 1 receives $y_t \in (0, 1)$ unit of endowment, then agent 2 receives $1 - y_t$. The support of y_t is $\{y^l, y^m, y^h\}$, where

$$\{y^l, y^m, y^h\} = \{1 - y, 0.5, y\} \text{ with } y \in (0.5, 1).$$

Also, $\text{prob}\{y_t = y^s\} = \pi^s$, where $\{\pi^l, \pi^m, \pi^h\} = \{\pi/2, 1 - \pi, \pi/2\}$.

The social planner aims to provide insurance by reallocating goods between the two agents. The design of insurance is constrained by a lack of commitment; each agent is free to walk away from any social arrangement at any time, but must then live in autarky afterward.

(10) (a) Formulate the recursive problem of optimal sustainable risk-sharing such that no agent has an incentive to renege.

(15) (b) State a necessary and sufficient condition on the parameters of the model β, y, π and $u(\cdot)$ under which a first-best allocation is sustainable. Prove your claim.

(20) (c) Suppose that a first-best allocation is not sustainable. What can you say about the properties of possible limiting consumption sequences of the constrained efficient allocation? Your discussion should include descriptions of the qualitative features of the constrained efficient allocation such as the support of the consumption set, whether consumption is history dependent, and whether the distribution of consumption is stationary.